Section 2.1.9 – Work & Energy (Part 1)

Work

In Physics terms, work is done when a **force is applied** to an object causing it to **displace** in the **direction of the force**.

$W = Fscos\theta$

Where ***W*** represents Work (Joules)
F represents Force (N)
s represents displacement (m)
θ represents the angle between the force and displacement (°)

NB: 1 Joule = 1Nm

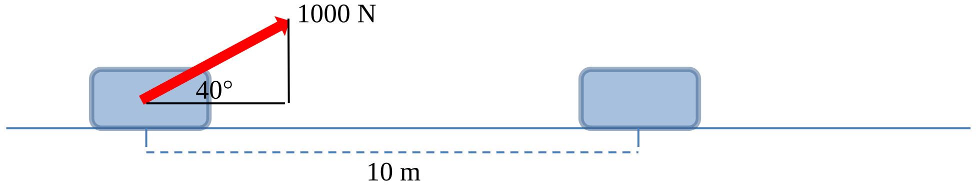
Example.1
Calculate the work done upon an object when a force of 200N is applied, causing it to displace 5m.

$W = ?$
$F = 200\ N$
$s = 5\ m$

$W = Fs$
$= 200 \times 5$
$= \mathbf{1000\ J}$

NB: As both the force and displacement are parallel, cos(0°) = 1

Example.2
Calculate the work done upon a box that is pulled by a 1000 N force at an angle of 40° from the horizontal, given it is displaced by 10 m.



$W = ?$
$F = 1000\ N$
$s = 10\ m$
$\theta = 40°$

$W = Fscos\theta$
$= 1000 \times 10 \times cos(40°)$
$= \mathbf{7660\ J\ or\ 7.66\ kJ}$

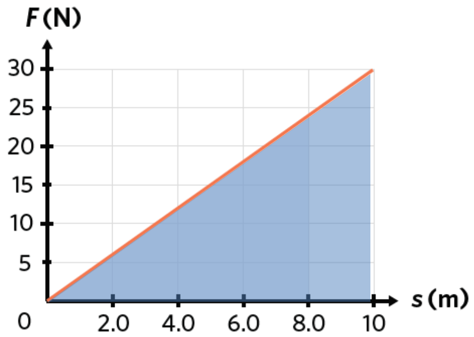
NB: As force and displacement are not parallel, cos(θ°) must be included

Force – Displacement Graph

Not every system involves a constant force being applied upon an object resulting in a displacement. In fact, often the applied force varies as an object is displaced. In which case the work done upon an object can be found via a force – displacement graph.

$$W = Area\ under\ graph$$

Where $\boldsymbol{W}$ represents Work (Joules)



Example.1

Calculate the work done on the object displayed in the above graph as it is displaced from 0 to 10.0 m.

$W = ?$
$W = \ Area\ under\ graph$
$= \frac{1}{2} BH\ [Area\ of\ a\ triangle]$
$= \frac{1}{2} \times 10.0 \times 30$
$= \mathbf{150\, J}$

Example.2

A stationary object has a varying force applied to it as shown in the below force – displacement graph. Calculate the work done on the object displayed in the above graph as it is displaced from 0 to 10.0 m.

$W = ?$
$W = \ Area\ under\ graph$
$= \frac{1}{2}(a + b)h\ \ [Area\ of\ a\ trapezium]$
$= \frac{1}{2} \times (2 + 10) \times 6$
$= \mathbf{36\, J}$

Kinetic Energy

Kinetic energy is the energy associated with an object's **motion**.

$$E_k = \frac{1}{2}mv^2$$

Where $\boldsymbol{E_k}$ represents the kinetic energy (J)
$\boldsymbol{m}$ represents the object's mass (kg)
$\boldsymbol{v}$ represents the object's speed (ms^{-1})

Example.1
Calculate the kinetic energy of the Falcon 9 rocket of mass 549 tonne at a top speed of 9.31×10^3 ms^{-1} upon launch.

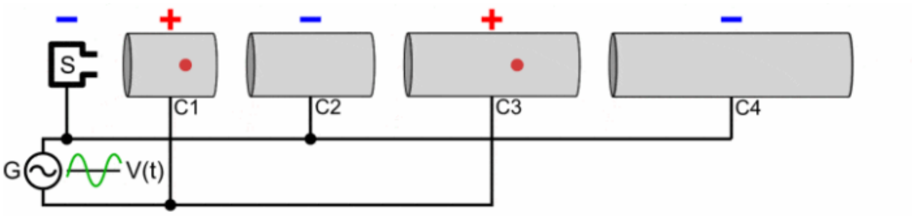
$E_k = ?$
$m = 549 \times 10^3\ kg$
$v = 9.31 \times 10^3\ ms^{-1}$

$$Ek = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 549 \times 10^3 \times (\mathbf{9.31 \times 10^3})^2$$
$$= \mathbf{2.38 \times 10^{13}\ J}$$

Example.2
Calculate the kinetic energy of an electron travelling at one tenth the speed of light.
NB: For this example we are ignoring relativistic mass.

$E_k = ?$
$m_e = 9.1 \times 10^{-31}\ kg$
$v = 3.0 \times 10^7\ ms^{-1}$

$$Ek = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (3.0 \times 10^7)^2$$
$$= \mathbf{4.1 \times 10^{-16}\ J}$$



A linear accelerator – LINAC

Example.3
Calculate the work done by friction to stop a 900 kg car rolling initially at 10 ms^{-1}.

$W = ?$
$m = 900\ kg$
$v = 10\ ms^{-1}$

$$W_{fric} = \Delta Ek$$
$$= Ek(f) - Ek(i)$$
$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= \frac{1}{2}m(v^2 - u^2)$$
$$= \frac{1}{2} \times 900 \times (0^2 - 10^2)$$
$$= \mathbf{-45\ 000\ J}$$

NB: The neg. sign shows the net force is in the opposite direction to that of the displacement.

Gravitational Potential Energy

Gravitational potential energy is the energy associated with an object's **position** in a **gravitational field.**

$$E_g = mg\Delta h$$

Where $\boldsymbol{E_g}$ represents the gravitational potential energy ($\boldsymbol{J}$)
$\boldsymbol{m}$ represents the object's mass ($\boldsymbol{kg}$)
$\boldsymbol{g}$ represents the gravitational field strength = $\mathbf{9.8}\ \boldsymbol{Nkg^{-1}}$ on Earth
$\boldsymbol{\Delta h}$ represents the change in height (elevation) ($\boldsymbol{m}$)

Example.1

Calculate the gravitational potential energy of a 100 kg man who climbs a 5 m tall ladder.

$E_g = ?$
$m = 100\ kg$
$g = 9.8\ Nkg^{-1}$
$\Delta h = 5\ m$

$$E_g = mg\Delta h$$
$$= 100 \times 9.8 \times 5$$
$$= \mathbf{4900\ J}$$

Example.2

A pick-up truck is hoisted a height of 1.5 m above the ground in order for it to be serviced. Given it gains 36 750 J of gravitational potential energy in the process, what is the mass of the pick-up?

$m = ?$
$E_g = 36\ 750\ J$
$g = 9.8\ Nkg^{-1}$
$\Delta h = 1.5\ m$

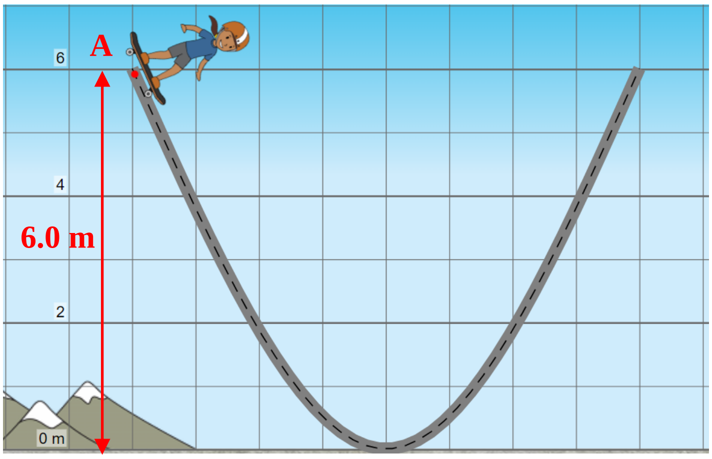
$$E_g = mg\Delta h$$
$$\therefore m = \frac{E_g}{g\Delta h}$$
$$= \frac{36\ 750}{9.8 \times 1.5}$$
$$= \mathbf{2500\ kg}$$

Conservation of Energy

A fundamental law of Physics is that **energy is always conserved** in a **closed or isolated system**. That is, it **cannot be created or destroyed**. Rather is can be converted into another energy form.

Scenario 1: Gravitational Potential Energy ↔ Kinetic Energy Conversion

Consider the following scenario: A skater of mass 60 kg is elevate a height of 6.0 m above the ground on a skateboard track, as shown below. Assume there is no friction on the track, and that the skater is initial stationary.



Question. 1
What is the total energy of the skater at the top of the track @ Point A?

$$\Sigma E_A = ?$$
$$\Sigma E_A = E_g(A) + E_k(A)$$
$$= mg\Delta h + \frac{1}{2}mv^2$$
$$= 60 \times 9.8 \times 6.0 + \frac{1}{2} \times 60 \times 0^2$$
$$= \mathbf{3528\ J}$$

NB: @A $E_k = 0\ J$

Question. 2
What is the total energy of the skater at the bottom of the track @ Point B?

$$\Sigma E_B = ?$$
$$\Sigma E_B = \Sigma E_A$$
$$= \mathbf{3528\ J}$$

NB: ΣE remains constant at all locations

Question. 3
What is the speed of the skater at the bottom of the track @ Point B?

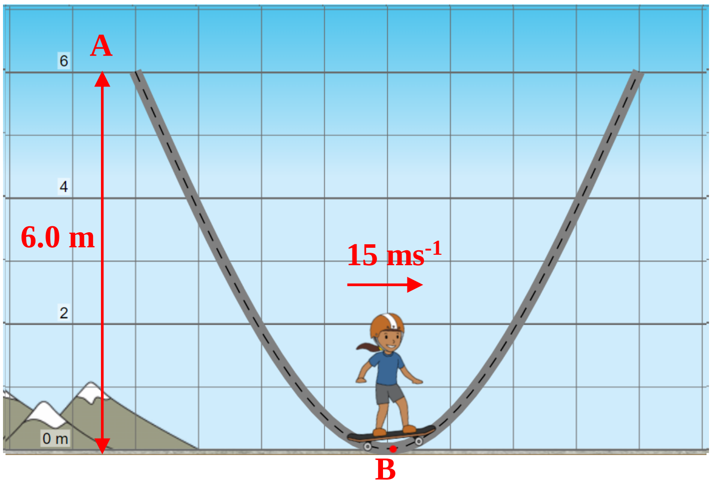
$v_B = ?$
$\Sigma E_B = 3528\ J$
$m = 60\ kg$

$$\Sigma E_B = E_g(B) + E_k(B)$$
$$3528 = 0 + \frac{1}{2}mv^2$$
$$\therefore v = \sqrt{\frac{3528 \times 2}{60}} = \mathbf{10.8\ ms^{-1}}$$

NB: @B $E_g = 0\ J$

Scenario 2: Gravitational Potential Energy ↔ Kinetic Energy Conversion

Consider the following scenario: A skater of mass 60 kg is elevate a height of 6.0 m above the ground on a skateboard track, and skates to the bottom of the track as shown below.
Assume there is no friction on the track, and that the skater has a speed of 15 ms^{-1} at the bottom of the track (Point B).



Question. 1
What is the total energy of the skater at the bottom of the track @ Point B?

$\Sigma E_B = ?$

NB: @B $E_g = 0\,J$

$$\Sigma E_B = E_g(B) + E_k(B)$$
$$= mg\Delta h + \frac{1}{2}mv^2$$
$$= 60 \times 9.8 \times 0 + \frac{1}{2} \times 60 \times 15^2$$
$$= \mathbf{6750\,J}$$

Question. 2
What is the total energy of the skater at the top of the track @ Point A?

$\Sigma E_A = ?$

NB: ΣE remains constant at all locations

$$\Sigma E_A = \Sigma E_B$$
$$= \mathbf{6750\,J}$$

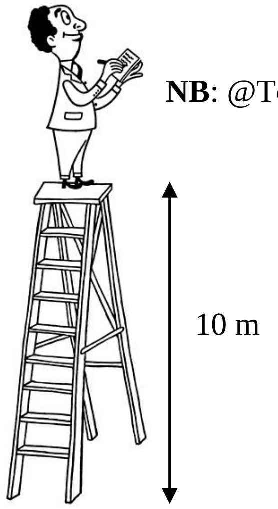
Question. 3
What was the initial speed of the skater at the top of the track @ Point A?

$v_A = ?$
$\Sigma E_A = 6750\,J$
$m = 60\,kg$

$$\Sigma E_A = E_g(A) + E_k(A)$$
$$6750 = mg\Delta h + \frac{1}{2}mv^2$$
$$6750 = 60 \times 9.8 \times 6.0 + \frac{1}{2} \times 60 \times v^2$$
$$6750 = 3528 + 30 \times v^2$$
$$\therefore v = \sqrt{\frac{6750 - 3528}{30}} = \mathbf{10.4\,ms^{-1}}$$

Scenario 3: Gravitational Potential Energy ↔ Kinetic Energy Conversion

Consider the following scenario: A100 kg man falls from the top of a 10 m tall ladder.



Question. 1
What is the total energy of the man at the top of the ladder?

NB: @Top $E_k = 0\,J$

$\Sigma E_{top} = ?$
$\Sigma E_{top} = E_g(top) + E_k(top)$
$= mg\Delta h + \frac{1}{2}mv^2$
$= 100 \times 9.8 \times 10 + \frac{1}{2} \times 100 \times 0^2$
$= \mathbf{9800\,J}$

Question. 2
What is the total energy of the man at the ground following his fall?

$\Sigma E_{ground} = ?$
$\Sigma E_{ground} = \Sigma E_{top}$
$= \mathbf{9800\,J}$

NB: ΣE remains constant at all locations

Question. 3
What is the "impact speed" of the man upon hitting the ground?

$v_{ground} = ?$
$\Sigma E_{ground} = 9800\,J$
$m = 100\,kg$

$\Sigma E_{ground} = E_g(Ground) + E_k(Ground)$

NB: @Ground $E_g = 0\,J$

$9800 = 0 + \frac{1}{2}mv^2$
$9800 = 0 + \frac{1}{2} \times 100 \times v^2$

$$\therefore v = \sqrt{\frac{9800 \times 2}{100}} = \mathbf{14\,ms^{-1}}$$

Option: Use the "suvat" equations to solve Question 3.

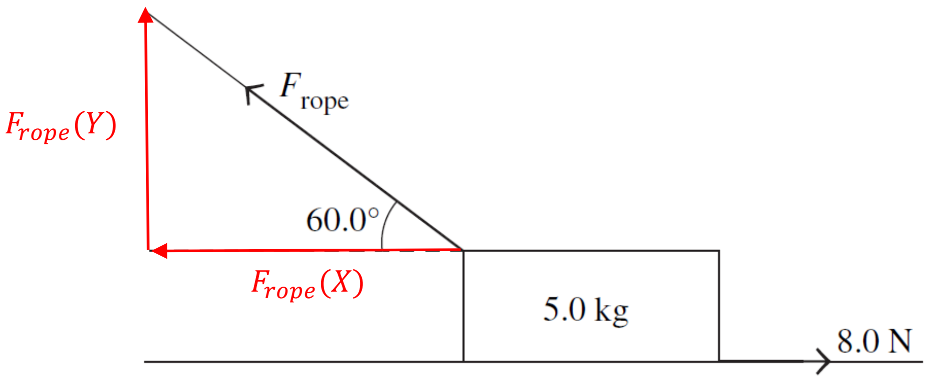
$v = ?$
$u = 0\,ms^{-1}$
$a = 9.8\,ms^{-2}$
$s = 10\,m$

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2 \times 9.8 \times 10$
$v^2 = 196$
$\therefore v = \sqrt{196}$
$= \mathbf{14\,ms^{-1}}$

Exam Styled Questions

Questions 1 & 2 refer to the following information

The figure below shows a box of mass 5 kg being pulled along at a constant velocity by a rope held at an angle of 60° to the horizontal.



Question 1
What force must be acting on the box along the rope if the frictional force on the box is 8.0 N to the right?

$F_{rope} = ?$
$F_{net} = 0$
$\therefore F(\leftarrow) = F(\rightarrow)$
$\therefore F_{rope}(X) = F_f$
$\therefore F\cos(\theta) = F_f$

$$F\cos(\theta) = F_f$$
$$\therefore F = \frac{F_f}{\cos(\theta)} = \frac{8.0}{\cos(60)} = 16.0\ N$$

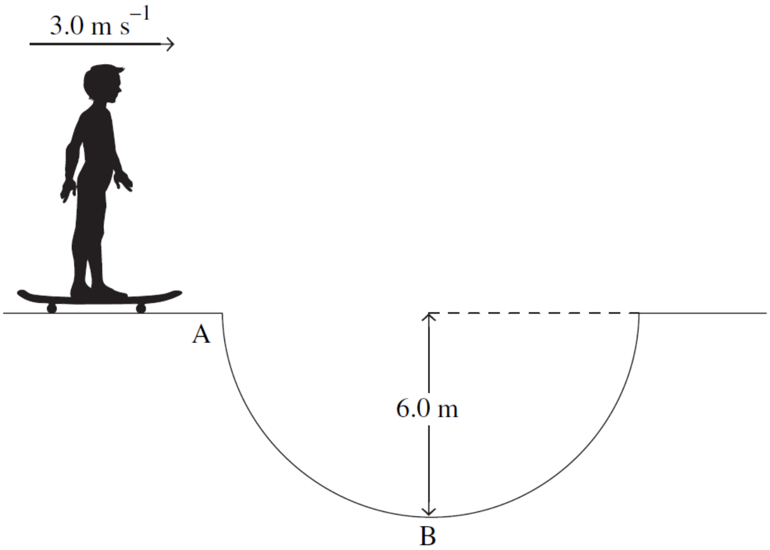
16.0 N

Question 2
What is the work done against friction if the box is dragged 5.0 m?

$W = ?$
$W = F_f s$
$= 8.0 \times 5.0$
$= 40.0\ J$

40.0 J

A boy and a skateboard have a total mass of 60.0 kg. The boy is moving on the skateboard at a constant speed of 3.0 ms^{-1} along a horizontal path. He then travels through a dip in the path. The dip is an arc of a circle with a radius of 6.0 m, as shown below.



Question 3
What is the speed of the boy and the skateboard at point B?

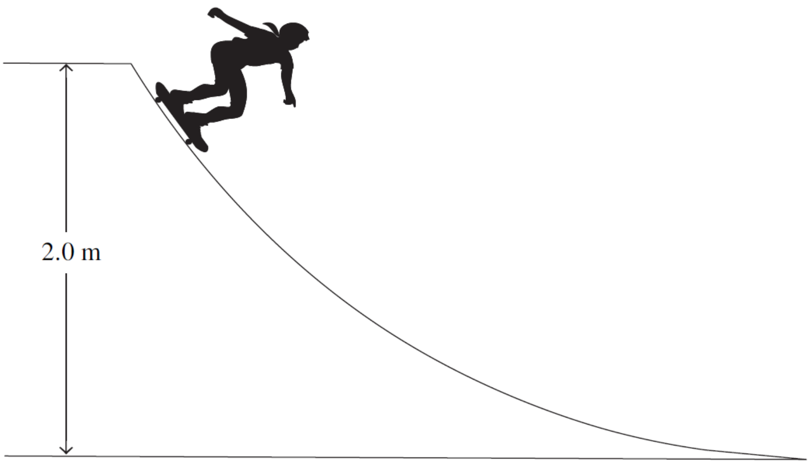
$$v_B = ?$$
$$\Sigma E_A = \Sigma E_B$$
$$E_g(A) + E_k(A) = E_g(B) + E_k(B)$$
$$mg\Delta h + \frac{1}{2}mv^2 = 0 + \frac{1}{2}mv^2$$
$$g\Delta h + \frac{1}{2}v^2 = \frac{1}{2}v^2$$

$$9.8 \times 6 + \frac{1}{2} \times 3^2 = \frac{1}{2} \times v^2$$
$$63.3 = \frac{1}{2} \times v^2$$
$$v = \sqrt{2 \times 63.3}$$
$$= 11.25\ ms^{-1}$$

11.25 ms^{-1}

Questions 4 & 5 refer to the following information

Zelda is skateboarding at a skate park. She rolls from rest down a curved ramp with a vertical drop of 2.0 m, as shown below.



Question 4
Calculate Zelda's speed at the bottom of the curved ramp. Ignore the effects of friction.
Show your working.

$V_{bottom} = ?$
$\Delta E_g = \Delta E_k$
$mg\Delta h = \frac{1}{2}mv^2$
$g\Delta h = \frac{1}{2}v^2$
$v = \sqrt{2g\Delta h}$

$v = \sqrt{2 \times 9.8 \times 2.0}$
$= 6.26\ ms^{-1}$

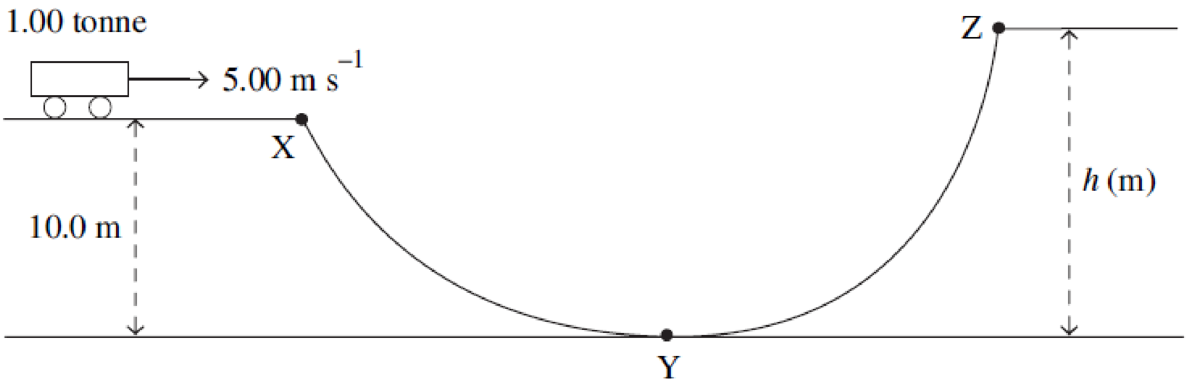
6.26 ms^{-1}

Question 5
If friction were significant, explain how the result from **Question 4** would change.

- *If friction were significant, it would do work against the motion and some useful mechanical energy would be converted to heat.*
- *This would mean that less energy would be available as kinetic energy.*
- *So Zelda would reach the bottom of the curved ramp at a lower speed.*

Questions 6 - 8 refer to the following information

A roller coaster cart with a mass of 1.00 tonne is moving along a horizontal section of the track at a speed of 5.00 ms^{-1}, as shown below.



Point X is at the edge of the horizontal section of the track and has a height of 10.0 m. Point Y is the lowest point of the track. The track is designed so that the roller coaster cart will come to rest at point Z. Ignore the effects of friction.

Question 6
What is the kinetic energy of the roller coaster cart at point X?
Show your working.

$$E_k = ?$$
$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 1000 \times 5.00^2$$
$$= 12500\, J \text{ } or \text{ } 1.25 \times 10^4\, J$$

$\mathbf{1.25 \times 10^4}$ **J**

Question 7
What is the speed of the cart at point Y?
Show your working.

$$v_Y = ?$$
$$\Sigma E_X = \Sigma E_Y$$
$$E_g(X) + E_k(X) = E_g(Y) + E_k(Y)$$
$$mg\Delta h + \frac{1}{2}mv^2 = 0 + \frac{1}{2}mv^2$$
$$g\Delta h + \frac{1}{2}v^2 = \frac{1}{2}v^2$$

$$9.8 \times 10 + \frac{1}{2} \times 5^2 = \frac{1}{2} \times v^2$$
$$110.5 = \frac{1}{2} \times v^2$$
$$v = \sqrt{2 \times 110.5}$$
$$= 14.87\, ms^{-1}$$

14.87 $\mathbf{ms^{-1}}$

Question 8

What is the vertical height, h, of point Z above point Y? Show your working.

$v_Y = ?$
$\Sigma E_X = \Sigma E_Z$
$E_g(X) + E_k(X) = E_g(Z) + E_k(Z)$
$mg\Delta h + \frac{1}{2}mv^2 = mg\Delta h + 0$
$g\Delta h + \frac{1}{2}v^2 = g\Delta h$
$9.8 \times 10 + \frac{1}{2} \times 5.00^2 = 9.8 \times \Delta h$
$110.5 = 9.8 \times \Delta h$
$\therefore \Delta h = \frac{110.5}{9.8}$
$= 11.28\ m$

11.28 m