Section 2.1.7 – Inclined Planes and Connected Bodies

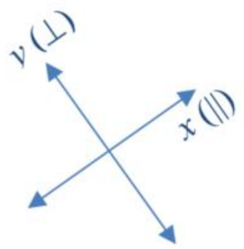
Newton's 2nd Law of Motion – Inclined Planes

Up until this point in the course forces have been examined in flat planes. Effectively we have been able to examine force experienced left, right, up and down.

Let us now examine an **inclined plane**.

When considering an inclined plane it is best to examine forces in terms of:

1. **Forces parallel** (||) to the inclined plane; &
2. **Forces perpendicular** ($\perp$) to the inclined plane



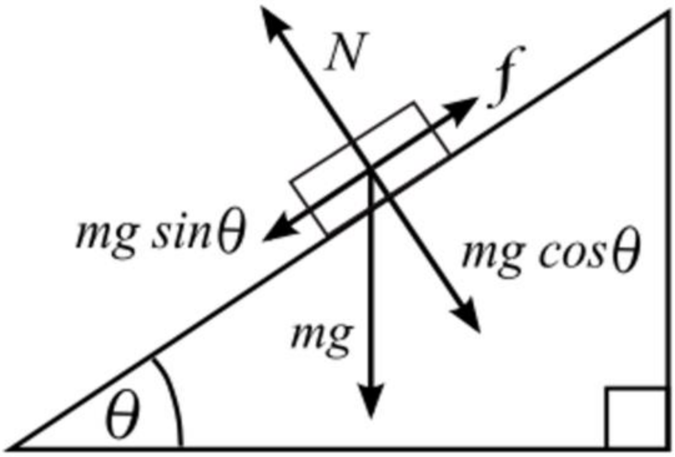


***Figure 1** - An inclined plane*

As can be seen in Figure 1 above, the gravitational force of the object on the plane (mg) can be broken into a component acting **parallel/down** the plane ($\boldsymbol{mgsin\theta}$) and another **perpendicular** to the plane ($\boldsymbol{mgcos\theta}$).

$\boxed{F_g = mg}$

$\boxed{F_{gx} = F_{||} \text{ down the plane } = mgsin\theta}$

$\boxed{F_{gy} = F_{\perp} = mgcos\theta}$

$\boxed{a = gsin\theta}$

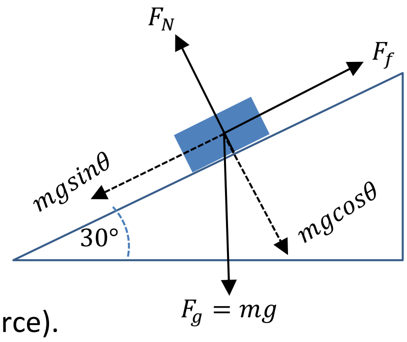
Where F_g is the force due to gravity (N)
F_{gx} is the || component of F_g down the plane
F_{gy} is the $\perp$ component of F_g to plane
a is the rate of acceleration down the incline
(for a frictionless surface)

NB: N = - $mgcos\theta$

Example.1

The object shown in Figure 1 has a mass of 100 kg and the plane is inclined at an angle of 30°.

$F_g \ = ?$
$m \ = 100\ kg$
$g = 9.8\ Nkg^{-1}$
$\theta = 30\,^{o}$



Question.1

Calculate the objects force due to gravity (ie. weight force).

$$\begin{aligned} F_g &= mg \\ &= 100 \times 9.8 \\ &= 980\ N \end{aligned}$$

Question.2

Calculate the component of the object's weight that is <u>down the slope</u> of the inclined plane.

$$\begin{aligned} F_{gx} &= F_{\parallel}\ down\ the\ plane \\ &= mgsin\theta \\ &= 100 \times 9.8 \times sin(30) \\ &= 490\ N \end{aligned}$$

Question.3

Calculate the component of the object's weight that is <u>perpendicular to the slope</u> of the inclined plane.

$$\begin{aligned} F_{gy} &= F_{\perp} \\ &= mgcos\theta \\ &= 100 \times 9.8 \times cos(30) \\ &= 849\ N \end{aligned}$$

Given the object slides down the inclined plane at a constant speed.

Question 4.

Calculate the magnitude of the friction force experienced by the object.

As the object is travelling at a constant speed, acceleration is zero (a = 0 ms^{-2}).
$\therefore F_{net} = 0\ N$

$$\begin{aligned} \therefore F_{up\ the\ plane} &= F_{down\ the\ plane} \\ \therefore F_f &= F_{gx} \\ &= 490\ N \end{aligned}$$

Example.2

The object shown in Figure 2 below represents a 40 kg object that is sliding down a 25°.

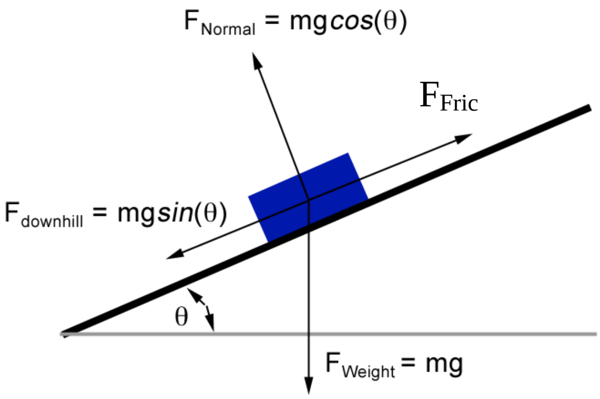


Figure 2 *– Object sliding down an inclined plane*

Question.1

If there were <u>no friction</u>, what would be the object's rate of acceleration down the plane?

$$a = gsin\theta$$
$$a = 9.8 \times \sin(25)$$
$$= 4.1\ ms^{-2}$$

Question.2

Given that friction is now considered and the object is now accelerating at a rate of 4 ms^{-2}, calculate the magnitude of the Frictional Force (F_{Fric}).

<u>Step.1</u> Construct an equation:

$$F_{net} = F_{downhill} - F_f$$
$$ma = mgsin\theta - F_f$$

<u>Step.2</u> Substitute values into equation and solve:

$$(40 \times 4) = (40 \times 9.8 \times sin(25)) - F_f$$
$$160 = 166 - F_f$$
$$\therefore F_f = 6\ N$$

Question.3

If the object was initially stationary and maintained its rate of acceleration of 4 ms^{-2} for 5 seconds, what distance would it travel down the inclined plane?

$$u = 0\ ms^{-1} \qquad s = ut + \frac{1}{2}at^2$$
$$a = 4\ ms^{-2} \qquad = (0 \times 5) + \frac{1}{2} \times 4 \times (5)^2$$
$$t = 5\ sec$$
$$s = ? \qquad = 50\ m$$

Newton's 2nd Law of Motion – Connected Multiple Body Systems

The next scenario that we need to consider is one involving **connected bodies**. Typical scenarios include ship and tug boat joined by cable, train and carriages joined by a connection, or a car and trailer joined via a tow bar.

Example.3

A truck of mass 1500 kg towing a boat of mass 500 kg accelerates at a constant rate on a horizontal road. A thrust of 5000 N is provided by the truck's engine. The Road friction of the truck is 1500 N, while that on the boat is 500 N. The air resistance on both the truck and the boat is negligible. The configuration is shown below in Figure 3.

Calculate:

1. The acceleration of both the truck and boat
2. *The tension force in the tow bar between the truck and boat**Figure 3** – Truck towing boat*

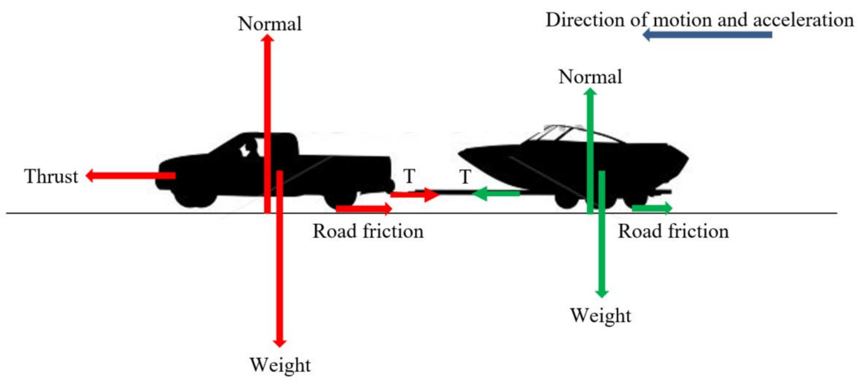
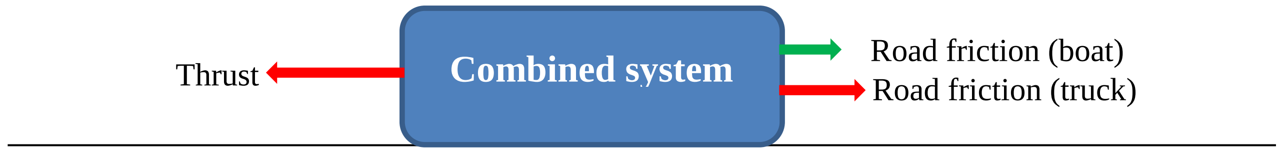


***Figure 3** – Truck towing boat*

Task.1 Calculate the acceleration of both the truck and boat

<u>Analyse the truck and boat as a single system.</u>

NB: vertical forces are balanced, examine the horizontal forces on the system, excluding the tension force (T) between the two objects.



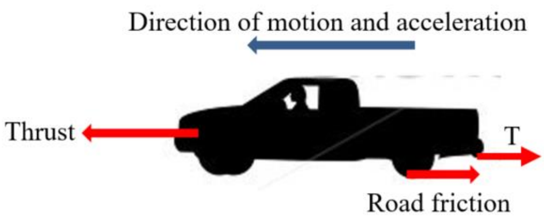
Construct an equation (truck & boat combination):

$$F_{net} = F_{thrust} - F_{f(truck)} - F_{f(boat)}$$
$$ma = F_{thrust} - F_{f(truck)} - F_{f(boat)}$$
$$(1500 + 500) \times a = 5000 - 1500 - 500$$
$$2000 \times a = 3000$$
$$\therefore a = \frac{3000}{2000}$$
$$= 1.5\ ms^{-2}$$

Task.2 Calculate the tension force in the tow bar between the truck and boat

Option 1. Analyse the truck

NB: vertical forces are balanced, examine the horizontal forces on the system, including the tension force (T).



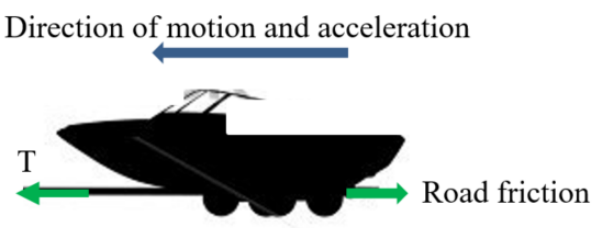
Construct an equation (truck alone):

$$F_{net} = F_{thrust} - F_f - T$$
$$ma = F_{thrust} - F_f - T$$
$$1500 \times 1.5 = 5000 - 1500 - T$$
$$2250 = 3500 - T$$
$$\therefore T = 3500 - 2250$$
$$= 1250\ N\ (\rightarrow)$$

OR

Option 2. Analyse the boat

NB: vertical forces are balanced, examine the horizontal forces on the system, including the tension force (T).



Construct an equation (boat alone):

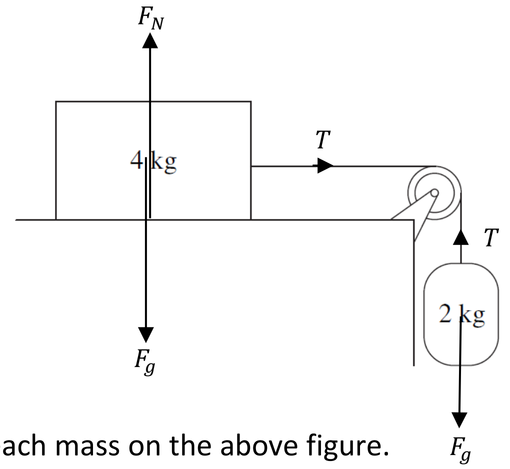
$$F_{net} = T - F_f$$
$$ma = T - F_f$$
$$500 \times 1.5 = T - 500$$
$$750 = T - 500$$
$$\therefore T = 750 + 500$$
$$= 1250\ N\ (\leftarrow)$$

NB: By analysis of either the truck or boat the same value for the tension (T) is found.
The truck experiences a tension of 1250 N to the right.
The boat experiences a tension of 1250 N to the left.

Exam Style Question

Questions 1 - 4 refer to the following information

A 4.0 kg box rests on a smooth horizontal table. It is attached to a 2.0 kg mass by a light string via a frictionless pulley, as shown below.



4 marks
1 mark for both F_W forces.
1 mark for correct direction of T on 4.0 kg box.
1 mark for correct direction of T on 2.0 kg weight.
1 mark for F_N on 4.0 kg box.

Question.1
Sketch the forces acting on each mass on the above figure.

Question.2
Determine the acceleration of the 4.0 kg box.

Analyse the 4 kg box

$F_{net} = ma$

$T = 4a \longleftarrow$ Equation 1

Analyse the 2 kg box

$F_{net} = ma$

$mg - T = 2a$

$2 \times 9.8 - T = 2a \longleftarrow$ Equation 2

Sub Equation 1 into Equation 2

$2 \times 9.8 - (4a) = 2a$

$19.6 - 4a = 2a$

$19.6 = 6a$

$\therefore a = \frac{19.6}{6}$

$a = 3.3\ ms^{-2}$

Question.3
What is the tension in the string?

Analyse the 4 kg box

$T = ?$

$T = F_{net} = ma$

$= 4 \times 3.3$

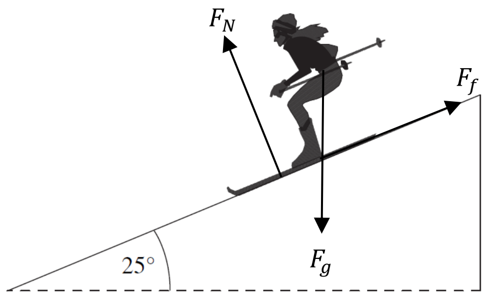
$= 13.2\ N$

Question.4
If the table provides a frictional force to prevent the box from moving, what is the minimum size of this force?

As 13.2 N is acting on the 4.0 kg block, F_{fric} would have to be 13.2 N to prevent the box from moving.

Questions 5 - 7 refer to the following information

The figure below shows a skier skiing down a slope at an angle of 25.0° to the horizontal. She has a mass of 70.0 kg. The resistance force of the slope is 10.0 N parallel to and up the slope.



3 marks

1 mark for showing force due to weight.
1 mark for showing force due to normal reaction.
1 mark for showing frictional forces.
Note: Deduct 1 mark if a driving force is shown.

Question.5
Sketch and label the forces acting on the skier.

Question.6
Calculate the normal reaction force acting on the skier.

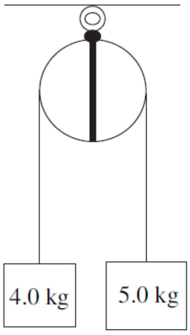
$F_N = ?$
$F_N = mgcos\theta$
$\quad = 70 \times 9.8 \times cos25$
$\quad = 621.7\ N$

Question.7
Calculate the magnitude of the acceleration of the skier.

$a = ?$
$F_{net} = ma$
$mgsin\theta - F_f = ma$
$70 \times 9.8 \times sin25 - 10 = 70 \times a$
$$a = \frac{70 \times 9.8 \times sin25 - 10}{70}$$
$a = 4.0\ ms^{-2}$

Questions 8 & 9 refer to the following information

The pulley system shown below is set up with a 4.0 kg block on one side and a 5.0 kg block on the other side, connected with fishing wire. The two blocks are held stationary and then released to move.



Question.8
Calculate the acceleration of the 4.0 kg block. Include a direction in your answer. Show your working.

Analyse the 4 kg block
$F_{net} = ma$
$T - mg = 4.0 \times a$
$T - 4.0 \times 9.8 = 4a$
$T - 39.2 = 4a$
$T = 4a + 39.2 \longleftarrow$ Equation 1

Analyse the 5 kg block
$F_{net} = ma$
$mg - T = 5.0 \times a$
$5.0 \times 9.8 - T = 5.0 \times a$
$49 - T = 5a \longleftarrow$ Equation 2

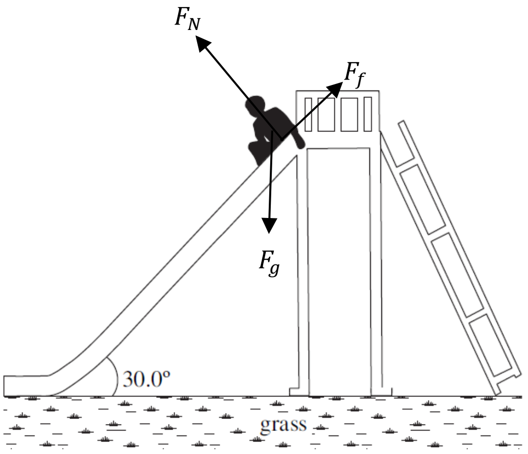
Sub Equation 1 into Equation 2
$49 - (4a + 39.2) = 5a$
$9.8 - 4a = 5a$
$9.8 = 9a$
$\therefore a = \frac{9.8}{9}$
$\therefore a = 1.1\ ms^{-2} (Upwards)$

Question.9
Calculate the tension in the fishing wire. Show your working.

Analyse the 4 kg block
$F_{net} = ma$
$T - mg = 4.0 \times a$
$T - 4.0 \times 9.8 = 4.0 \times 1.1$
$T - 39.2 = 4.4$
$\therefore T = 4.4 + 39.2$
$\quad = 43.6\ N$

Questions 10 - 13 refer to the following information

Kegan is sliding down a slide inclined at 30.0° to the ground, as shown below. There is a constant frictional force of 100.0 N that acts on Kegan. Kegan has mass of 80 kg.



3 marks
1 mark for showing force due to weight (w).
1 mark for showing force due to normal reaction (N).
1 mark for showing frictional forces (Ff).

Question.10
Sketch and label the forces acting on Kegan on the above figure.

Question.11
What is the magnitude of the normal force acting on Kegan? Show your working.

$F_N = ?$

$F_N = mgcos\theta$

$= 80 \times 9.8 \times cos30$

$= 679\ N$

Question.12
What is the magnitude of the net force acting on Kegan? Show your working.

$F_{net} = ?$

$F_{net} = mgsin\theta - F_f$

$= 80 \times 9.8 \times sin30 - 100$

$= 292\ N$

Question.13
What is the magnitude of Kegan's net acceleration? Show your working.

$a = ?$

$F_{net} = ma$

$\therefore a = \frac{F_{net}}{m}$

$= \frac{292}{80}$

$= 3.65\ ms^{-2}$