Section 4.2.1 – Introduction to Network Basics

VCAA "Dot Points"

Graphs and networks, including:

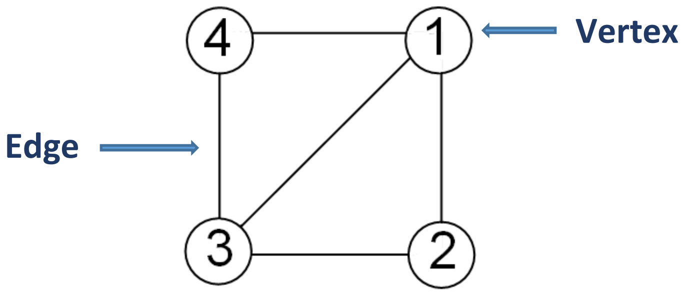
- a review of the concepts, conventions and terminology of graphs including planar graphs and Euler's rule, and directed (digraphs) and networks
- use of matrices to represent graphs, digraphs and networks and their application.

Introduction to Networks

A **network** is a collection of objects **connected to each other** in some specific way.
Some examples of networks include:

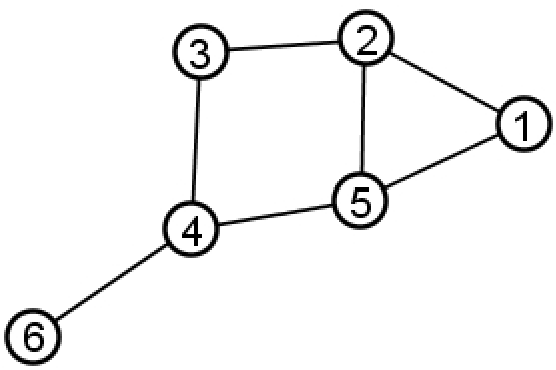
1. The world wide web (www)
2. Powerlines
3. Airline networks
4. Food webs

Networks are graphically presented via features/symbols called vertices and edges.
Objects within a network are called **vertices (V)** and the connection between objects are called **edges (E).**



The degree of a vertex

The **degree of a vertex** is the number of edges connected to it.



degree (1) = 2
degree (2) = 3
degree (3) = 2
degree (4) = 3
degree (5) = 3
degree (6) = 1

NB: A loop contributes 2 towards the degree of a vertex.

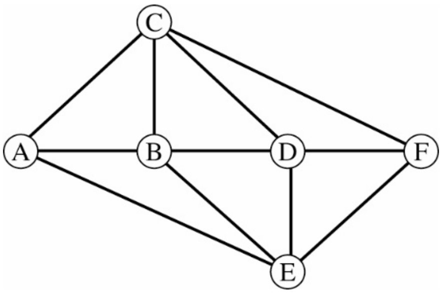
Representation of networks

Information about a network can be summarised using the following technique:

1. To label vertices, simply list them
2. To label edges , identify the vertices that that connect the edges

Example.1

Consider the following network:



Tasks:

1. Label vertices
2. Label edges

1. Label vertices: V = {A, B, C, D, E, F}

2. Examine each edge in turn:

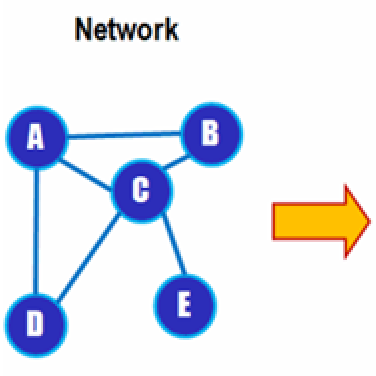
Vertex A – vertex B	(A, B)
Vertex A – vertex C	(A, C)
Vertex A – vertex E	(A, E)
Vertex B – vertex C	(B, C)
Vertex B – vertex D	(B, D)
Vertex B – vertex E	(B, E)
Vertex C – vertex D	(C, D)
Vertex C – vertex F	(C, F)
Vertex D – vertex E	(D, E)
Vertex D – vertex F	(D, F)
Vertex E – vertex F	(E, F)

Label edges:

E = {(A, B), (A, C), (A, E), (B, C), (B, D), (B, E), (C, D), (C, F), (D, E), (D, F), (E, F)}

Matrix Representation of Networks

Matrices can be used to summarise or represent a network.



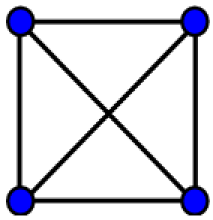
Connectivity Matrix

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

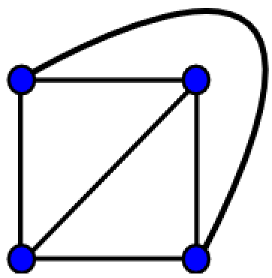
A "1" is entered when two vertices are connected via an edge. So in the above example a "1" is entered between Column A and Row B as these two vertices are connected. Note also, that a "1" is also entered between Column B and Row A. Accordingly the resultant connectivity matrix is symmetrical in nature.

Planar Graphs

A **planar graph** is a graph that can be drawn in the plane (ie. a flat surface such as a page) that has **no edges (paths) that cross**.



Non planar graph
(contains crossed edges)

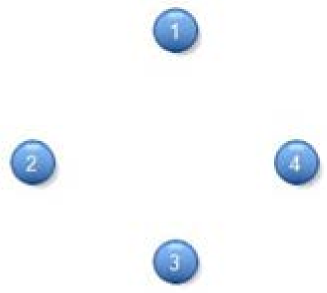


Redrawn planar graph
(doesn't contain crossed edges)

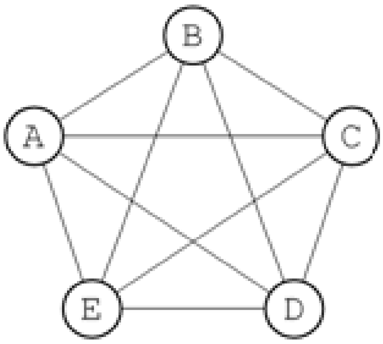
NB: Whilst a network may appear non-planar, often it can be modified, as in the above example to be clearly planar.

A graph with no edges is called a **degenerate**, or **null graph**.

A graph where all vertices are connected directly to all other vertices without parallel edges or loops is called a **complete graph**.



null graph



complete graph

For a complete graph:

$$\mathbf{E} = \frac{\mathbf{V(V-1)}}{\mathbf{2}}$$

Where E = no. of edges
V = no. of vertices

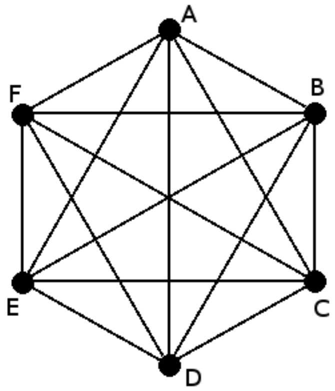
Example.2
How many edges would be required to construct a complete graph consisting of six (6) vertices?

E = ?
V = 6

$$E = \frac{6 \times 5}{2}$$

$$E = \frac{V(V-1)}{2}$$

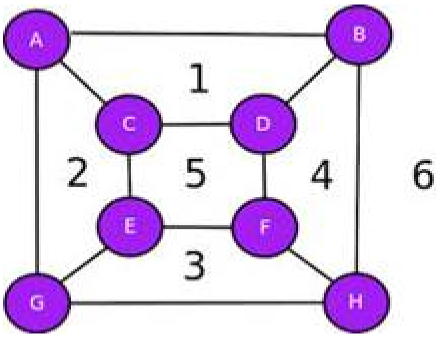
$$E = \frac{6(6-1)}{2}$$



$$E = 15$$

It would take 15 edges to connect a complete a six vertex complete graph.

Regions of a planar graph



This planar graph has 6 regions (or faces)
NB: the "infinite" region (6) is also counted

NB: Face 1 can be described as having a **degree of 4** as it has **four bordering edges.**

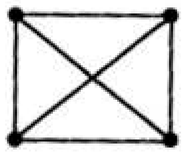
Face 1 is defined by edges(A,B), (A,C), (C, D) and (D,B).
So its degree = 4

Euler's Formula
Euler's formula states that:

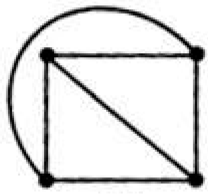
$V = E - F + 2$	Where	V = no. of vertices E = no. of edges F = no. of faces

This formula is upheld in every **connected planar graph**.

Example.3
Consider the following two examples.



Graph A



Graph B

Use Euler's formula to determine if both graphs are planar.

Graph A

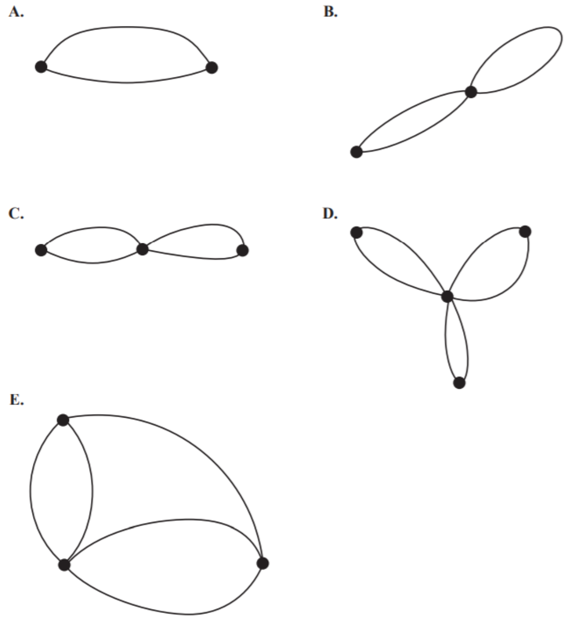
V = 4 $\quad$ $V = E - F + 2$ $\quad$ LHS = 4
E = 6 $\quad$ RHS = 6 – 5 + 2
F = 5 $\quad$ = 3
LHS ≠ RHS
∴ graph A is not a planar graph

Graph B

V = 4 $\quad$ $V = E - F + 2$ $\quad$ LHS = 4
E = 6 $\quad$ RHS = 6 – 4 + 2
F = 4 $\quad$ = 4
LHS = RHS
∴ graph B is a planar graph

Exam Styled Questions – Multiple Choice

Question 1
(2017 Exam 1, Module 2, Qn 1)
Which one of the following graphs contains a loop?



B

Option B is the only graph that shows a loop, that being a line from a vertex back to the same vertex.

Question 2
(2017 Exam 1, Module 2, Qn 2)
Two graphs, labelled Graph 1 and Graph 2, are shown below.

The sum of the degrees of the vertices of Graph 1 is

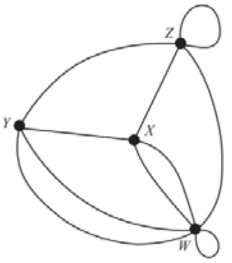
A. two less than the sum of the degrees of the vertices of Graph 2.
B. one less than the sum of the degrees of the vertices of Graph 2.
C. equal to the sum of the degrees of the vertices of Graph 2.
D. one more than the sum of the degrees of the vertices of Graph 2.
E. two more than the sum of the degrees of the vertices of Graph 2.

C

Graph 1 – Sum of all vertices degrees $= 3 + 3 + 3 + 3 = 12$
Graph 2 – Sum of all vertices degrees $= 2 + 2 + 2 + 2 + 2 + 2 = 12$

$\therefore$ *sum of degrees of vertices in Graph 1 = that of Graph 2*

Question 3
(2017 Exam 1, Module 2, Qn 3)
Consider the following graph.
The adjacency matrix for this graph, with some elements missing, is also shown below.



$$\begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 1 & \_ & \_ & \_ \\ \_ & 0 & \_ & \_ \\ \_ & \_ & 0 & \_ \\ \_ & \_ & \_ & 1 \end{bmatrix} \end{array}$$

This adjacency matrix contains 16 elements when complete. Of the 12 missing elements

A. eight are ‘1’ and four are ‘2’.
B. four are ‘1’ and eight are ‘2’.
C. six are ‘1’ and six are ‘2’.
D. two are ‘0’, six are ‘1’ and four are ‘2’.
E. four are ‘0’, four are ‘1’ and four are ‘2’.

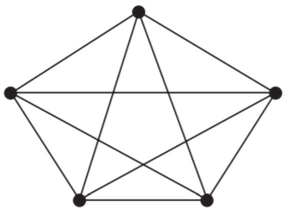
$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

A

The completed adjacency matrix shows eight “1s” added and four “2s” added

Question 4
(2016 Exam 1, Module 2, Qn 3)

The following graph with five vertices is a complete graph.



Edges are removed so that the graph will have the minimum number of edges to remain connected.
The number of edges that are removed is

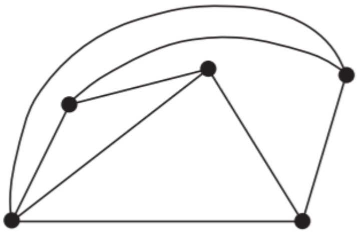
A. 4
B. 5
C. 6
D. 9
E. 10

C

There are currently 10 edges in the 5 vertices complete graph. In order to remain connected, you only need 4 edges (at the minimum). This is effectively a minimum spanning tree. Therefore 6 are removed.

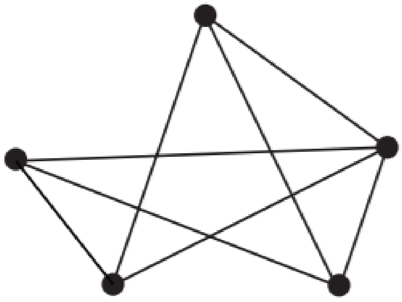
Question 5
(2016 Exam 1, Module 2, Qn 5)

Consider the planar graph below.

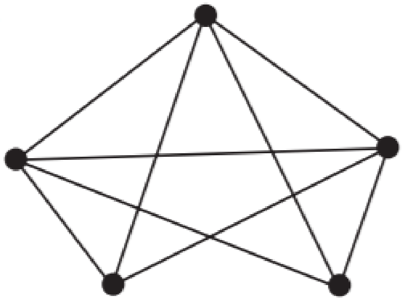


Which one of the following graphs can be redrawn as the planar graph above?

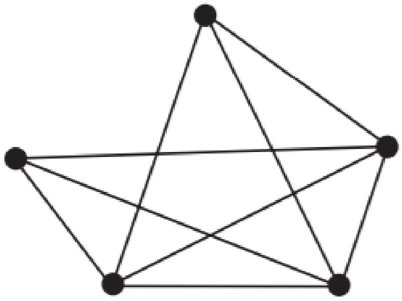
A.



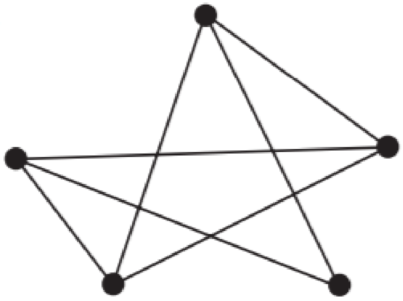
B.



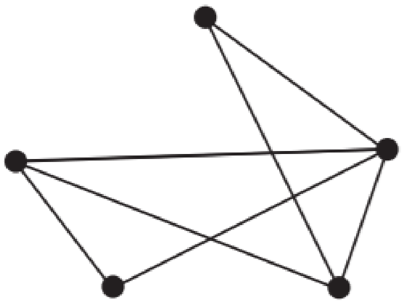
C.



D.



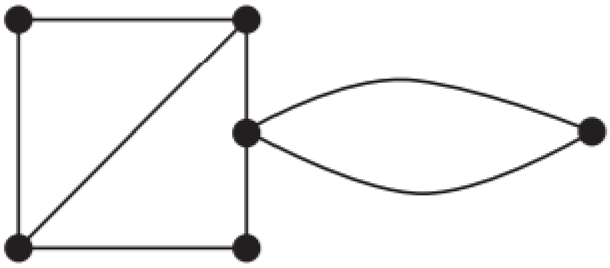
E.



A

Option A is the only graph consisting of 8 edges, the same as the planar graph shown.

Question 6
(2015 Exam 1, Module 5, Qn 1)



In the graph above, the number of vertices of odd degree is

A. 0
B. 1
C. 2
D. 3
E. 4

C

The two vertices in the square shape attached to the diagonal edge, each have a degree of 3. The rest are even. Therefore Option C.

Question 7
(2015 Exam 1, Module 5, Qn 2)

A planar graph has five vertices and six faces.
The number of edges is

A. 3
B. 6
C. 9
D. 11
E. 13

C

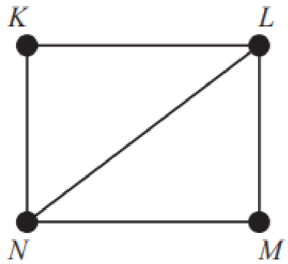
$E = ?$
$V = 5$
$F = 6$

$V = E - F - 2$
$\therefore E = V + F - 2$
$= 5 + 6 - 2$
$= 9$

$\therefore$ *Option C*

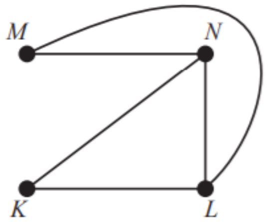
Question 8
(2015 Exam 1, Module 5, Qn 5)

The graph below represents a friendship network. The vertices represent the four people in the friendship network: Kwan (K), Louise (L), Milly (M) and Narelle (N). An edge represents the presence of a friendship between a pair of these people. For example, the edge connecting K and L shows that Kwan and Louise are friends.

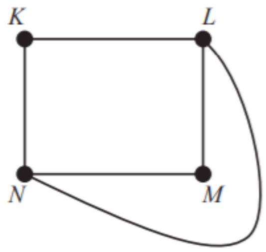


Which one of the following graphs does **not** contain the same information?

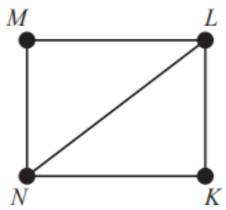
A.



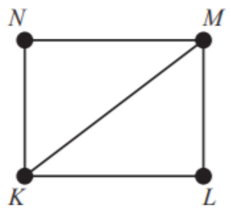
B.



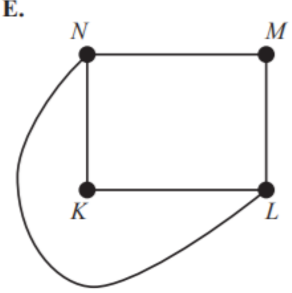
C.



D.



E.



D

In Option D: Vertex N is not connected to Vertex L, whereas it is in the original graph.