# Section 2.1.8 – Torque & Equilibrium

## Translational Equilibrium

Until this stage in the course, most force diagrams have been drawn originating from an objects centre of mass. This is depicted in Figure 1 below.

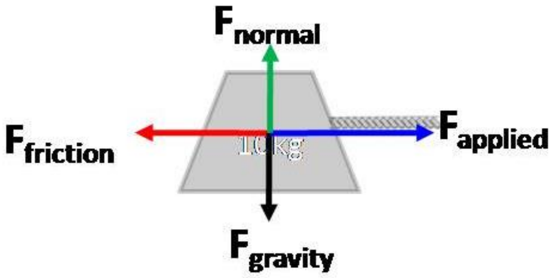



**Figure 1** – Centre of mass force diagram

In this situation the object is considered to be in **translational equilibrium** if it is not accelerating and its net force is zero.

Translational equilibrium means:

Forces right (→) = Forces left (←), &
Forces up (↑) = Forces down (↓)

## Torque (τ)

In many real life scenarios forces cannot simply be considered to operate from or upon the centre of mass. Consider the scenario shown in Figure 2 where a **force** is applied to an object at a **radius (r)** from its pivot point, or **fulcrum**.

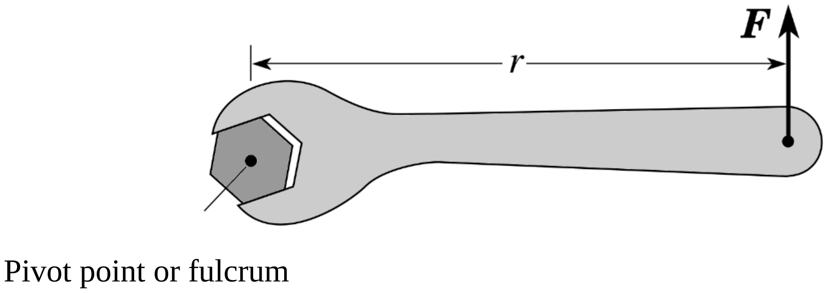



**Figure 2** – Torque generated by a spanner

The arrangement shown in Figure 2 above will clearly generate a **twisting force** or a **torque**. The magnitude of the torque generated is calculated using the following equation:

$$\tau = r_{\perp}F$$

Where $\tau$ τ represents torque (Nm)

$r_{\perp}$ represents the perpendicular distance from the applied force to the fulcrum (m)

$F$ represents the applied force (N)

## Example.1

A long armed spanner is used to undo wheel nuts on a truck. If a force of 500 N is applied perpendicular to the handle at a length of 80 cm from the fulcrum, what torque is generated? The arrangement is shown in Figure 3.

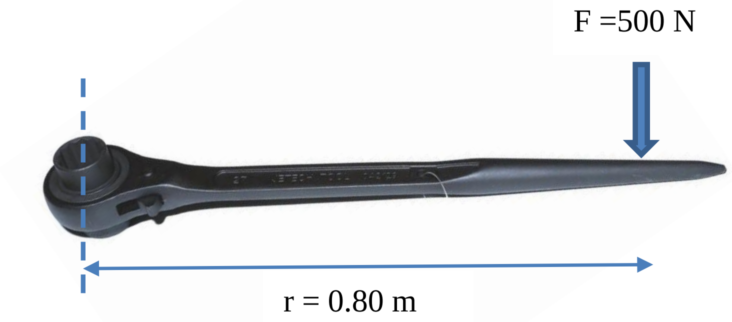



***Figure 3** – long armed spanner*

| | |
|---|---|
| $\tau = ?$ | $\tau = r_{\perp}F$ |
| $r_{\perp} = 0.80$ m | $= 0.80 \times 500$ |
| $F = 500$ N | $= 400$ Nm (↻) |

### Rotational Equilibrium

We have analysed many systems that have translational equilibrium. However, let us now consider rotational equilibrium. When a system has **rotational equilibrium**:

Torques clockwise (↻) = Torques anticlockwise (↺)

Recall as a child when you and a friend enjoyed the simple pleasures of a see-saw in the park. In the event that you and your friend were of similar mass, you could balance yourself approximately equal distance from the centre of the swing.

However, if your friend was considerably heavier or lighter than you, then it was important to locate positions on the swing that provided rotational equilibrium.

Refer to Figure 4 on the next page of notes.

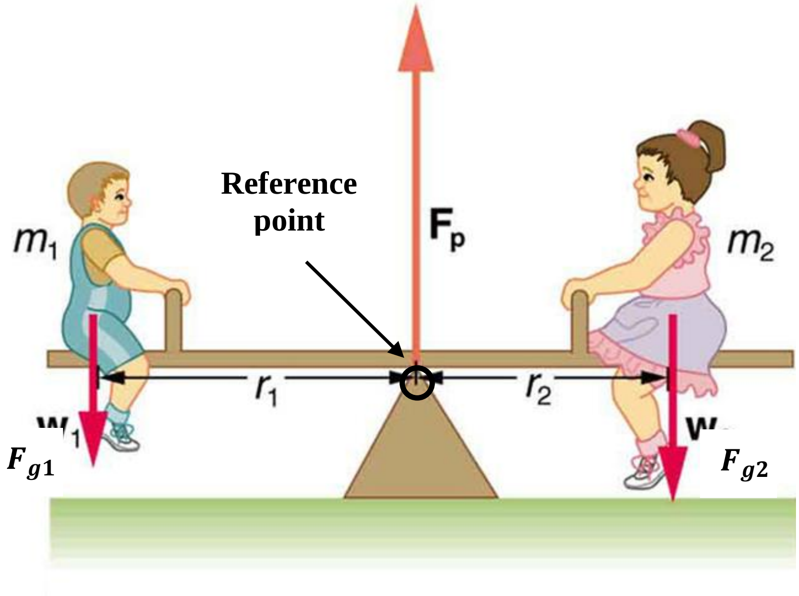



***Figure 4** – Rotational equilibrium on a see-saw*

If we consider the fulcrum of the swing (ie. the pivot point) as our reference point, then:
**NB:** The selection of the pivot point as our reference pint removes the torque generated by $F_P$

$\Sigma$ Torques (↻) = $F_{g2}r_2$
$\Sigma$ Torques (↺) = $F_{g1}r_1$

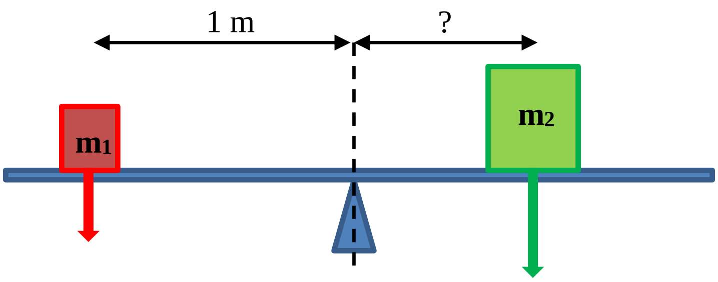
So for rotational equilibrium to exist:

$\Sigma$ Torques (↻) = $\Sigma$ Torques (↺)

$\therefore F_{g2}r_2 = F_{g1}r_1$

## Example.2

Examine Figure 4 above. If the smaller child had a mass of 30 kg and the larger child a mass of 50 kg, what distance would the larger child need to be from the centre of the see-saw if she was to balance out rotation of the smaller child located 1 m form the centre of the see-saw?




$m_1 = 30$ kg
$m_2 = 50$ kg
$r_1 = 1.0$ m
$r_2 = ?$

Rotational equilibrium (using the pivot point as the reference for the system)

$\therefore \tau(↺) = \tau(↻)$

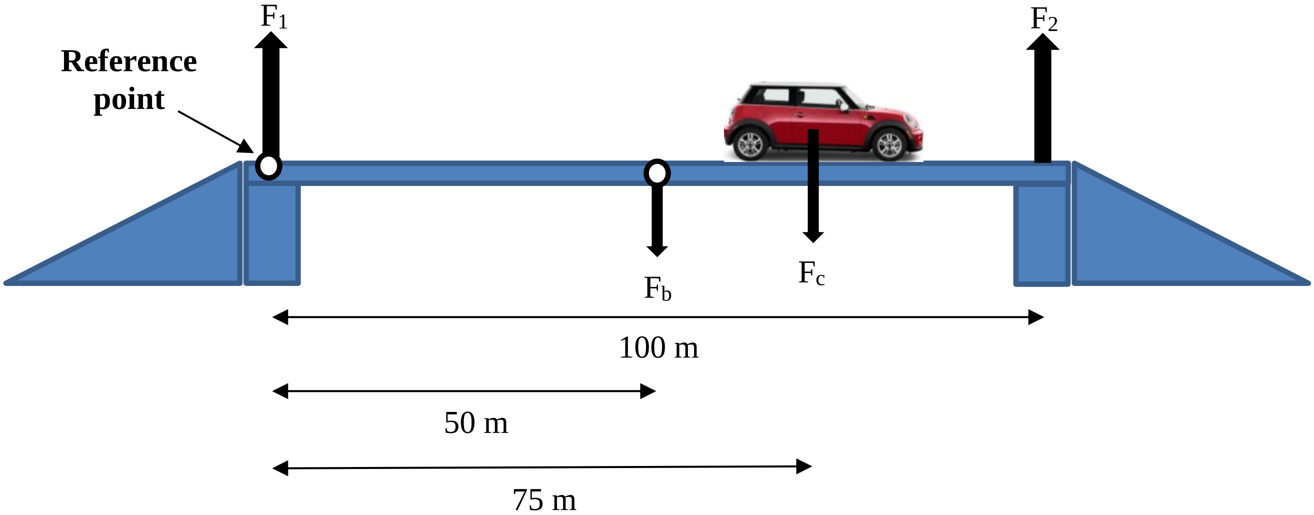
$$m_1 g r_1 = m_2 g r_2$$

$$30 \times 9.8 \times 1 = 50 \times 9.8 \times r2$$

$$\therefore r_2 = \frac{30 \times 9.8 \times 1}{50 \times 9.8} = 0.6\, m$$

## Example.3

A 75 m long bridge of mass 1 tonne is supported by two concrete bases which provide upwards forces $F_1$ and $F_2$ respectively. A 800 kg car is located 75 m from the left have concrete base. Calculate the two unknown forces $F_1$ and $F_2$.




<u>Translational equilibrium</u>

Forces up (↑) = Forces down (↓)

$\therefore F_1 + F_2 = F_b + F_c$

$F_1 + F_2 = (1000 \times 9.8) + (800 \times 9.8)$

$F_1 + F_2 = 17640$ [Equation.1]

<u>Rotational equilibrium</u>

**NB:** Take $F_1$ as the reference point

Therefore all radii are measured from the point $F_1$.

$\therefore \tau(\circlearrowleft) = \tau(\circlearrowright)$

$(F_2 \times 100) = (F_b \times 50) + (F_c \times 75)$

$(F_2 \times 100) = (m_b \times g \times 50) + (m_c \times 9.8 \times 75)$

$100F_2 = (1000 \times 9.8 \times 50) + (800 \times 9.8 \times 75)$

$100F_2 = 1078000$

$\therefore \boldsymbol{F_2} = \frac{\mathbf{1078000}}{\mathbf{100}} = 10780$ N

Substitute $F_2 = 10780$ N into Equation.1 above

$\therefore F_1 + 10780 = 17640$

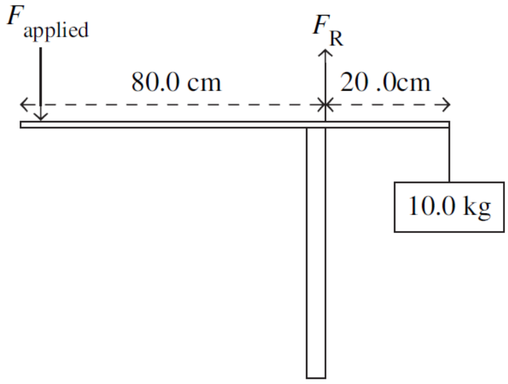
$F_1 = 6860$ N

So $F_1 = 6860$ N (↑) & $F_2 = 10780$ N (↑)

**Exam Styled Questions**

***Questions 1 & 2 refer to the following information***

A student holds a mass of 10.0 kg stationary over a fence using a metre ruler. The mass is suspended from the end of the ruler by a piece of string at a distance of 20.0 cm from the fence, as shown below. Assume that the mass of the ruler is negligible.




**Question 1**
Given that the student is standing at a distance of 80.0 cm from the fence, what is the magnitude of the force that the student must apply to the metre ruler to support the 10.0 kg mass? Show your working.

Σ Torques (↻) = Σ Torques (↺)

$$120.0 \times 9.8 \times 2 + 70 \times 9.8 \times 3.0 = F_Q \times 4.0$$

$$F_Q = 1102.5\ N$$

**Question 2**
What is the magnitude and direction of the reaction force, $F_R$, that the fence exerts on the metre ruler? Show your working.

$Taking\ up\ as\ positive\ gives$:

$\Sigma F_{vertical} = 0$

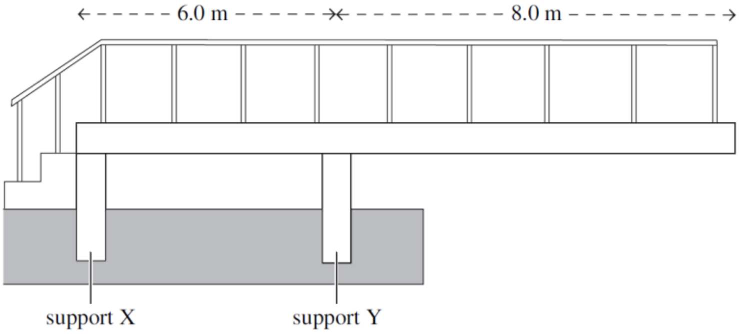
$F + 10.0 \times 9.8 - F_R = 0$

$\therefore F_R = 123\ N$

$direction = up$

**Question 3**

A viewing platform is installed at a national park. The platform can be considered as a long beam of uniform mass that extends 8.0 m beyond two vertical supports to which it is attached, as shown in the diagram below. The supports, X and Y, are 6.0 m apart.




Which one of the following statements is correct?

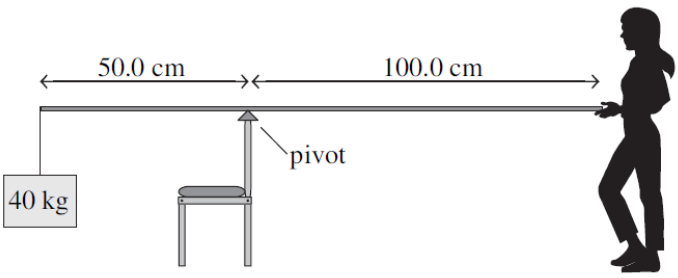
**A.** Support X is under compression, and support Y is under tension.
**B.** Support X is under tension, and support Y is under compression.
**C.** Both supports X and Y are under tension.
**D.** Both supports X and Y are under compression.

**B**

B is correct. The centre of mass of the beam is to the right of support Y, so it will have a clockwise torque due to its weight. Therefore, support X is under tension and support Y is under compression.

**Question 4**

Hannah holds a 40.0 kg hanging mass stationary with a 5.0 kg pole lever, using the back of a chair as the pivot, as shown below. The mass is suspended from the pole by a piece of string at a distance of 50 cm from the pivot. Hannah stands 100.0 cm from the pivot point.




The magnitude of the torque that Hannah must apply to keep the pole suspended is closest to

**A.** 35 N m
**B.** 184 N m
**C.** 196 N m
**D.** 392 N m

**B**

Take the pivot point as your reference point

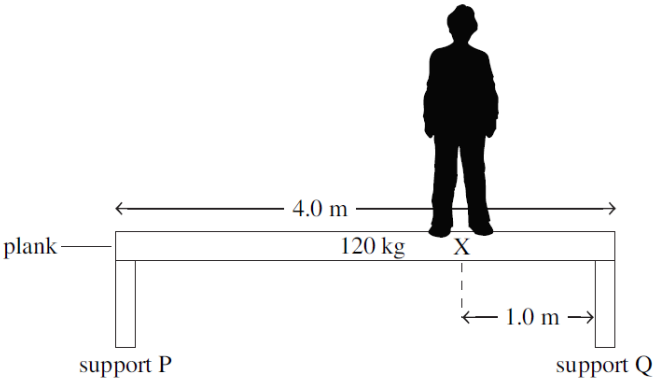
$$\tau \text{ anticlockwise } \tau = \text{clockwise}$$

$$40.0 \times 9.8 \times 0.5 = 5.0 \times 9.8 \times 0.25 + F_{\text{Hannah}} \times 1.0$$

$$F_{\text{Hannah}} = 184 \text{ N m}$$

**Question 5**

Tim stands on a 4.0 m plank of mass of 120 kg at position X, as shown in the below figure. Position X is 1.0 m from support Q. Tim has a mass of 70 kg.




Calculate the magnitude of the force exerted on the plank by support Q. Show your working.

Take Support P as your reference point

Σ Torques (↻) = Σ Torques (↺)

$$120.0 \times 9.8 \times 2 + 70 \times 9.8 \times 3.0 = F_Q \times 4.0$$

$$F_Q = 1102.5\,N$$