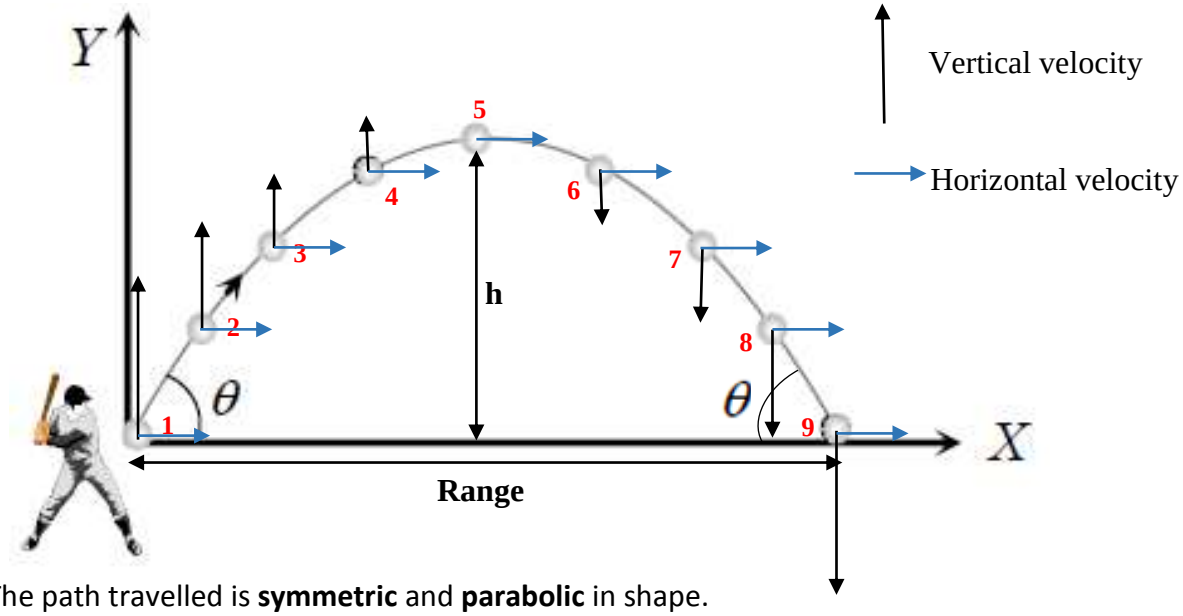
Section 3.3.5 – Oblique Projectile Motion

Oblique Projectile Motion

Scenario: Consider a baseball being hit in a parabolic path

NB: This example uses g of 9.8 ms^{-2}, takes the downwards direction as negative and ignores air resistance.



NB: The path travelled is **symmetric** and **parabolic** in shape.
The initial **angle of projection** is equal to the final **angle of impact**.
Time from projection to position.5 = Time from position.5 to impact

Horizontal motion

Upon leaving the bat, there is **no horizontal force** acting upon the ball, therefore **no acceleration.**

∴ horizontal velocity (v_H) remains constant for the entire motion.

Vertical motion

From projection to position.5

Upon leaving the bat, gravity decelerates the ball at a rate of **9.8 ms^{-2} (g).**

∴vertical velocity (v_V) decreases with time

At position.5

At the highest displacement, the v_V is 0 ms^{-1}

From position.5 to impact

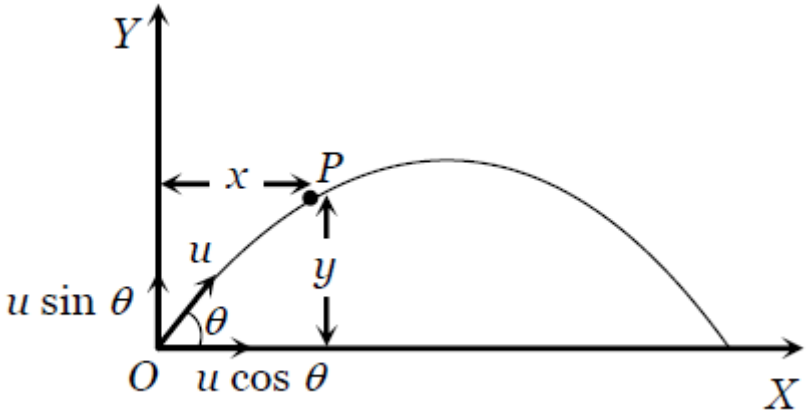
Gravity now accelerates the cannon ball at a rate of **9.8 ms^{-2} (g).**

∴v_V increases with time

NB: The horizontal and vertical components of motion **are independent of one another** and can be **treated separately**.

Resolving an initial velocity

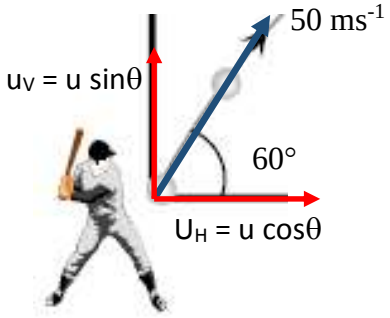
In order to make calculations upon an oblique projectile motion scenario, often the first step is to break an initial (projection) velocity into its vertical and horizontal components.



If the initial velocity of a projectile is *u*, then;

Horizontal initial velocity: $u_H = u\cos\theta$
Vertical initial velocity; $u_V = u\sin\theta$

Eg.1 If the baseball batter hits a ball with a velocity of 50 ms^{-1} at an angle of 60° to the horizontal, what is the balls vertical and horizontal initial velocity?

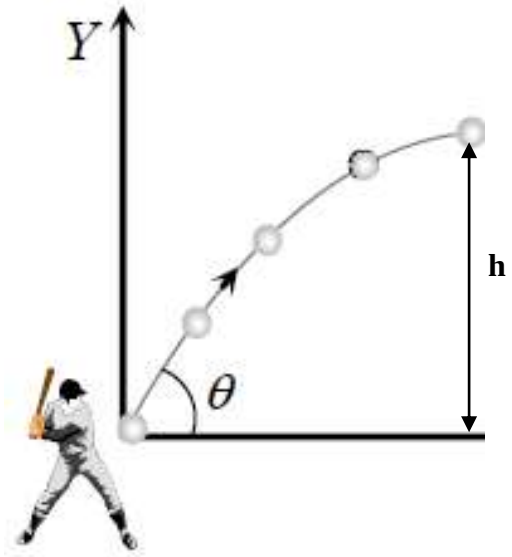


$u = 50\ ms^{-1}$
$\theta = 60°$
$u_V = ?$
$u_H = ?$

$u_V = u\sin\theta$
$= 50 \times \sin(60°)$
$= \mathbf{43.3\ ms^{-1}}$

$u_H = u\cos\theta$
$= 50 \times \cos(60°)$
$= \mathbf{25\ ms^{-1}}$

Eg.2 What is the maximum height the ball will reach?



NB: The vertical motion can be considered independently of the horizontal motion.

The ball will rise and fall vertically just like a ball hit straight up with no horizontal velocity.

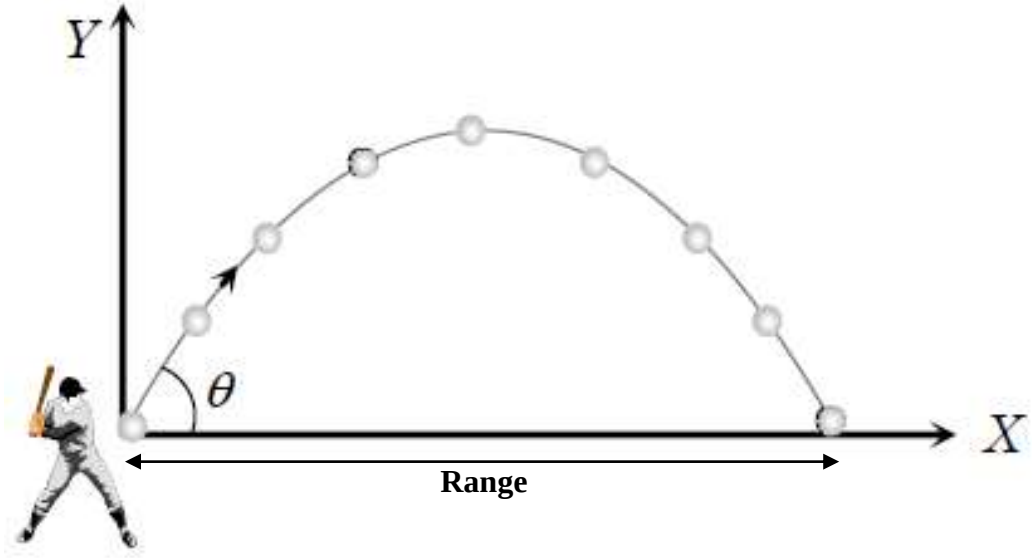
Analyse vertically (first half of motion only, to the maximum height)

s = ?
u = 43.3 ms^{-1}
a = -9.8 ms^{-2}
v = 0 ms^{-1}

take the upwards direction of motion as positive

$v^2 = u^2 + 2as$
$\therefore s = (v^2 - u^2)/2a$
$s = (0^2 - 43.3^2)/(2 \times -9.8)$
$s =$ **95.7 m**

Eg.3 What is the range of the ball's projection?



Step.1 Analyse vertically (to calculate time of ball flight)

t = ?
u = 43.3 ms^{-1}
v = -43.3 ms^{-1}
a = -9.8 ms^{-2}

take the upwards direction of motion as positive

$v = u + at$
$\therefore t = (v - u)/a$
$t = (-43.3 - 43.3)/-9.8$
$t = (-86.6)/-9.8$
$=$ **8.84 sec**

Step.2 Analyse horizontally (to calculate the range of the ball's projection) **Check online**

d = ?
t = 8.84 sec
v = 25 ms^{-1}

$v = d/t$
$\therefore d = v\,t$
$d = 25 \times 8.84 =$ **221.0 m**

Additional oblique projectile motion equations

The following equations can also be used for oblique projectile motion, providing you know both the initial velocity (u) and the angle (θ) of projection

Time of flight, $t = \frac{\mathbf{2usin\theta}}{\mathbf{g}}$

Max height reached, $h = \frac{(\boldsymbol{usin\theta})^{\mathbf{2}}}{\mathbf{2g}}$

Horizontal range, $R = \frac{\boldsymbol{u^2 sin2\theta}}{\mathbf{g}}$

Task: Check the previous calculations using the above equations

1. Max height (h) = ?

   $h = \frac{(\boldsymbol{usin\theta})^{\mathbf{2}}}{\mathbf{2g}}$

   $= \frac{(\mathbf{50}\times \boldsymbol{sin}\mathbf{60}^\circ)^{\mathbf{2}}}{\mathbf{2}\times\mathbf{10}}$

   = **93.75 m**

2. Time of flight (t) = ?

   $t = \frac{\mathbf{2usin\theta}}{\mathbf{g}}$

   $= \frac{\mathbf{2}\times\mathbf{50sin60}}{\mathbf{10}}$

   = **8.66 sec**

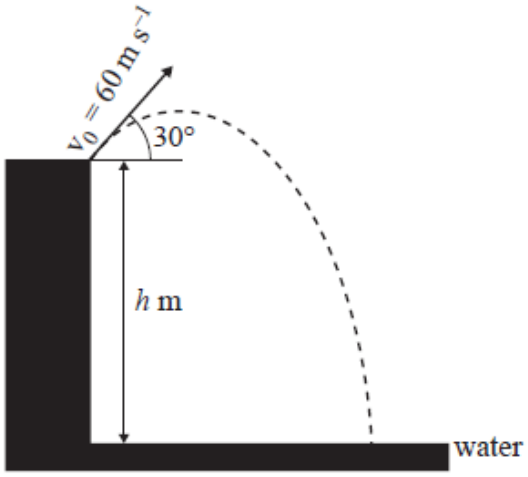
3. Horizontal range, $R = \frac{\boldsymbol{u^2 sin2\theta}}{\mathbf{g}}$

   $= \frac{\mathbf{50^2 sin}(\mathbf{2}\times\mathbf{60})}{\mathbf{9.8}}$

   = **220.9 m**

Exam Style Questions

Max hits a ball from the edge of a cliff. The ball has an initial speed of 60 ms^{-1} at an angle of 30° to the horizontal as shown below. Ignore the effects of air resistance.



Question 1

Calculate the height above the top of the cliff that the ball rises to.
(Consider the 1st half of the parabolic motion to max height)

Analyse vertically

u = 60 x sin(30°)
= 30 ms^{-1}
a = -10 ms^{-2}
v = 0 ms^{-1}
s = ?

$v^2 = u^2 + 2as$
$\therefore s = (v^2 - u^2)/2a$
$= (0^2 - 30^2)/(2 \times -9.8)$
$= -900/-19.6$
= **45.9 m** (above the cliff)

45.9 m

Question 2

The ball takes 9.0 s from the time that Max hits it until it strikes the water.
Calculate the height *h* of the cliff.

Analyse vertically

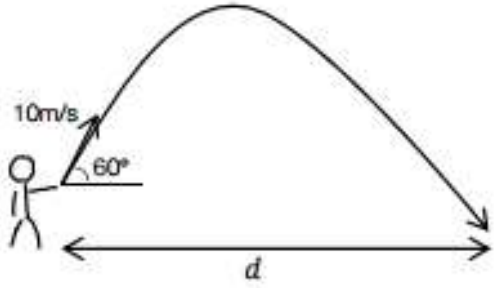
s = ?
u = 60 x sin(30°) = 30 ms^{-1}
a = -9.8 ms^{-2}
t = 9 sec

$s = ut + \frac{1}{2}at^2$
$= (30 \times 9) + (\frac{1}{2} \times -9.8 \times 9^2)$
$= 270 + (-396.9)$
$= -126.9$m
= **126.9 m** (below the height of the cliff)

126.9 m

The following information relates to questions 3 to 7

The string holding a yo-yo breaks and the yo-yo flies off at 10 meters per second at an angle of 60 degrees to the ground, as illustrated in the diagram below. Air resistance can be ignored. The experimenter is 1.6 m tall.



Question 3

State whether the following quantities are increasing, remain constant, or are constant and zero for the duration of the yo-yo's motion.

a. The horizontal component of the yo-yo's velocity

Remains constant

b. The vertical component of the yo-yo's acceleration

Remains constant

Question 4

What is the maximum height reached by the yo-yo in motion?

Analyse vertically, 1st half of motion

$u_{ver} = u\sin\theta$

$= 10 \times \sin(60)$

$= 8.66\ ms^{-1}$

$a = -9.8\ ms^{-1}$

$v = 0\ ms^{-1}$

$s = ?$

$v^2 = u^2 + 2as$

$0 = 8.66^2 + 2.(-9.8).s$

$\therefore s = \frac{-(8.66^2)}{2 \times -9.8}$

$= \underline{\underline{3.83\ m}}$

5.43 m

∴ the maximum height reached (from the ground) is 3.83 m + 1.6 m (height of the experiment) = ***5.43 m***

Question 5
What is the object's velocity the moment before it hits the ground?

First using symmetry properties we know that on the way down when it is 1.6 m above the ground it will also be travelling at 10 m/s at an angle of 60° to the horizontal. Therefore, we can now calculate the magnitude of the final vertical velocity.

Analyse vertically
v = ?
a = -9.8 ms^{-2}
u = 8.66 ms^{-1}
s = -1.6 m

$v^2 = u^2 + 2as$
$\therefore v = \sqrt{u^2 + 2as}$
$= \sqrt{8.66^2 + (2 \times -9.8 \times -1.6)}$
= 10.31 ms^{-1}

Next we add the horizontal velocity using vector addition

Analyse vertically
v = ?
$v = u\cos\theta$
= 10cos(60)
= 5 ms^{-1}

Velocity final^2 = Velocity Vert^2 + Velocity Hor^2
$v^2 = 10.31^2 + 5^2$
$v^2 = 131.3$
$\therefore v = \sqrt{131.3}$
*v = **11.46 ms^{-1}***

11.46 ms^{-1}

Question 6
What is the value of ***d*** (the range of the yo-yo)?

To find the value of the range you need to figure out the time for the total journey.
First we calculate the time it takes to rise and then fall back to the same height, t_1

Analyse vertically, 1st half of motion
u = 8.66 ms^{-1}
v = 0 ms^{-1}
a = -9.8 ms^{-2}
t = ?

v = u + at
0 = 8.66 + (-9.8 x t)
$\therefore t = \frac{-8.66}{-9.8}$ = 0.884sec $\therefore t_1$ = 2 x 0.884 = 1.768 sec

Then we calculate the time it takes to reach the ground, t_2

Analyse vertically
$v = 10.34\ ms^{-1}$
$a = 10\ ms^{-2}$
$u = 8.66\ ms^{-1}$
$t = ?$

$v = u + at$
$10.34 = 8.66 + 9.8t$
$\therefore 9.8t = 10.34 - 8.66$
$9.8t = 1.68$
$\therefore t = \frac{1.68}{9.8} = 0.171sec \ \therefore t_2 = 0.171\ sec$

$\therefore t_{total} = t_1 + t_2$
$= 1.768 + 0.171$
$= 1.939\ sec$

Next we calculate the range

Analyse horizontally
$v = 5\ ms^{-1}$
$t = 1.939\ sec$
$x = ?$

$x = vt$
$= 5 \times 1.939$
$=$ ***<u>9.695 m</u>***

The range of the yo-yo is 9.5 m.

9.695 m

Question 7
On the diagram on the previous page, draw a dotted line to indicate how the trajectory of the projectile would be different if air resistance was not ignored.

- *Correct general shape of line, fitting inside the curve of the original trajectory*
- *Shorter range*