Section 3.2.3 – Lenz's Law

Lenz's Law

In our previous notes (Section 3.2.2) we introduced Faraday's Law:

$$\varepsilon_{avg} = -\frac{N\Delta\varphi_B}{\Delta t}$$

Up until this point we haven't as yet discussed the significance of the "-" sign in Faraday's Law.

You would recall that Faraday discovered that an electromotive force, or potential difference, was **induced** when there is a **changing magnetic field** relative to a conducting coil.

In a single coil (ie. N = 1) generator, as shown below in Figure 1, the EMF induced equal to:

$$\varepsilon_{avg} = -\frac{\Delta\varphi_B}{\Delta t}$$

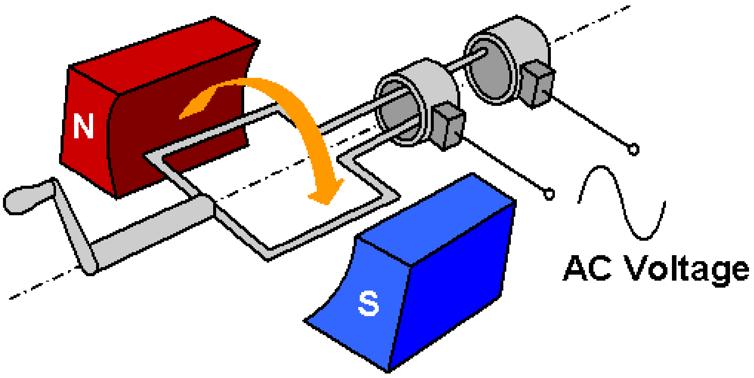


Figure 1

Therefore it can be stated that the induced EMF (ε) is equal to the **negative rate of change of the magnetic flux**, relative to time.

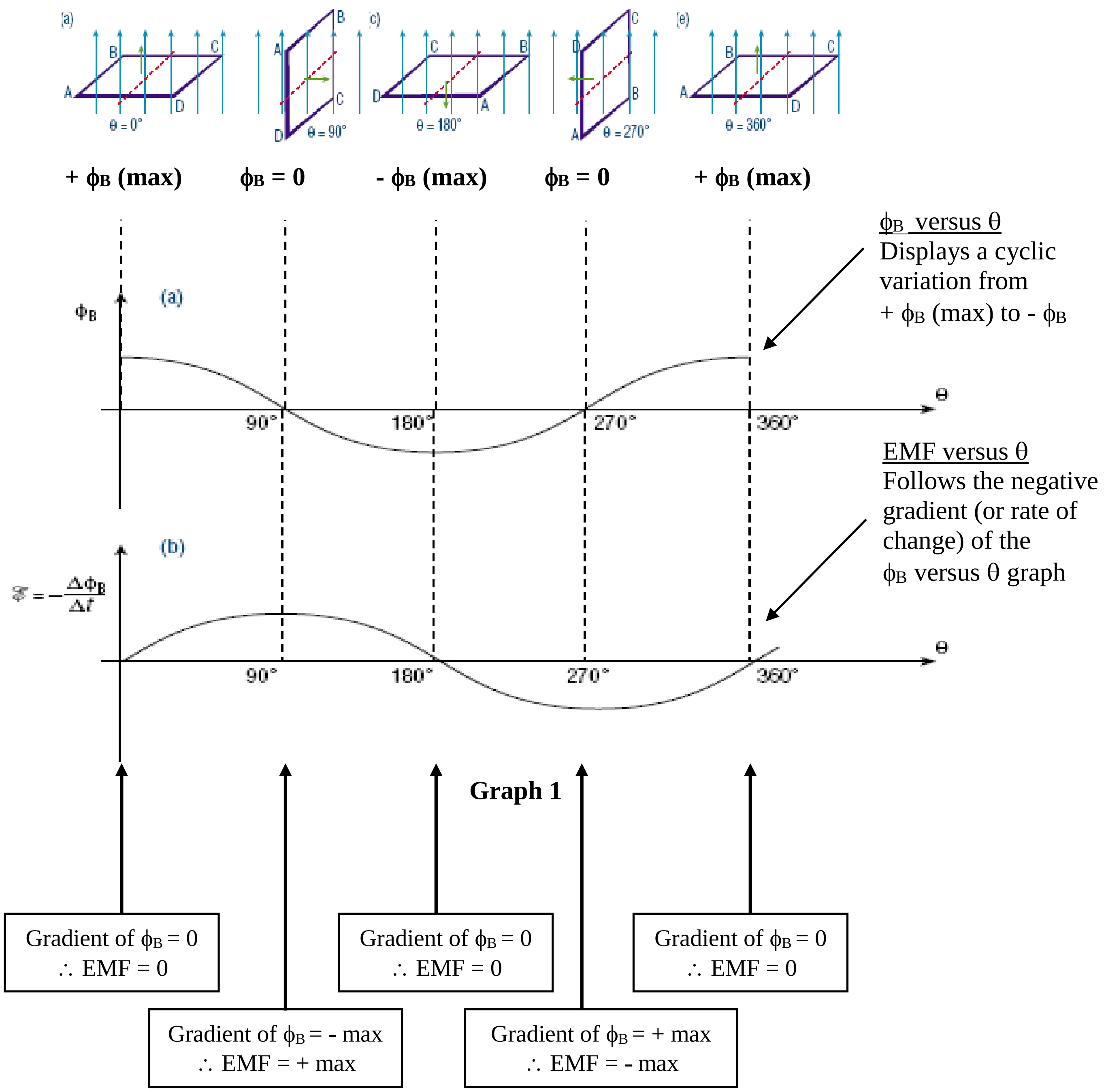
Lenz's Law states that:

"The induced current in a loop will be in such a direction that its magnetic flux will oppose the change in magnetic flux that produced it"

Graphical Representation of Lenz's Law

As a coil rotates through a magnetic field, its magnetic flux and indeed induced EMF varies in a cyclical fashion, that is it **alternates**.

Mathematically, the EMF ($\varepsilon_{avg} = -\frac{\Delta\varphi_B}{\Delta t}$) can be found as the **negative gradient** of a magnetic flux (Φ_B) versus time (t) graph. To further examine this relationship consider the following graph, Graph 1.

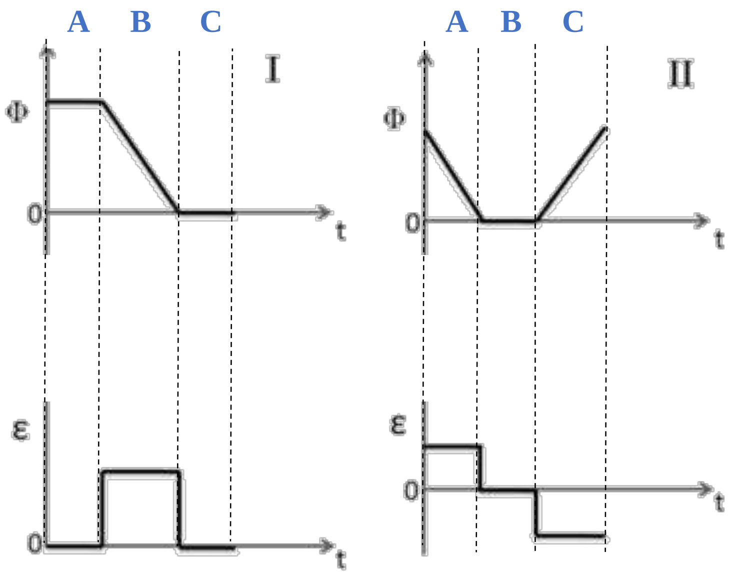


Graph 1

NB: An EMF is induced during the times that flux is actually changing.

Example 1

Consider the following two magnetic flux (Φ_B) versus time (t) graphs. Using your knowledge of Lenz's Law and induced EMF (ε), sketch a corresponding EMF (ε) versus time (t) graph.



NB: The steepness of the magnetic flux (Φ_B) versus time (t) **gradient**, determines the magnitude of the EMF (ε) produced.

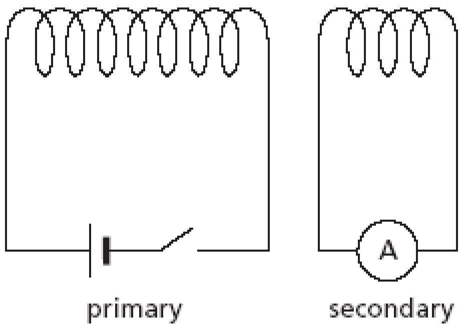
Task 1

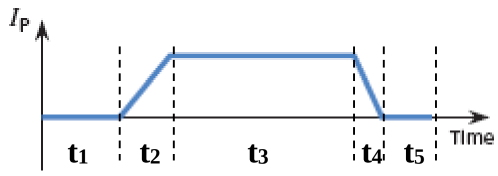
Complete the following summary table.

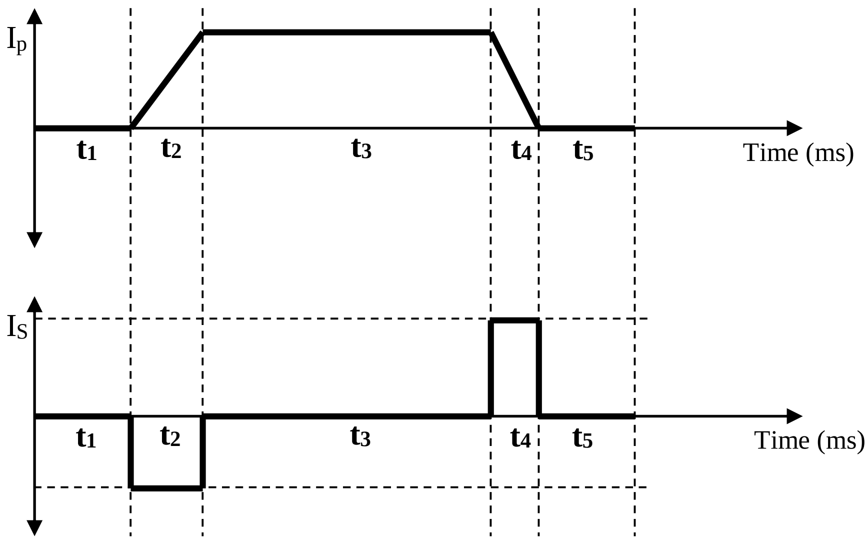
	Graph 1		Graph 2	
	Φ_B V t	ε V t	Φ_B V t	ε V t
Section A	Gradient of 0	∴0 EMF	Constant negative gradient	∴ Constant positive EMF
Section B	Constant negative gradient	∴ Constant positive EMF	Gradient of 0	∴ 0 EMF
Section C	Gradient of 0	∴ 0 EMF	Constant positive gradient	∴ Constant negative EMF

Example 2

The below circuit is used to induce a current from the first coil within the secondary coil. Given the graph of current in the primary (I_P) versus Time, construct a graph of current in the secondary (I_S) versus Time







NB: An induced current is also proportional to the negative rate of change of the magnetic flux.

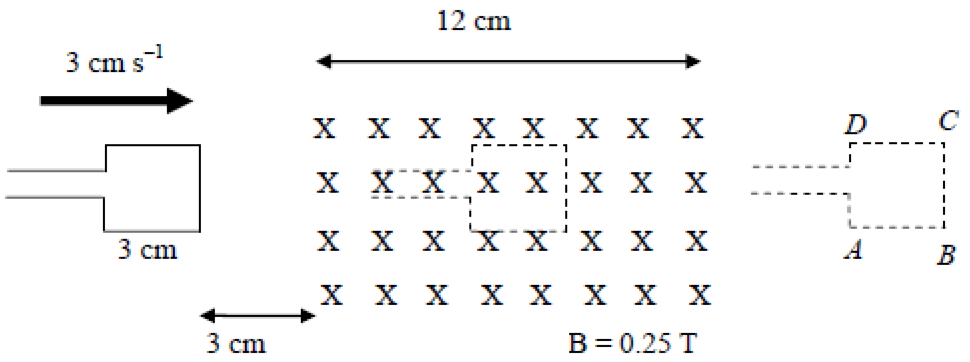
$$I \propto -\frac{\Delta\varphi_B}{\Delta t}$$

The magnitude of the induced I_S is greater during t_4 than at t_2 as the magnitude of the gradient of the magnetic flux (Φ_B) versus time (t) is also greater (ie. steeper) at the corresponding times

Example 3

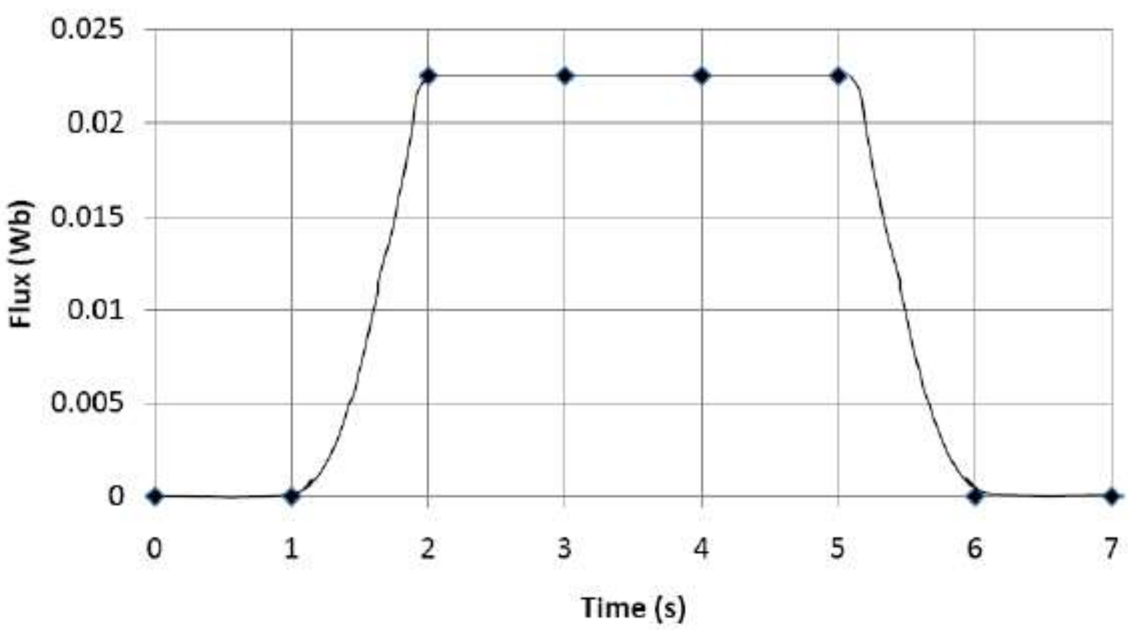
Use the following information to answer Questions 1 to 3.

A square coil, ABCD, with sides of 3 cm and consisting of a single turn, is moved at a uniform speed of 3 cm s^{-1} into a region of constant magnetic field and continues to move through it to the other side. The magnetic region is 12 cm long. For simplicity of analysis, assume the magnetic field drops from a uniform magnitude of 0.25 T to zero at the end of the field.



Question 1

On the axes below, draw a graph showing the flux through the coil as a function of time for the first 7 seconds of its motion, from the time it starts at the initial location (shown on the left side of the figure) and continues through the magnetic field emerging from the other side (shown on the right).



Mark allocation

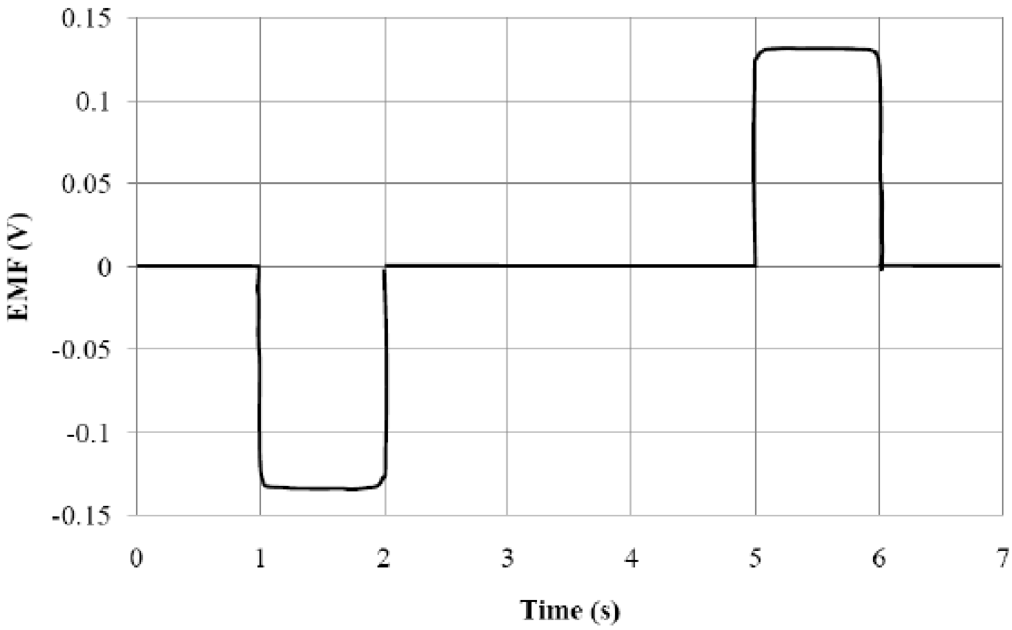
- 1 mark for correct magnitude of maximum flux.
- 1 mark for correct values of time.
- 1 mark for correct shape of graph.

Question 2

The coil is now replaced with another coil of the same size but consisting of six turns instead of one. Again, the coil is moved with the same constant speed through the magnetic region.
On the axes below, draw the induced EMF in the coil for the first 7 seconds of its motion.

Worked solution

The shape of the induced EMF is given as the negative gradient of the flux–time graph. The maximum value of induced EMF is = $-\boldsymbol{N}\frac{\Delta\emptyset}{\Delta t}$ = 0.135V.



Mark allocation

- 1 mark for correct magnitude of flux.
- 1 mark for correct values of times.
- 1 mark for correct shape.

Question 3

When the coil first enters the magnetic field, will the direction of induced current be from A to B or from D to C? Explain your reasoning clearly.

Worked solution

When the coil first enters the field, it encounters magnetic field lines going into the page. According to Lenz's law, the induced EMF will oppose this change and thus generate a current that would create magnetic field lines coming out of the page. This would be an anti-clockwise direction; i.e. in the direction A–B–C–D.

Mark allocation

- 1 mark for the correct answer.
- 1 mark for giving the correct explanation based on Lenz's law.

Tips

- *Whenever possible, draw the flux–time graph first and this will help when drawing the EMF–time graph.*
- *Remember: Lenz's law states that induced EMF will oppose the change in flux. Hence, induced EMF will be the negative gradient of flux.*

Using Lenz's Law to Predict the Direction of Current in a Coil

Recall:

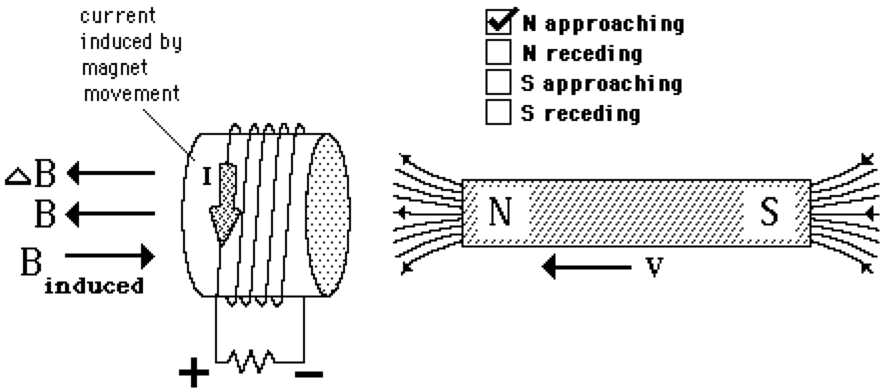
"The induced current in a loop will be in such a direction that its magnetic flux will oppose the change in magnetic flux that produced it"

Let's now use Lenz's Law to predict the direction of current within a coil as the surrounding magnetic field changes.

Scenario 1

A North pole is approaching towards the coil.

- External magnetic field (ie bar magnet) increases in strength from right to left (←)
- Lenz's Law states that a current will be induced in the coil to establish a magnetic field to oppose the change of the external magnetic field, ie. from left to right (→)
- RH Rule for solenoids is used upon the coil to predict the current shown in the below diagram

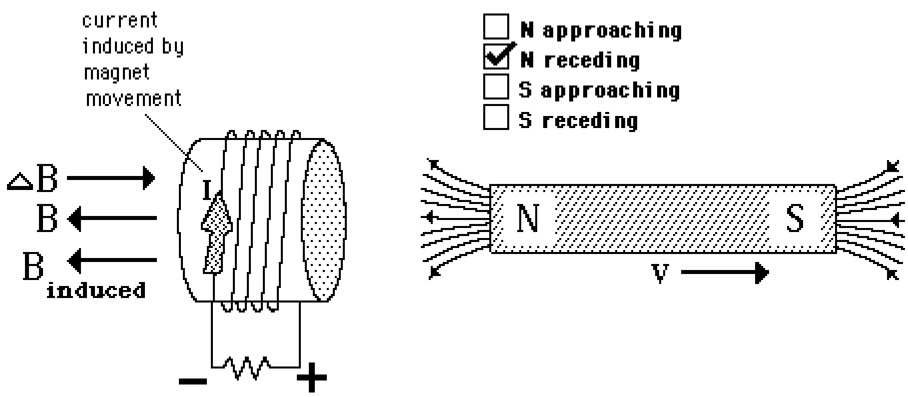


NB: Alternatively you can think of the coil creating a North pole to <u>oppose</u> the introduction of the external North pole

Scenario 2

A North pole is receding away from the coil.

- External magnetic field (ie bar magnet) decreasing in strength from right to left (←)
- Lenz's Law states that a current will be induced in the coil to establish a magnetic field to oppose the change of the external magnetic field, ie. from right to left (←)
- RH Rule for solenoids is used upon the coil to predict the current shown in the below diagram

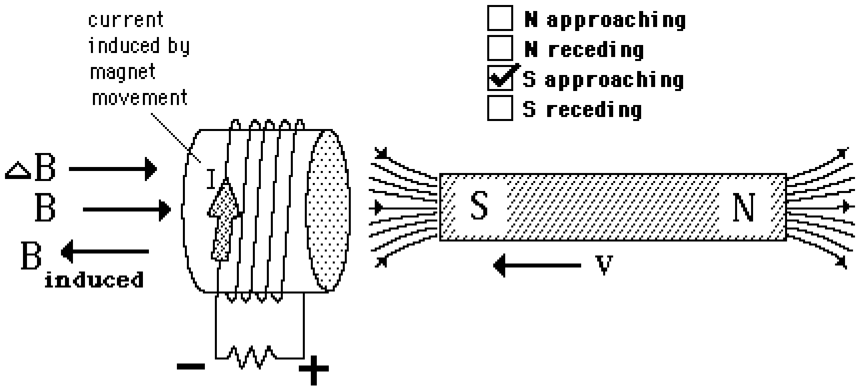


NB: Alternatively you can think of the coil creating a South pole to <u>oppose</u> the removal of the external North pole

Scenario3

A Sorth pole is approaching towards the coil.

- External magnetic field (ie bar magnet) increases in strength from left to right (→)
- Lenz's Law states that a current will be induced in the coil to establish a magnetic field to oppose the change of the external magnetic field, ie. from right to left (←)
- RH Rule for solenoids is used upon the coil to predict the current shown in the below diagram

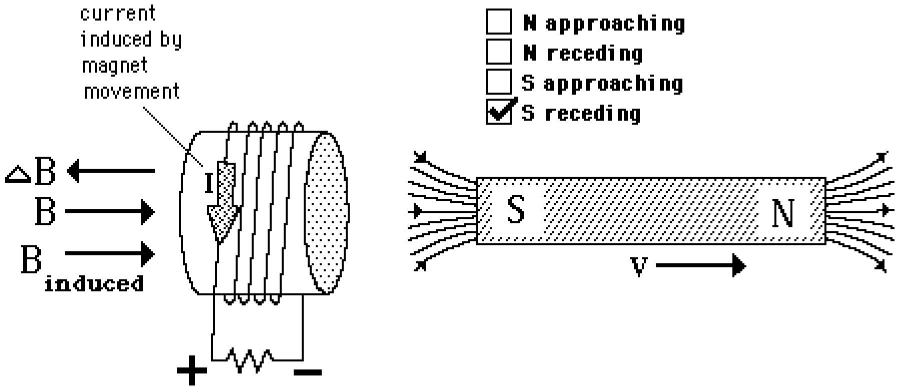


NB: Alternatively you can think of the coil creating a South pole to <u>oppose</u> the introduction of the external South pole

Scenario4

A South pole is receding away from the coil.

- External magnetic field (ie bar magnet) decreasing in strength from left to right (→)
- Lenz's Law states that a current will be induced in the coil to establish a magnetic field to oppose the change of the external magnetic field, ie. from left to right (→)
- RH Rule for solenoids is used upon the coil to predict the current shown in the below diagram

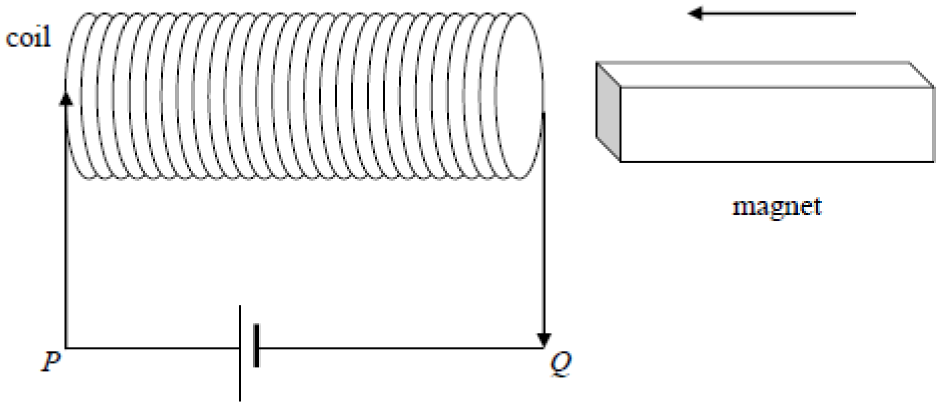


NB: Alternatively you can think of the coil creating a North pole to <u>oppose</u> the removal of the external South pole

Example 4

Use the following information to answer Questions 1 and 2.

A coil of wire is connected to a DC power supply, as shown in the figure below.



Question 1

In the first instance, a DC current flows in the coil. To determine the direction of the current, a magnet is brought to the coil and is found to be repelled. Is it a north or a South pole of the magnet that is inserted into the coil? Explain your answer.

Worked solution

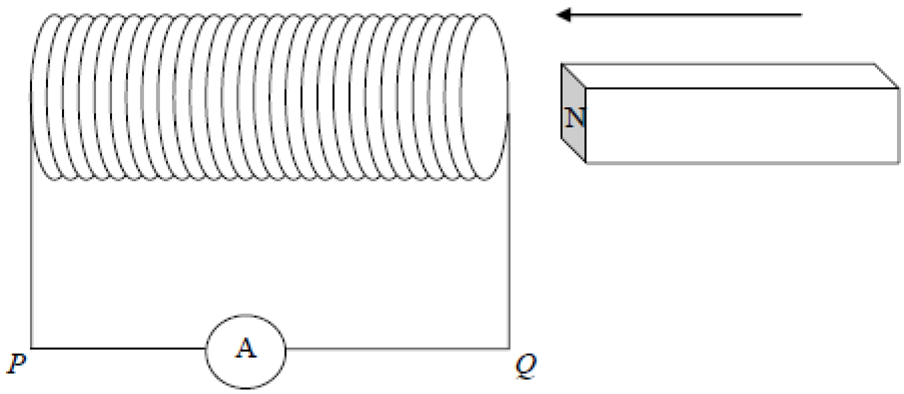
According to the right-hand grip rule, the coil is an electromagnet with the coil facing towards the magnet, being a south pole. Since the magnet is repelled, the incoming face of the magnet must be a south pole as well.

Mark allocation

- 1 mark for the correct answer.
- 1 mark for an explanation based on right-hand rule.

Question 2

With the current now switched off and the power supply replaced by an ammeter, a current is found to be induced when the magnet is re-inserted with the North pole facing towards the coils. See the figure below.



Will the current now flow from *P* to *Q* or from *Q* to *P* through the ammeter? Explain your answer.

Worked solution

Current will flow from *P* to *Q* through the ammeter. The coil will repel the incoming magnet by inducing a current such that the face of the coil facing the incoming magnet is north pole, as in accordance with Lenz's law. This will occur when the current in that face is anti-clockwise and the current in the external circuit is from *P* to *Q*.

Mark allocation

- 1 mark for the correct answer.
- 1 mark for the correct determination of current based on Lenz's law.
- 1 mark for the correct conclusion.

Tip

- *Remember: Induced current will be such that it creates a magnetic field to oppose the changing flux that caused it.*

Alternators & DC Generators

A rotating coil in a magnetic field induces a sinusoidal EMF. This EMF can be can be used to either produce an AC output voltage or a DC output voltage depending on the connection between the coil and the external circuit.

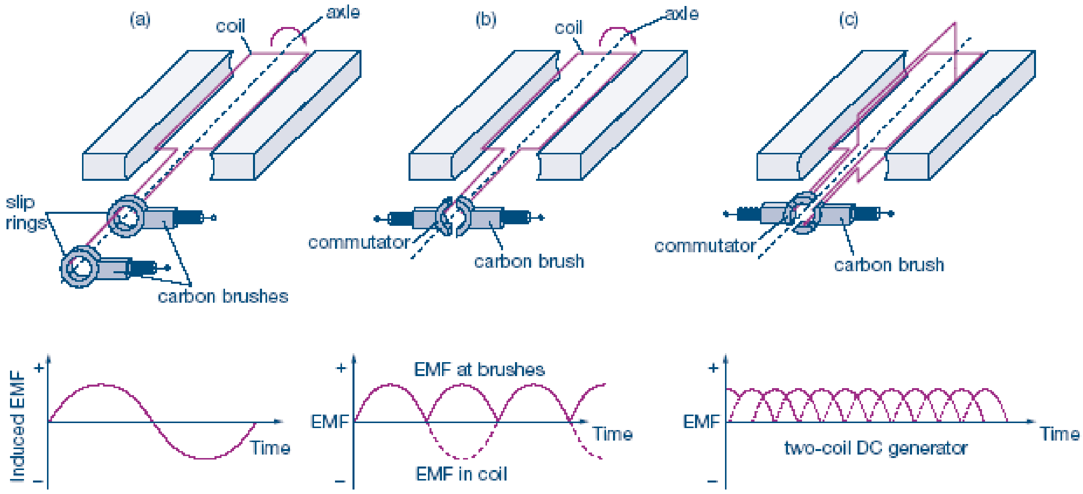


Diagram A - Alternator

Each side of the coil is connected to a **slip ring**. As the slip rings rotate with the coil, the spring loaded **carbon brushes** maintain contact with them to produce an alternating current (AC) and voltage that goes in one direction for half a cycle, then reverses direction for the next half cycle.

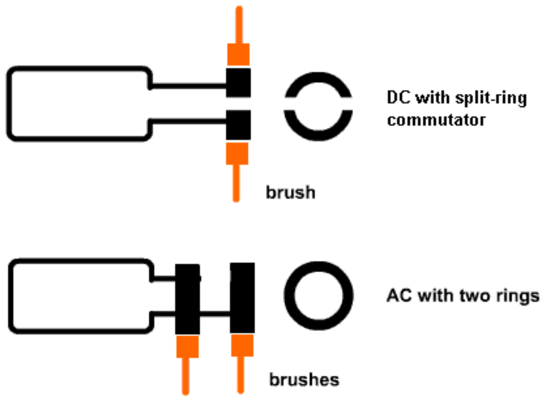
Diagram B – DC Generator

Each side of the coil is connected to one half of a **split-ring commutator**. This arrangement reverses the connection of the coil to the output terminals every half-turn. As the commutator rotates past the **carbon brushes**, the output current is **reversed** every **½ cycle** to form a **DC current and voltage.**

NB: A DC generator is essentially the same as a DC motor run in reverse.

Diagram C

With **multi coil** DC generators a **smoother DC** current is produced. Also known as a **ripple current**.



A top profile view of a DC generator and AC alternator

Example 5

Use the following information to answer Questions 1 - 3

A group of students is studying DC generators and AC alternators. They use a coil which rotates at a constant speed in the magnetic field as shown in Figure 1a. The apparatus has slip rings and a commutator. Either the slip rings or the commutator can be connected to an oscilloscope to observe the output voltage.

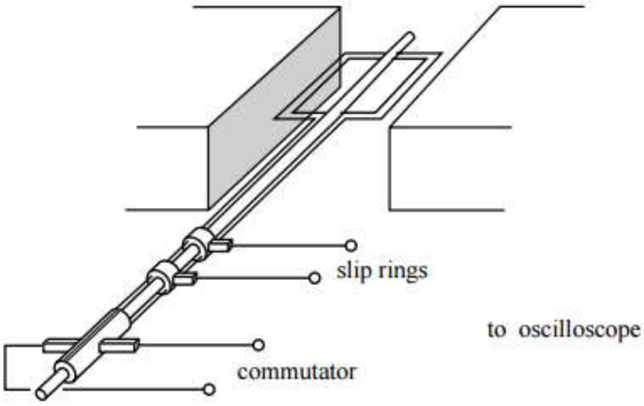
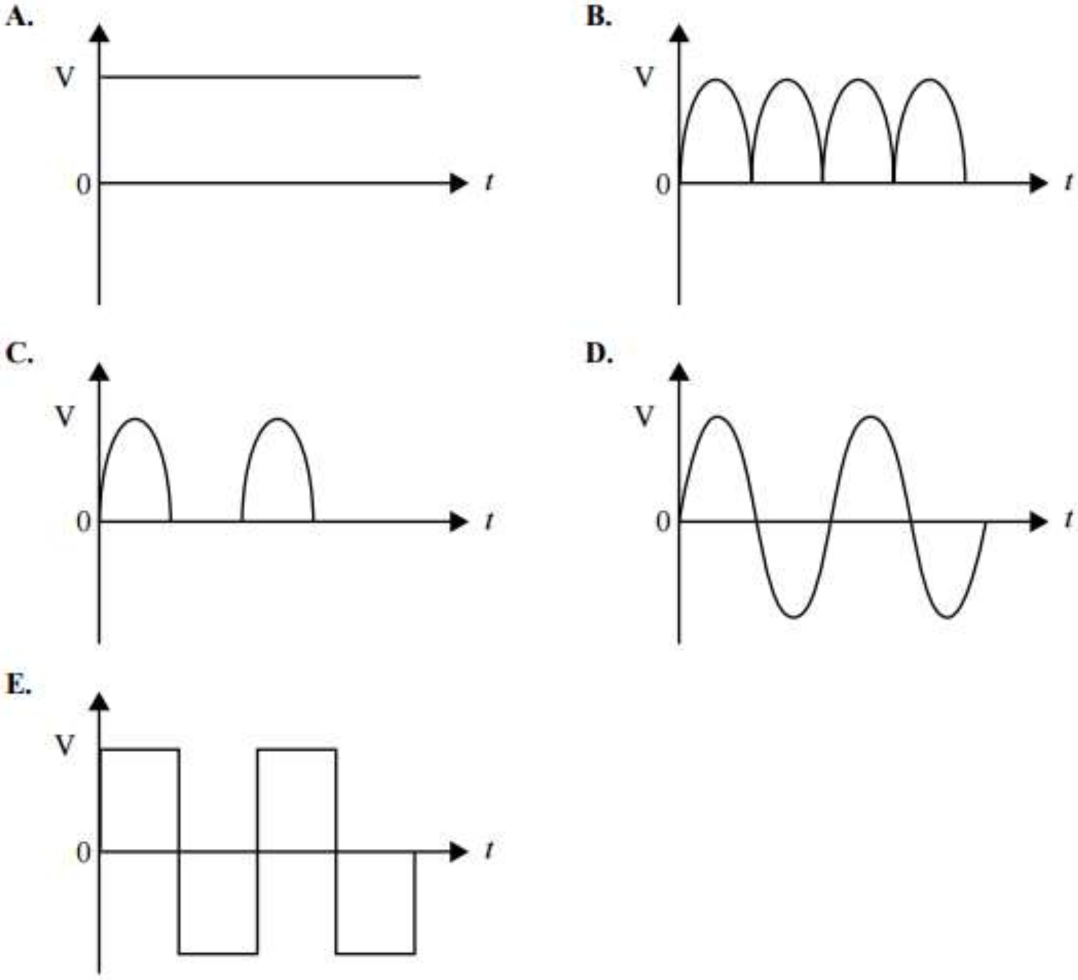


Figure 1a

Use the graphs in Figure 3b to answer Question 1 – 3



Question 1
The oscilloscope is connected to the commutator. Which one of the graphs (A.–E.) best shows the shape of the output as observed on the oscilloscope?

The output from the coils will be AC, but the commutator reverses the voltage each half cycle.

B

Question 2
The oscilloscope is next connected to the slip rings. Which one of the graphs (A.–E.) best shows the shape of the output as observed on the oscilloscope?

The output from the coils will be AC.

D

Question 3
In diagram D. of Figure 3b, the peak-to-peak voltage is observed to be 8.0 V. What will the RMS voltage be? Show working.

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$

$$V_{RMS} = \frac{4}{\sqrt{2}}$$

= 2.8 Volts

2.8 V

Exam Styled Questions

The following information relates to Questions 1 and 2.

Emily and Gerry have been studying generators and alternators. They have constructed the device shown in Figure 1. The rectangular coil, which is a single loop of area of 9.0×10^{-4} m^2, is rotated in the direction shown, in a uniform magnetic field with a direction indicated by B. The coil is completely contained in the magnetic field.

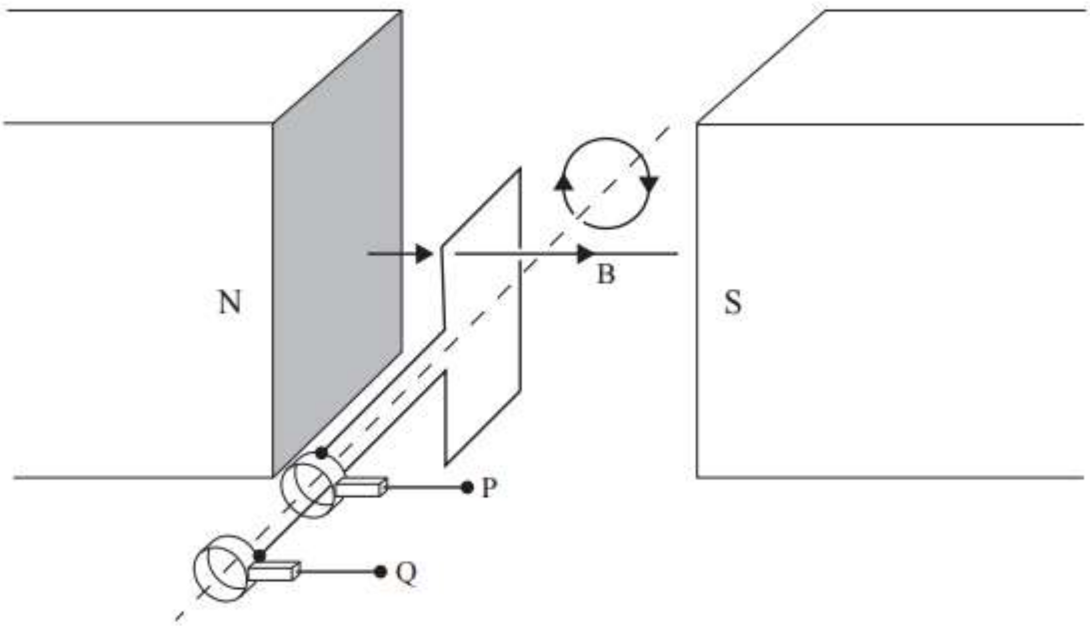


Figure 1

They tested the device by firstly connecting an oscilloscope between the terminals P and Q, and then rotating the coil at a constant rate, in the uniform field B, in the direction shown. Figure 2 shows graphs of the magnetic flux through the coil and of the voltage measured between the terminals.

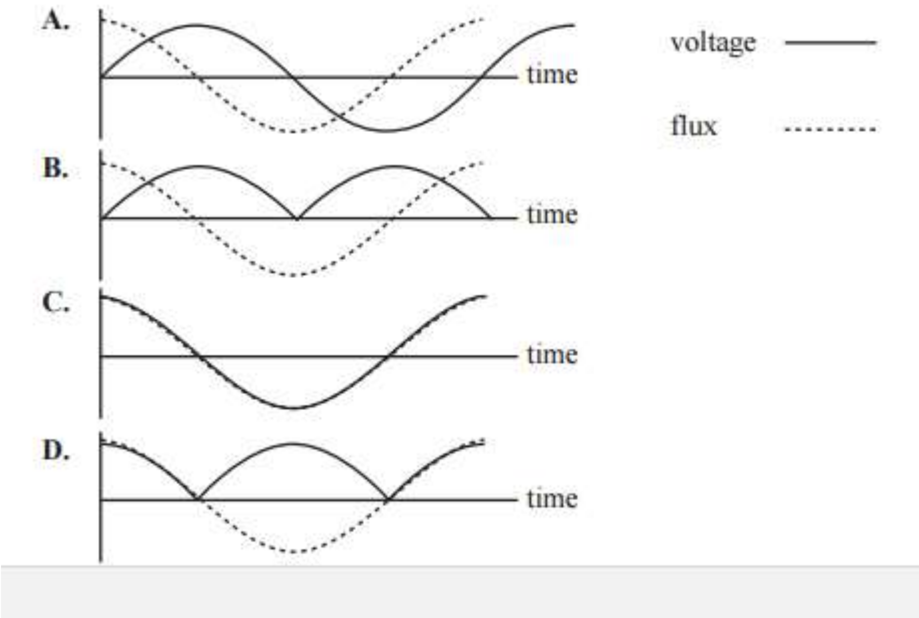


Figure 2

Question 1

Which one of the graphs in Figure 2 best represents the voltage observed on the oscilloscope?

When the flux is at a maximum, the induced voltage is zero (A or B). When the flux is passing through zero, the voltage is a maximum (A or B). The voltage for decreasing flux will have the opposite polarity to that for increasing flux (A).

A

Question 2

Explain the difference in function between a split-ring commutator and slip rings. Describe the situations in which a split-ring commutator and slip rings are used.

Slip rings are shown in Figure 1, two are required, each ring stays connected to either P or Q throughout the cycle. (1) A split ring is one ring with two gaps, it would swap the connection to P and Q twice every cycle. (1) Slip rings produce AC, a split ring produces DC. (1)

The following information relates to Questions 3 – 6.

Figure 3 shows an experiment where the voltage induced in a coil by a time-dependent magnetic field is measured. The voltmeter measures the voltage induced in the coil as a function of time. The coil has 120 turns.

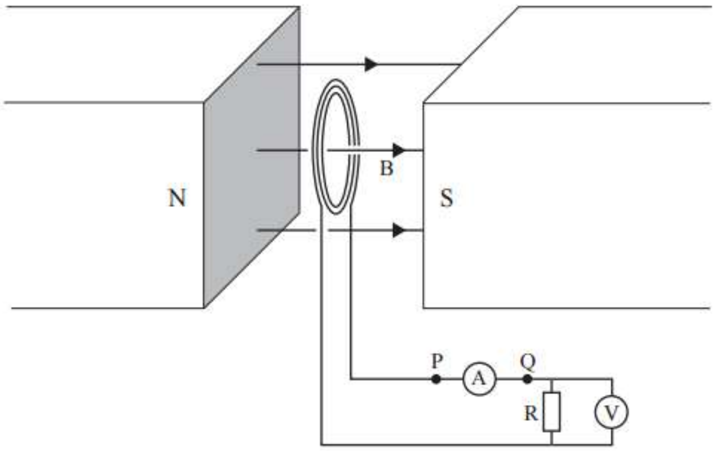


Figure 3

The magnetic field varies with time as shown in Figure 4.

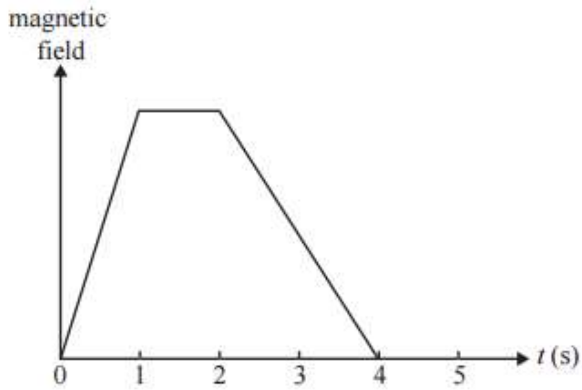
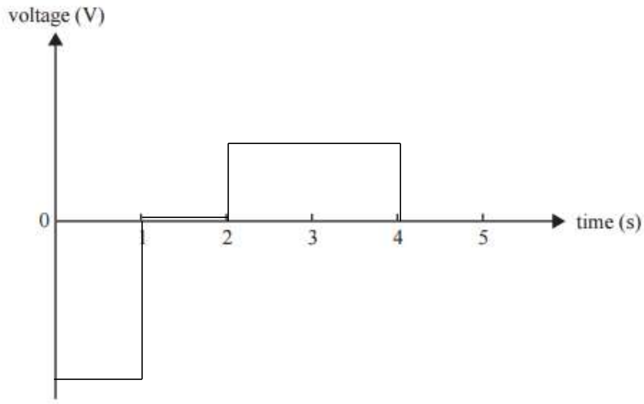


Figure 4

Question 3

Sketch a graph of voltage against time as measured by the voltmeter



Question 4

Identify the physical law you used for constructing your graph.

Faraday's law for magnitude, Lenz' law for direction. The combined law is also known as Neumann's Law.

At another time, the magnetic flux through the 120 turns coil is a constant 3.0×10^{-4} Wb.

Question 5

The magnetic field is now reduced to zero over a period of 0.012 s. What is the average EMF induced in the coil during that 0.012 s interval? Show your working.

$\varepsilon = ?$
N = 120
$\Delta\Phi = 3.0 \times 10^{-4}$ Wb
$\Delta t = 0.012$ s

$$\varepsilon_{avg} = -\frac{N\Delta\varphi_B}{\Delta t}$$

$$\varepsilon_{avg} = \frac{120 \times 3.0 \times 10^{-4}}{0.012}$$

= 3.0 V

3.0 V

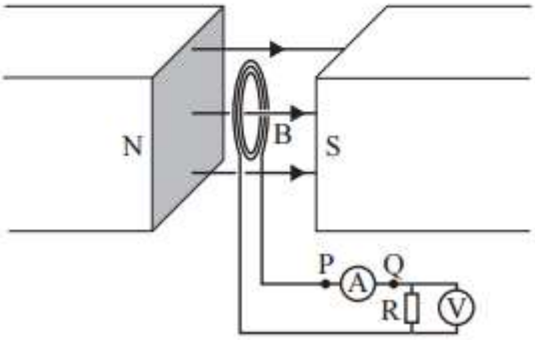


Figure 3 (repeated)

Question 6

As the field is being reduced, in what direction (P → Q or Q → P) will the current flow through the ammeter A in Figure 3 above? Explain your answer.

Direction Q to P

The magnetic field to the right is decreasing, so the induced magnetic field is to the right (1) and using the right hand grip rule, the induced current will be from Q to P

Use the following information to answer Questions 7 – 9.

A small bar magnet is moved through a circular wire loop, as shown in Figure 5. The magnet moves with constant speed through the centre of the loop, in the direction shown by the arrow. An emf is generated in the wire loop. The wire loop is connected to an oscilloscope, as shown in Figure 5.

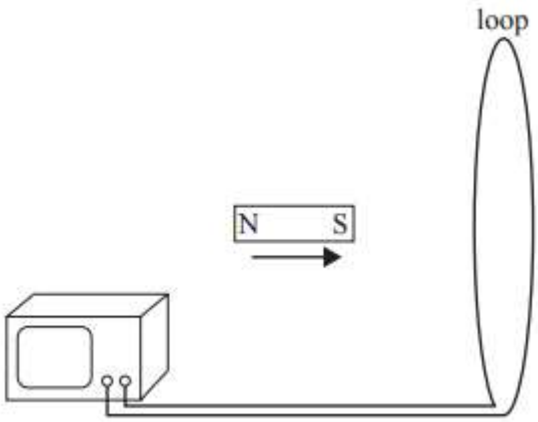


Figure 5

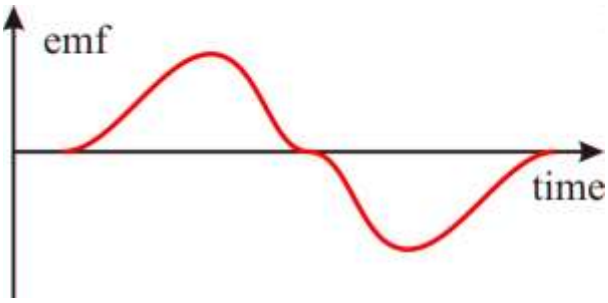
Question 7

Explain why an emf is generated in the wire loop.

The moving magnet produces a changing magnetic flux in the loop (1) which induces an EMF

Question 8

On the graph axes below, sketch the variation of the emf with time, from when the magnet is a long way to the left of the loop, through the loop, to when it is a long way to the right of the loop. Note that you can take either upwards or downwards as positive.



Question 9

After the magnet has passed through the wire loop, and is moving away from the loop, current flows around the loop in an **anticlockwise** direction, as viewed from the left of in Figure 5.
Use Lenz's law to explain why the current flows in an **anticlockwise** direction.

As the magnet moves away to the right, the magnetic flux in the loop is pointing to the left and decreasing (1). The induced EMF in the loop acts to oppose the change in the magnetic flux (1), that is, in this case its magnetic effect will be to the left (1). For the coil to produce a magnetic field to the left, the current would need to flow in a counter clockwise direction as viewed from the left.

Students are using an AC generator with a coil that rotates at a steady speed in a uniform magnetic field. The generator is producing a sinusoidal voltage signal that varies with time, as shown in Figure 6.

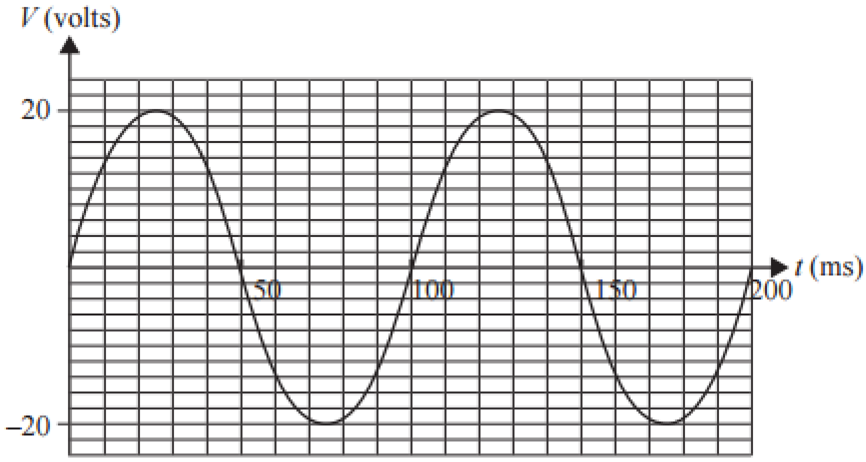


Figure 6

Question 10

Calculate the frequency of the AC signal shown in Figure 6.

Period = 100 ms, so the frequency = 1/(100 × 10^{-3}) (1) = 10 Hz. (1)

10 Hz

Question 11

Calculate the RMS voltage of the signal shown in Figure 6.

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$

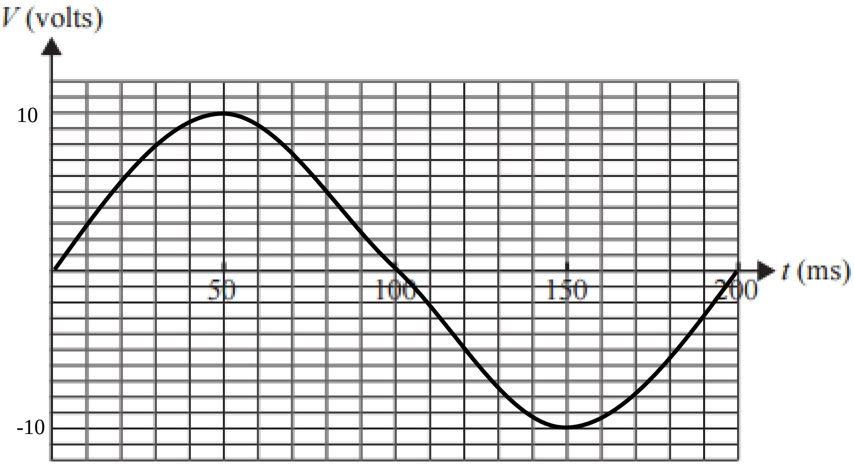
$$V_{RMS} = \frac{20}{\sqrt{2}}$$

= <u>14 Volts</u>

14 V

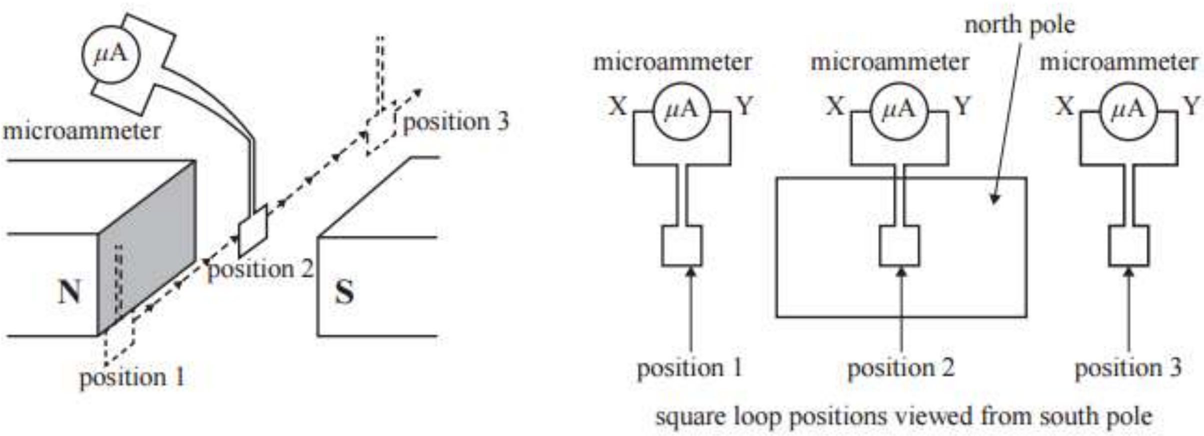
Question 12

The students halve the speed of rotation of the generator. On the axes below, sketch the voltage signal that the AC generator will now produce. Label the voltage axis carefully with the correct numbers.



A sine curve with a period of 200 ms (1) with a peak voltage of 10 V (1) on a graph with scale. (1) Halving the speed doubles the period, but also halves the peak voltage as the same change in flux occurred in twice the time.

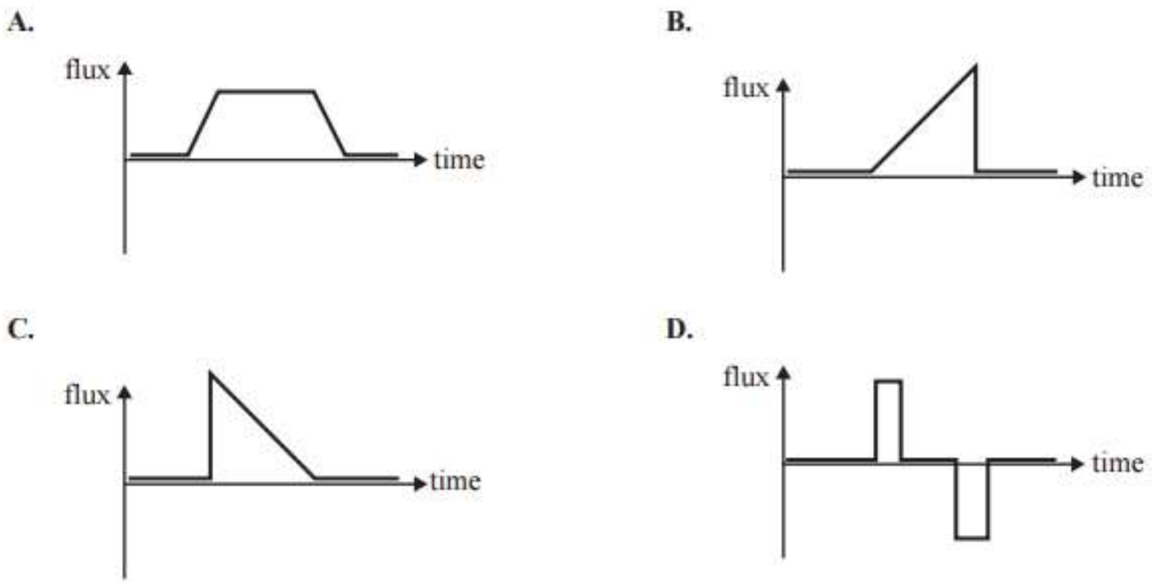
Figures 7a and 7b show a square loop being moved between the poles of a magnet. In the space between the poles there is a uniform magnetic field. The loop moves at a steady speed from position 1 to position 3. The loop is connected to a sensitive microammeter. The area of the loop is much less than the area of the magnetic field. You may assume that the only magnetic field present is located directly between the north and south poles.



Figures 7a & 7b

Question 13

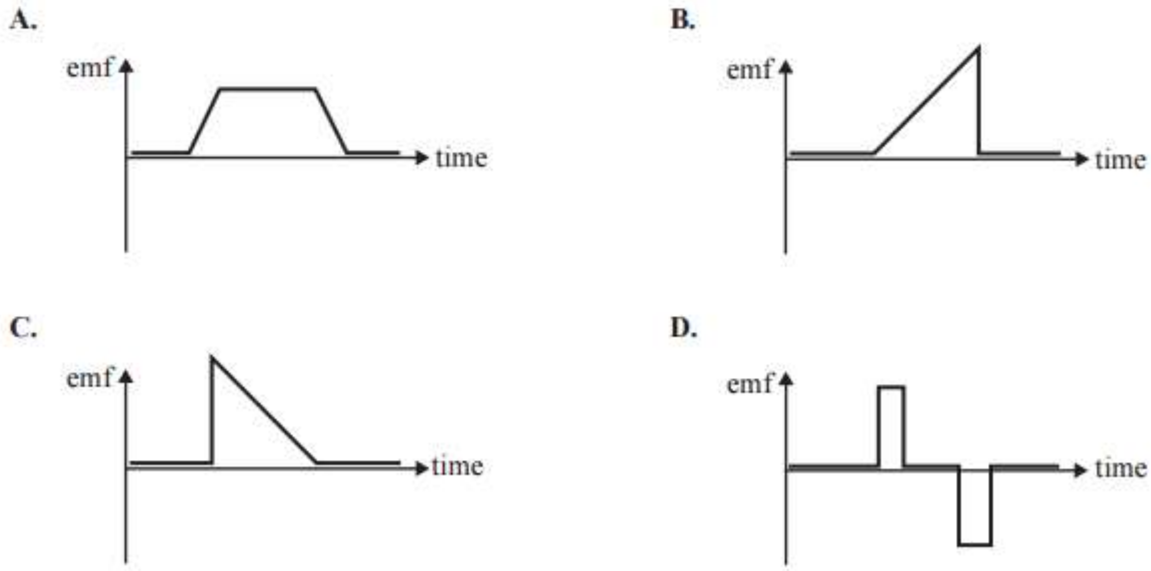
Which of the following graphs best shows how the flux through the square loop varies with time as it moves from position 1 through to position 3? Write your answer in the box provided.



A

Flux increases steadily in going from position 1 to 2, constant while in position 2 and decreases steadily when going from position 2 to position 3

Question 14
Which of the following graphs best shows how the magnitude of the emf in the square loop varies with time as it moves from position 1 through to position 3? Write your answer in the box provided.



D

The magnitude of the induced emf is given by the gradient of the flux / time graph. He steady increases and decreases mentioned in 7a produce a constant emf with opposite polarities.

Question 15
Describe the direction of the current in the square loop as it moves from position 2 to position 3, as viewed from the South pole (see Figure 7b). You may use a sketch in your answer. Explain the reasons for your answer.

Anti-clockwise (1) As viewed from the South pole, the magnetic flux is outwards and decreasing, (1) so there will be an induced current in the loop such that its magnetic affect opposes the change, that is, the magnetic field of the induced current will point to the South Pole. (1) To achieve this, the current will need to be an anti-clockwise current or from Y to X through the meter.