

## Section 3.2.5 – Power Transmission

### Power Transmission

The sources of power (eg. coal & natural gas) are usually located some distance from major cities. As a result, the power produced needs to be transmitted from the supplier to the consumer.

When power is transmitted over 100's – 1000's of kilometres there is considerable power loss due to the resistance of the cable.

$$R = \frac{\rho L}{A}$$

Where R = Resistance of cable ( $\Omega$ )  
 $\rho$  = Resistivity of cable ( $\Omega\text{m}$ )  
 L = Length of cable (m)  
 A = Cross sectional area ( $\text{m}^2$ )

**NB:** A large cross sectional area actually reduces the resistance of a cable!  
 (much like a large hose allows water to flow easier)

$$P_{\text{loss}} = I^2 R$$

Where  $P_{\text{loss}}$  = Power loss in cable (W)  
 I = Current (A)  
 R = Resistance of cable ( $\Omega$ )

The above equation clearly shows that a **low current** is essential to **reducing power loss** (as  $P_{\text{loss}} \propto I^2$ ). It is for this reason that long distance cables transfer at:

- High voltages (220 - 500 kV)
- Low currents (relatively)

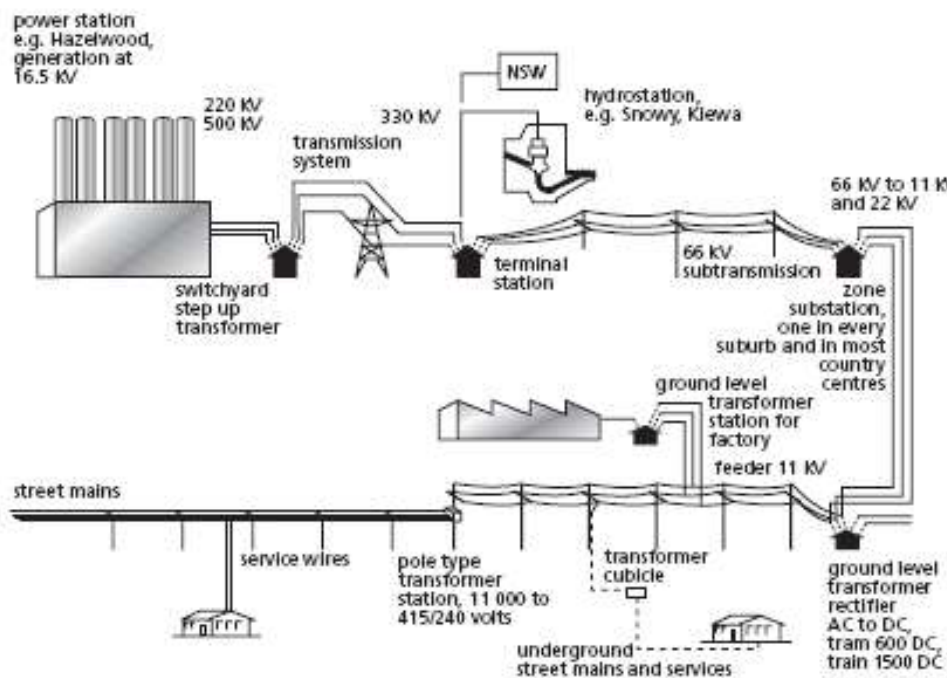


Figure 1. A typical power grid arrangement

**Fact:** When power leaves the generator at “Yallourn W”, it is **stepped up** from 20 kV to 220 – 500 kV via a transformer at the switchyard.

Eg. A power cable has a total resistance of  $10\Omega$  and a power supply of 2 kW. Calculate the power loss when it is transmitted at:

a. 200 V

b. 1000 V

c. 2000 V

**Step. 1** calculate trans<sup>n</sup> I

$$P_{\text{tot}} = VI \therefore I = P_{\text{tot}}/V$$

$$I = 2000/200 \\ = 10 \text{ A}$$

**Step. 2** calculate  $P_{\text{loss}}$

$$P_{\text{loss}} = I^2R \\ = 10^2 \times 10 \\ = 1000 \text{ W}$$

ie. 50% power loss!

**Step. 1** calculate trans<sup>n</sup> I

$$P_{\text{tot}} = VI \therefore I = P_{\text{tot}}/V$$

$$I = 2000/1000 \\ = 2 \text{ A}$$

**Step. 2** calculate  $P_{\text{loss}}$

$$P_{\text{loss}} = I^2R \\ = 2^2 \times 10 \\ = 40 \text{ W}$$

ie. 2% power loss!

**Step. 1** calculate trans<sup>n</sup> I

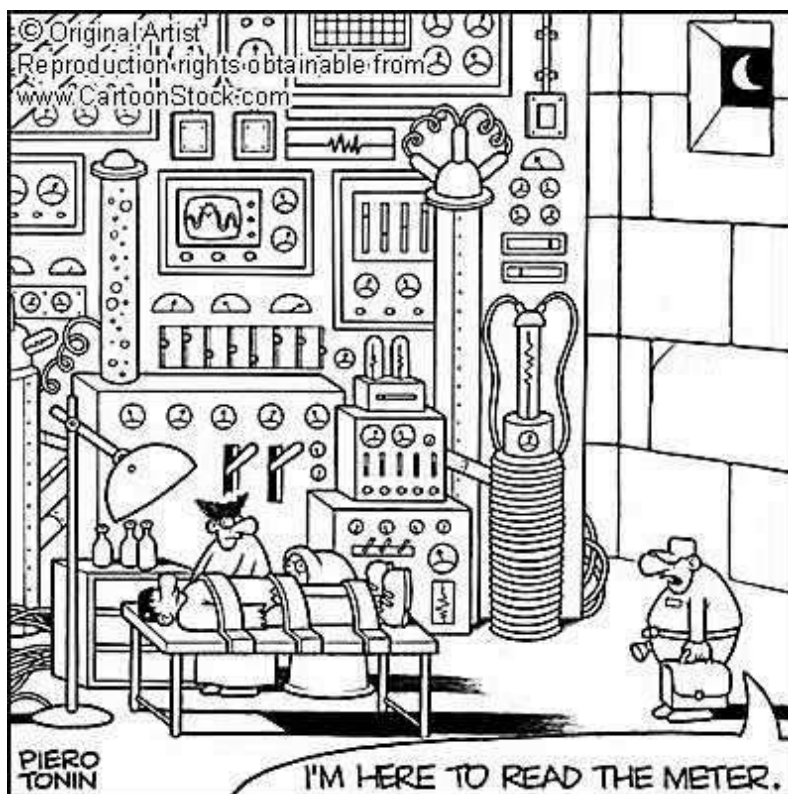
$$P_{\text{tot}} = VI \therefore I = P_{\text{tot}}/V$$

$$I = 2000/2000 \\ = 1.0 \text{ A}$$

**Step. 2** calculate  $P_{\text{loss}}$

$$P_{\text{loss}} = I^2R \\ = 1.0^2 \times 10 \\ = 10.0 \text{ W}$$

ie. 0.5% power loss!



## Power loss within transmission

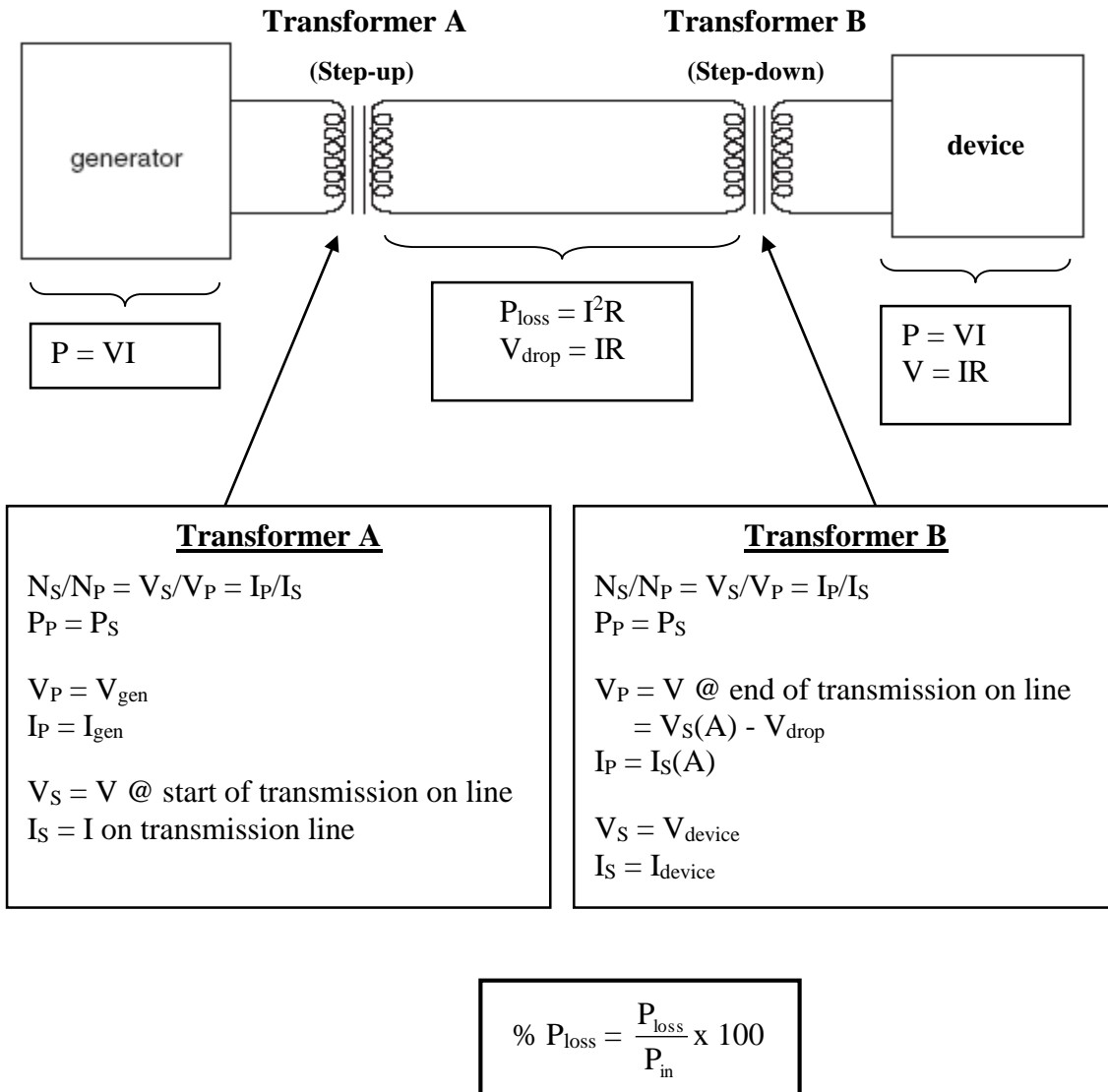
Equations available:

$$P = VI$$

$$V = IR$$

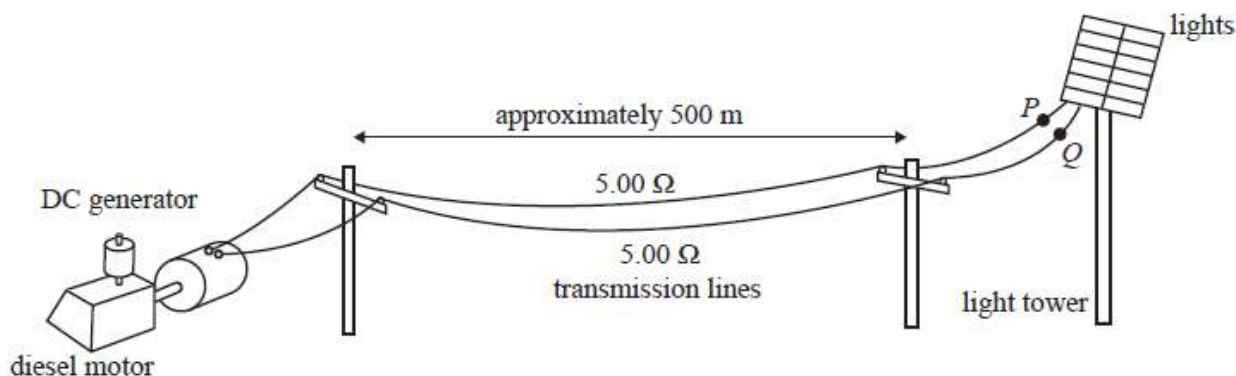
$$P_{\text{loss}} = I^2R$$

As  $P_{\text{loss}} \propto I^2$ , transmission of power over considerable distances is optimised at relative **high voltages** and **low currents**. The standard arrangement used consists of a pair of transformers:



## Exam Styled Questions

A diesel-powered portable DC generator is used to power lights on a light tower at an isolated football ground. The generator produces a constant 500 V DC. The generator is approximately 500 m from the light tower, and a two wire transmission line connects the generator to the lights. Each of the wires in the transmission lines has a resistance of 5.00  $\Omega$ . Ignore the resistance of the other connecting wires. The system is shown below.



With the lights on, the generator has an output of 20.0 A.

### Question 1

What is the power output of the generator?

$$\begin{aligned}
 P &= ? & P &= VI \\
 V &= 500 \text{ V} & &= 500 \times 20 \\
 I &= 20.0 \text{ A} & &= \underline{10000 \text{ W}} \text{ (} 1.0 \times 10^4 \text{ W)}
 \end{aligned}$$

### Question 2

What is the **total** power loss in the transmission lines?

$$\begin{aligned}
 P_{\text{loss}} &= ? & P_{\text{loss}} &= I^2 R \\
 I &= 20.0 \text{ A} & &= 20.0^2 \times (2 \times 5.00) \\
 R &= 5 \Omega & &= \underline{4000 \text{ W}} \text{ (} 4\text{kW)}
 \end{aligned}$$

### Question 3

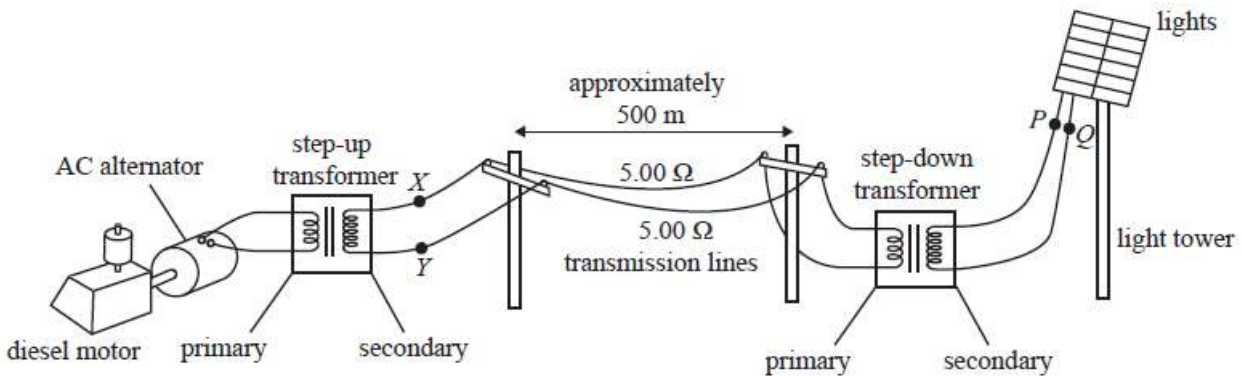
What is the voltage (potential difference) across the terminals (PQ) of the light tower?

$$\begin{aligned}
 V_{PQ} &= ? & V_{PQ} &= V_{\text{gen}} - V_{\text{drop in lines}} \\
 V_{\text{gen}} &= 500 \text{ V} & &= 500 - (IR_{\text{lines}}) \\
 I &= 20.0 \text{ A} & &= 500 - (20 \times 10) \\
 R_{\text{lines}} &= 5.00 \times 2 & &= 500 - 200 \\
 &= 10.0 \Omega & &= \underline{300 \text{ V}}
 \end{aligned}$$

The players find the lights too dim. They call in an electrician. She suggests the following.

- retain the diesel motor
- replace the DC generator with an AC alternator producing 500 VRMS
- insert a 1:10 step-up transformer between the alternator and the power lines, and a 10:1 step-down transformer between the power lines and the light tower

The transformers can be considered ideal. The arrangement is shown below.



The lights are on.

The resistance of each wire in the transmission lines is still  $5.00 \Omega$ . Ignore the resistance of the other connecting wires.

The output of the alternator is  $20.0 \text{ A RMS}$ .  
(The generator output was  $20.0 \text{ A DC}$ .)

#### Question 4

What will the RMS voltage (potential difference) now be at the input to the transmission lines (at point XY) at the alternator end?

At the “step up” transformer

$$\begin{aligned} N_S/N_P &= 10:1 & V_{XY} &= V_P \times (N_S/N_P) \\ V_P &= 500 \text{ V} & &= 500 \times (10) \\ V_S &= V_{XY} = ? & &= \underline{5000 \text{ V}} \end{aligned}$$

#### Question 5

The primary of the step-down transformer has 4800 turns. How many turns are in the secondary winding?

At the “step down” transformer

$$\begin{aligned} N_S/N_P &= 1:10 & N_S &= 10 \times N_P \\ N_P &= 4800 \text{ turns} & &= 1/10 \times 4800 \\ N_S &= ? & &= \underline{480 \text{ turns}} \end{aligned}$$

**Question 6**

What will be the power loss in the transmission lines now?

$$\begin{aligned}
 P_{\text{loss}} &= ? & P_{\text{loss}} &= I^2 R \\
 I &= 2.0 \text{ A} & &= 2.0^2 \times (2 \times 5.00) \\
 & \text{(1/10 its original value)} & &= \underline{40 \text{ W}} \\
 R &= 5 \Omega & &
 \end{aligned}$$

**Question 7**

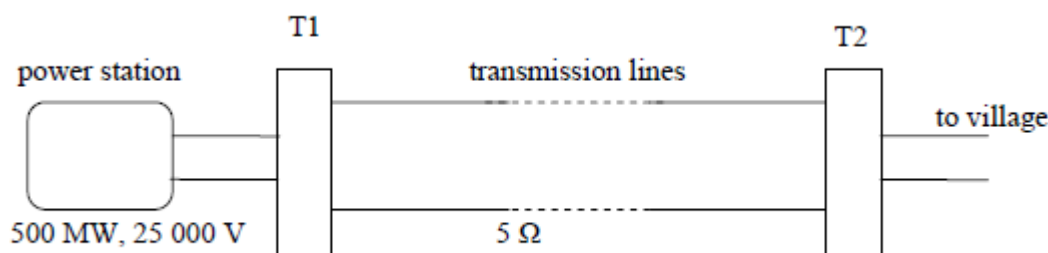
What will be the voltage at the output of the step-down transformer? Give your answer correct to three significant figures.

At the “step down” transformer

$$\begin{aligned}
 V_P &= 5000 - 20 & V_S &= V_P \times (N_S/N_P) \\
 &= 4980 \text{ V} & &= 4980 \times (1/10) \\
 N_S/N_P &= 1:10 & &= \underline{498 \text{ V}} \\
 V_S &= V_{PQ} = ? & &
 \end{aligned}$$

Use the following information to answer Questions 8 to 11.

A power station generates AC power of 500 MW at 25 000 V to power a village 80 km away. Two ideal transformers, T1 and T2, are used to assist with power transmission. Transformer T1 has a turn ratio of 1 : 100. The total resistance of the power lines between the two transformers is a total of 5  $\Omega$ .

**Question 8**

What is the current in the transmission lines?

**Worked solution**

$$\text{Current in the primary coil of T1} = \frac{500 \times 10^6}{25000} = 20\,000 \text{ A.}$$

$$\text{Current in the transmission lines} = \frac{20000}{100} = 200 \text{ A}$$

**200 A**

**Mark allocation**

- 1 mark for correctly calculating the current in the primary coil of T1.
- 1 mark for correctly evaluating the current in the transmission lines.

### Question 9

Calculate the power loss in the transmission lines.

#### Worked solution

$$\text{Power loss} = I^2 R = 200^2 \times 5 = 200\,000 \text{ W} = 0.2 \text{ MW}$$

**0.2 MW**

#### Mark allocation

- 1 mark for correct use of the power loss formula.
- 1 mark for the correct answer.

### Question 10

What is the voltage in the primary coil of transformer T2? Show your working.

#### Worked solution

Power remaining =  $500 \text{ MW} - 0.2 \text{ MW} = 499.8 \text{ MW}$   
Current in the transmission lines =  $200 \text{ A}$

$$\therefore \text{Voltage in the primary coil of T2} = \frac{P}{I} = 2499 \text{ kV}$$

**2499 kV**

#### Mark allocation

- 1 mark for correctly estimating the power remaining.
- 1 mark for correctly estimating the voltage as  $= \frac{P}{I}$
- Consequential marks awarded if incorrect value of current used from Question 8 or incorrect value of power loss used from Question 9.
- Full marks awarded if an alternative method of voltage loss is used to get the correct answer.

### Question 11

Suggest two ways to reduce power loss in the transmission lines, explaining clearly why the amount of power lost will be reduced.

#### Worked solution

Suggestion 1: Increase voltage in the transmission lines by increasing the relative number of turns in the secondary coil of T1. This will decrease the current, thereby reducing power loss, which is  $I^2R$ .

Suggestion 2: Check the quality of the transmission lines. If there is corrosion of wires, then resistance could increase. If there is corrosion, then segments of the corroded wire should be replaced.

#### Mark allocation

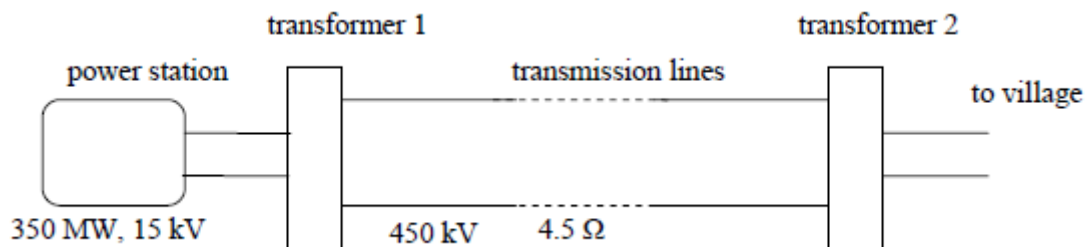
- 1 mark for each suggestion.
- 1 mark for the explanation.
- Deduct 1 mark if the explanation is inadequate or does not refer to the power loss expression.

#### Tips

- *Develop a systematic method to calculate transmission line problems, such as estimating transmission current  $\rightarrow$  power loss  $\rightarrow$  remaining power  $\rightarrow$  voltage output and so on.*
- *Take care to check units for quantities such as kV and MW.*

Use the following information to answer Questions 12 – 16.

A power station produces 350 MW at 15 kV. Power lines transmit the power to a small village 10 km away using overhead power transmitting cables, which have a total resistance of  $4.5 \Omega$ . Two transformers are used in the process. From the first transformer, called transformer 1 and which is at the site of the power station, the voltage output is 450 kV. A schematic diagram of the station, transformers and transmission lines to the village is shown in the figure below.



### Question 12

Calculate the ratio of number of primary turns to that of secondary turns for the transformer 1 used at the site of the power station.

Transformer 1 is a step up transformer

$$\frac{N_P}{N_S} = \frac{V_p}{V_S} = \frac{15000}{450000} = 1:30$$

**1:30**

### Mark allocation

- 1 mark for correct use of formula with correct substitution.
- 1 mark for correct answer.

### Question 13

What is the current in the transmission lines between transformer 1 and transformer 2?

Worked solutions

$$I_p = \frac{P}{V_p} = \frac{350 \times 10^6}{15\,000} = 23\,333.3 \text{ A}$$

$$I_s = \frac{I_p}{30} = \frac{23\,333.33}{30} = 777.78 \text{ A}$$

**777.8 A**

### Mark allocation

- 1 mark for correctly estimating the current in the primary coil.
- 1 mark for correctly estimating the current in the secondary coil.

### Question 14

What is the power lost by the transmission lines from transformer 1 to transformer 2? Show your working.

### Worked solution

Current in the transmission lines = 777.78 A

Therefore, power lost =  $I^2R = (777.8)^2 \times 4.5 = 2.7 \text{ MW}$

**2.7 MW**

### Mark allocation

- 1 mark for correct use of primary current (no need to complete calculation if values are substituted into expression for secondary current) in the expression for power lost.
- 1 mark for correct answer.

### Question 15

What is the voltage at the primary coil of transformer 2?

#### Worked solution

Current in the transmission lines = 777.78 A and

power lost =  $I^2R = (777.8)^2 \times 4.5 = 2.7 \text{ MW}$

Therefore, power at primary end of transformer 2 (i.e. the end of transmission lines) is

$350 - 2.7 = 347.3 \text{ MW}$

Therefore,  $V = \frac{P}{I} = \frac{347.3 \times 10^6}{777.8} = 446.5 \text{ kV}$

<b>446.5 kV</b>
-----------------

#### Mark allocation

- 1 mark for using power loss to work out power at transformer 2.
- 1 mark for correctly estimating power at transformer 2 by subtracting power loss from power supplied.
- 1 mark for correctly estimating voltage at transformer 2.
- Give 1 mark if all answers are incorrect but a value of power loss is subtracted from power generated.
- Give 2 marks maximum for consequential when correct formula is used but incorrect value of  $P$  or  $I$  is substituted. Give 1 mark consequential only if *both*  $P$  and  $I$  have incorrect values.