Section 3.2.2 – Induced EMF (ε)

Induced EMF

An electromotive force (**EMF or ε**) is produced in a conducting loop in which there is a **changing magnetic flux**. The magnitude of the produced EMF is as follows:

$$\varepsilon_{avg} = -\frac{\Delta\varphi_B}{\Delta t}$$, or in a coil with several turns

$$\boxed{\varepsilon_{avg} = -\frac{N\Delta\varphi_B}{\Delta t}}$$

Where ε_{avg} = Average electromotive force (V)
N = No. of turns in coil
$\Delta\phi_B$ = Change in magnetic flux (Wb)
Δt = Change in time (sec)

So in order to increase the EMF (ε) generated you must either:

1. Increase the number of turns of wire in the coil (ie. ↑N)
2. Increase the change in flux (ie. $\uparrow\Delta\Phi_B$)
   - $\uparrow\Delta\Phi_B = \uparrow\Delta A \times B$ or $\uparrow\Delta\Phi_B = A \times \uparrow\Delta B$
     (you can either increase the change of cross sectional area of the coil or increase the change of of magnetic field strength)
3. Increase the rate at which the flux changes = do it quicker (ie. ↓Δt)

NB: $\varepsilon_{avg} \; \alpha - \frac{\Delta\varphi_B}{\Delta t}$

The average emf generated is **proportional** to the **negative rate of the change of flux.**

NB: The negative (-ve) symbol relates to Lenz's Law (which will be studied later in the semester) Often you are only asked to calculate the magnitude of the EMF (ε), in which case the "-" sign can be ignored.

Example 1

A coil containing 1000 turns of copper wire is threaded by a magnetic flux of 9.6×10^{-4} Wb. What EMF is generated if this flux is reduced to 0 in 0.015 seconds?

N = 1000 turns
$\phi_B(1) = 9.6 \times 10^{-4}$ Wb
$\phi_B(2) = 0$ Wb
$\Delta t = 0.015$ sec
$EMF_{avg} = ?$

$\Delta\phi_B = \phi_B(2) - \phi_B(1)$
$= 0 - 9.6 \times 10^{-4}$
$= -9.6 \times 10^{-4}$ Wb

$$EMF_{avg} = -\frac{N\Delta\varphi_B}{\Delta t}$$

$$= \frac{(-1000 \times -9.6 \times 10^{-4})}{0.015}$$

$= \underline{64 \text{ Volts}}$

Example 2

Figure 1 shows an experiment where the voltage induced in a coil by a time dependant magnetic field is measured. The voltmeter measures the voltage induced in the coil as a function of time.

The coil has 200 turns

The magnetic flux through the 200 turn coil is a constant 5 mWb.

The magnetic field is now reduced to zero over a period of 20 ms.

What is the average EMF induced in the coil during the 20 ms interval?

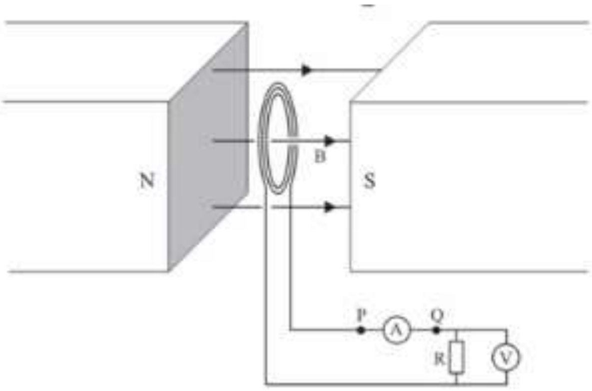


Figure 1

$\varepsilon = ?$
$N = 200$
$\Delta\Phi_B = 5$ mWb
$= 5 \times 10^{-3}$ Wb
$\Delta t = 20$ ms
$= 20 \times 10^{-3}$ sec

$$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$$
$$= \frac{(200 \times 5.0 \times 10^{-3})}{20 \times 10^{-3}}$$
$$= \underline{50 \text{ Volts}}$$

Example 3

Figure 2 shows an experiment where the voltage induced in a coil by a time dependant magnetic field is measured. The voltmeter measures the voltage induced in the coil as a function of time.

The coil has 200 turns

The magnetic flux through the 200 turn coil is a constant 5 mWb.

The magnetic field is now increased to 15 mWb over a period of 10 ms.

What is the average EMF induced in the coil during this 10 ms interval?

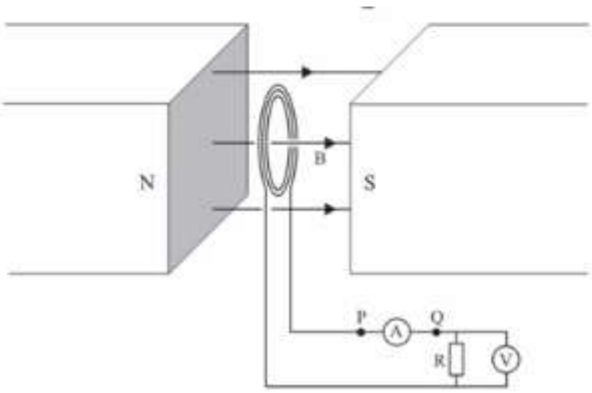


Figure 2

$\varepsilon = ?$
$N = 200$
$\Delta\Phi_B = 15$ mWb – 5 mWb
$= 10$ mWb
$= 10 \times 10^{-3}$ Wb
$\Delta t = 10$ ms
$= 10 \times 10^{-3}$ sec

$$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$$
$$= \frac{(200 \times 10.0 \times 10^{-3})}{10 \times 10^{-3}}$$
$$= \underline{200 \text{ Volts}}$$

Example 4

Figure 3 shows a generator consisting of a coil of area 0.05 m^2 and 125 turns. The coil is made to rotate at a frequency of 50 Hz in a magnetic field of strength 0.1 T.

What is the average EMF induced as the coil turns from parallel to the field to perpendicular to the field?

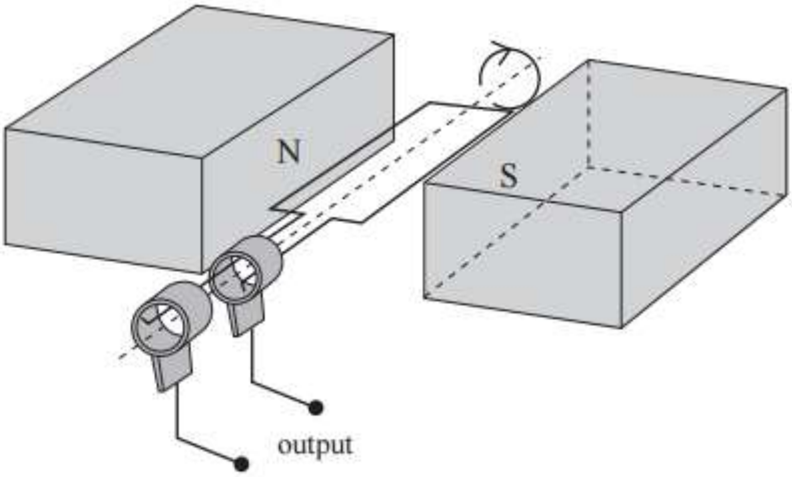


Figure 3

$\varepsilon = ?$
$N = 125$
$\Delta\Phi_B = 0\text{Wb} - (0.05 \times 0.1)$
$\quad = 0.005$ W
$F = 50$ Hz

Step.1

$T = \frac{1}{f}$
$\quad = \frac{1}{50}$
$\quad = 0.02$ sec per revolution

$\therefore$ for $\frac{1}{4}$ rotation
$\Delta t = 0.02/4$
$\quad = 0.005$ sec

Step.2

$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$
$\quad = -\frac{125 \times 0.005}{0.005}$
$\quad = \underline{125 \text{ Volts}}$

Sketching EMF Generation

$$EMF_{avg} = -N\frac{\Delta\varphi}{\Delta t}$$

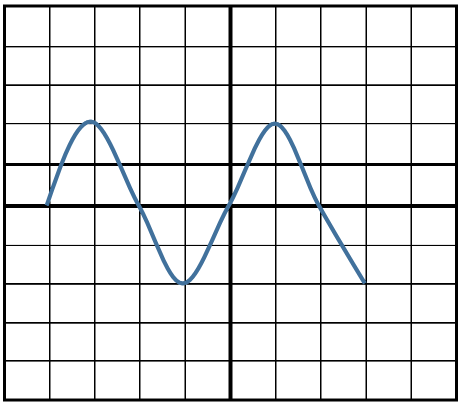
A generator consists of a magnetic field (B), a loop of area (A), number of turns (N) & change of time (Δt)

Axes Scale

Time base
= 0.05 sec/cm
(50 ms/cm)

Voltage gain
= 1V/cm

EMF (V)



V_p = 2 div × 1 V/div
= 2 Volt

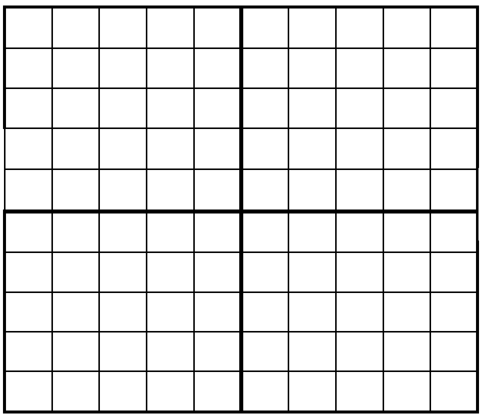
T = 4 div × .05 s/div
= 0.2 sec

Time(s)

Original trace

Sketch the EMF signal or the following generator arrangements:

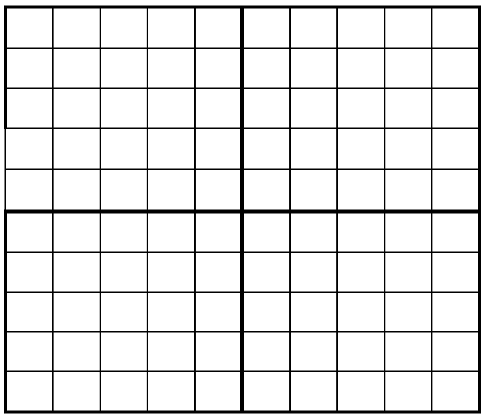
EMF (V)



Time(s)

Identical to original trace, but <u>B is doubled</u>

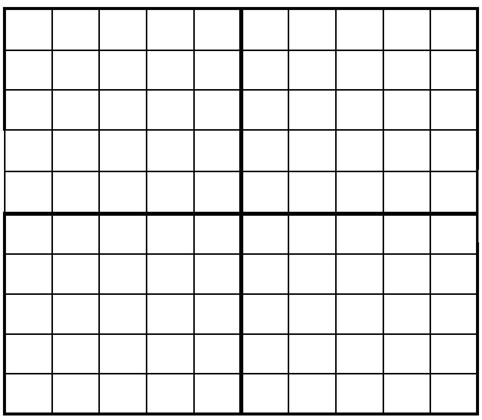
EMF (V)



Time(s)

Identical to original trace, but coil <u>area is halved</u>

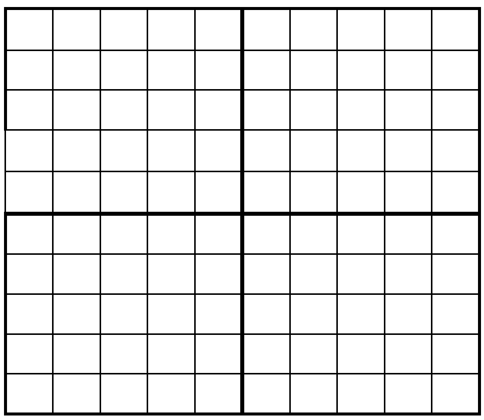
EMF (V)



Time(s)

Identical to original trace, but <u>B is doubled and number of turns is halved</u>

EMF (V)



Time(s)

Identical to original trace, but <u>the frequency of coil rotation is doubled</u>

Exam Styled Questions

Emily and Gerry are studying generators and alternators. They have constructed the device shown in Figure 1.

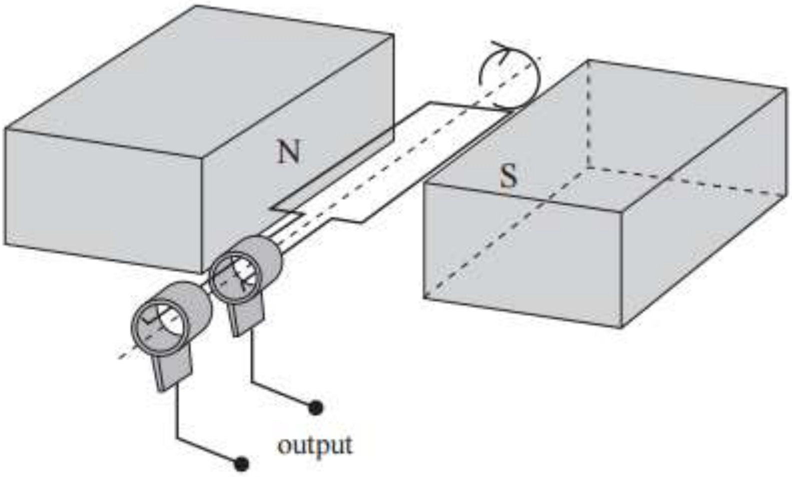


Figure 1

The coil can be rotated about an axis as shown, in a uniform magnetic field, B, larger in area than the coil.

The maximum magnetic flux that passes through the coil is 7.2×10^{-6} Wb.
In an interval of 0.020 s the coil is rotated by one quarter turn (90°) from the orientation shown in Figure 1.

Question 1
What is the magnitude of the average voltage generated?

$\varepsilon = ?$
$N = 1$
$\Delta\Phi_B = 0 - 7.2 \times 10^{-6}$
$\quad = 7.2 \times 10^{-6}$ Wb
$\Delta t = 0.020$ s

$$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$$
$$= \frac{(1 \times 7.2 \times 10^{-6})}{0.02}$$
$$= \underline{3.6 \times 10^{-4} \text{ Volts}}$$

3.6×10^{-4} Volts

NB: A quarter ($\frac{1}{4}$) rotation question is ideal as the $\Delta\Phi = \Phi_{max} - 0$.

Figure 2 shows an experiment where the voltage induced in a coil by a time-dependent magnetic field is measured. The voltmeter measures the voltage induced in the coil as a function of time. The coil has 120 turns.

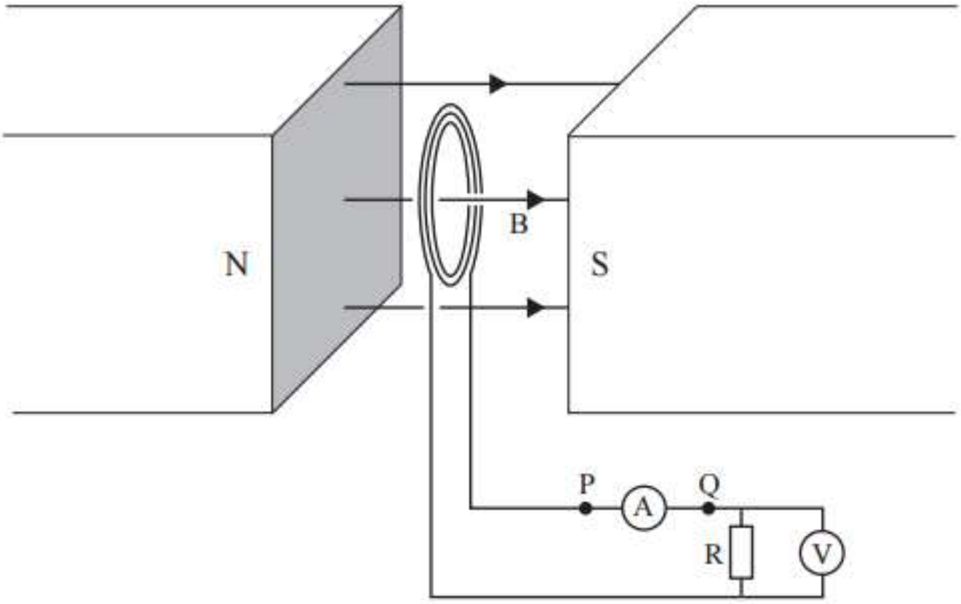


Figure 2

The magnetic flux through the 120 turns coil is a constant 3.0×10^{-4} Wb.

Question 2

The magnetic field is now reduced to zero over a period of 0.012 s. What is the average EMF induced in the coil during that 0.012 s interval?

$\varepsilon = ?$
$N = 120$
$\Delta\Phi_B = 3.0 \times 10^{-4} - 0$
$= 3.0 \times 10^{-4}$ Wb
$\Delta t = 0.012$ s

$$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$$
$$= \frac{(120 \times 3.0 \times 10^{-4})}{0.012}$$
$$= \underline{3.0\text{Volts}}$$

3.0 Volts

Figure 3 shows a 50 turn coil of area 0.020 m^2 that can rotate in a uniform magnetic field of 2.0 T. The coil is shown in three different orientations A, B and C.

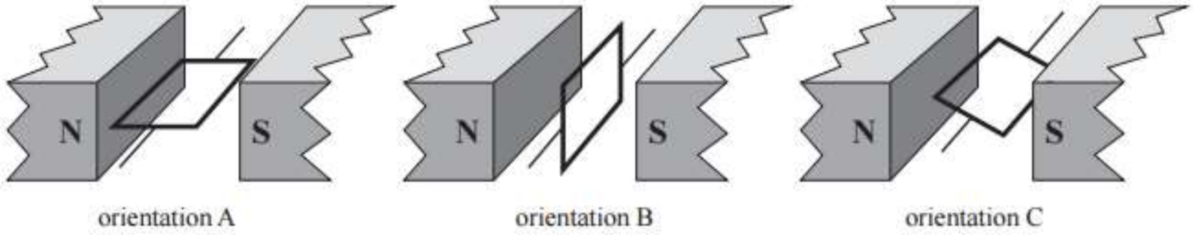


Figure 3

The 50 turn coil is rotated from orientation A to orientation B in a time of 0.15 seconds.

Question 3
Calculate the average EMF generated in the coil over this time period.

$\varepsilon = ?$
$N = 50$
$\Delta\Phi_B = 0 - (0.020 \times 2)$
$= 0.04$ Wb
$\Delta t = 0.15$ s

$$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$$
$$= \frac{(50 \times 0.04)}{0.15}$$
$$= \underline{13.3\text{Volts}}$$

13.3 Volts

The following information relates to Questions 4 & 5

Students are experimenting with a simple AC generator, as shown in Figure 4. It consists of a rectangular coil of n turns that rotates at a constant speed in a uniform magnetic field.

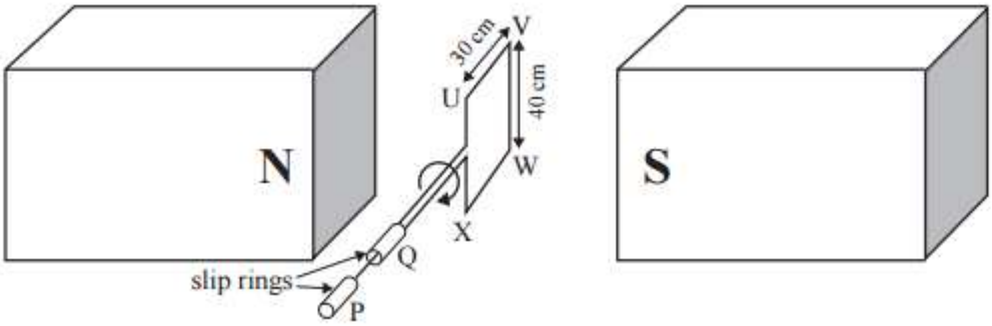


Figure 4

The uniform magnetic field in the region between the magnets is 0.030 T. The dimensions of the rectangular coil are 30 cm × 40 cm.
The coil is located completely within the uniform magnetic field of the two magnets.
The coil is rotating at a steady rate of 2 rotations per second.

Question 4
The generator produces a sinusoidal AC voltage. For a quarter rotation of the coil from the point shown in Figure 4, it generates an average emf of magnitude 3.6 V. Calculate the number of turns in the rotating coil.

N = ?
$\varepsilon = 3.6$
F = 2 Hz
$\Delta\Phi_B = (0.030 \times 0.30 \times 0.40) - 0$
$= 0.0036$ Wb

125 turns

Step.1

$T = \frac{1}{f}$
$= \frac{1}{2}$
= 0.5 sec per revolution

$\therefore$ for $\frac{1}{4}$ rotation
$\Delta t = 0.5/4$
= 0.125 sec

Step.2

$\varepsilon = -\frac{N\Delta\varphi_B}{\Delta t}$

$\therefore N = \frac{\varepsilon\Delta t}{\Delta\varphi_B}$

$= \frac{3.6 \times 0.125}{0.0036}$

= <u>125 turns</u>