Section 2.1.10 – Work & Energy (Part 2)

Springs

Hooke's Law describes the linear relationship between the force and displacement of an ideal spring.

$$F_s = -k\Delta x$$

Where F_s represents the spring restoring force (N)
k represents the spring constant (Nm^{-1})
Δx represents the extension/compression

Figure 1 shows that the extension of the spring (Δx) is proportional to the applied force (F).

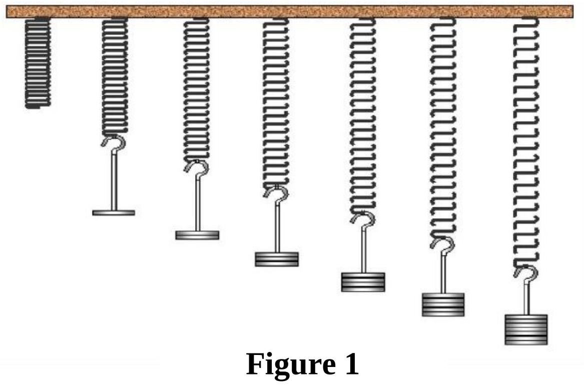
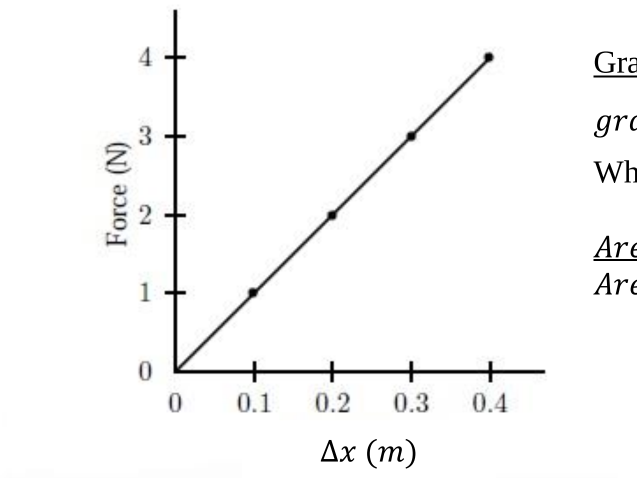


Figure 1

Strain Potential Energy

Consider when a spring is either stretched or compressed. In order for the length of the spring to change, work has to be done on the spring (ie. a force is applied over a displacement). This can be graphically represented as a $F - \Delta x$ graph as shown below in Graph.1.



Graph.1

<u>Gradient of a $F - \Delta x$ graph</u>

$$gradient\ (k) = \frac{F}{\Delta x}$$

Where k represents the spring constant (Nm^{-1})

<u>*Area under a $F - \Delta x$ graph*</u>

$Area = Work\ done = Strain\ Potential\ Energy\ (J)$

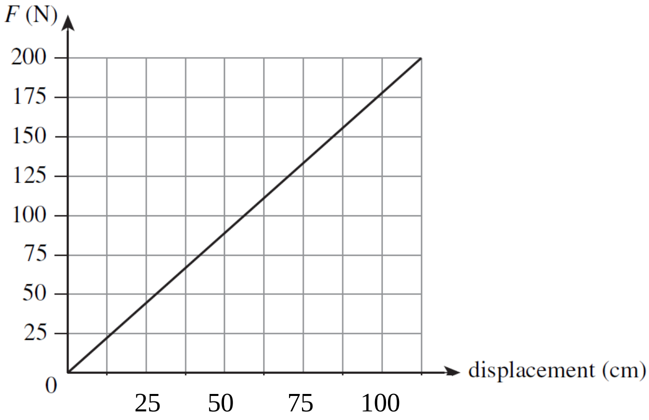
During extension or compression, work is stored in the spring in the form of **strain potential energy**, E_s.

$$E_s = \frac{1}{2}k(\Delta x)^2$$

Where E_s represents the strain potential energy (J)
k represents the spring constant (Nm^{-1})
Δx represents the extension/compression (m)

Example.1

The graph below shows the force required to extend a spring.



Question 1

What is the spring constant, k, of the spring?

$$k = ?$$
$$k = gradient$$
$$= \frac{175\ N}{1\ m}$$
$$= 175\ Nm^{-1}$$

Question 2

What is the strain potential energy stored in the spring at an extension of 100 cm?

<u>Option.1</u>

$$E_s = ?$$
$$E_s = Area\ under\ the\ curve$$
$$= \frac{1}{2}BH$$
$$= \frac{1}{2} \times 1.0 \times 175$$
$$= 87.5\ J$$

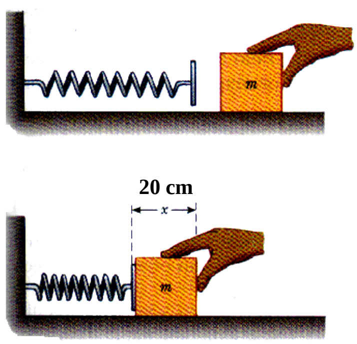
<u>Option.2</u>

$$E_s = ?$$
$$\Delta x = 1.0\ m$$
$$k = 175\ Nm^{-1}$$

$$E_s = \frac{1}{2}k(\Delta x)^2$$
$$= \frac{1}{2} \times 175 \times (1.0)^2$$
$$= 87.5\ J$$

Example.2

A spring of spring constant (k) $200Nm^{-1}$ is compressed by 20 cm. Calculate the strain potential energy stored in the spring.



$$E_s = ?$$
$$\Delta x = 0.2\ m$$
$$k = 200\ Nm^{-1}$$

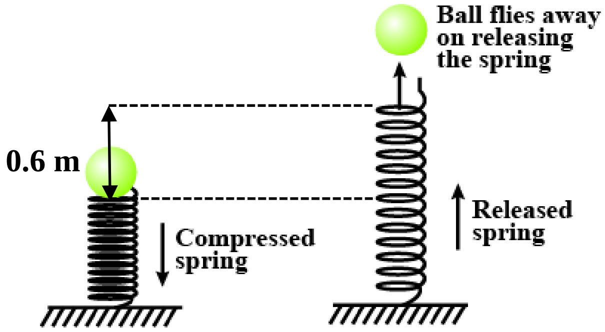
$$E_s = \frac{1}{2}k(\Delta x)^2$$
$$= \frac{1}{2} \times 200 \times (0.2)^2$$
$$= 4.0\ J$$

Conservation of Energy

A fundamental law of Physics is that energy is always conserved in a closed or isolated system. That is, it cannot be created or destroyed. Rather is can be converted into another energy form.

Scenario 1: Strain Potential Energy ↔ Kinetic Energy Conversion

An ideal spring is being used to launch a $500\ g$ ball vertically upwards.
The spring is initially compressed by $0.6\ m$ and has a spring constant of $200\ Nm^{-1}$



Question. 1
What is the strain potential energy initial stored in the compressed spring?

$E_s = ?$
$k = 200\ Nm^{-1}$
$\Delta x = 0.6\ m$

$$\begin{aligned} E_s &= \frac{1}{2}k(\Delta x)^2 \\ &= \frac{1}{2} \times 200 \times (0.6)^2 \\ &= 36\ J \end{aligned}$$

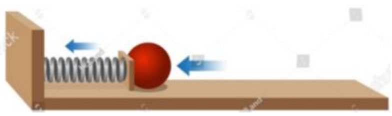
Question. 2
What is the speed of the ball upon release?

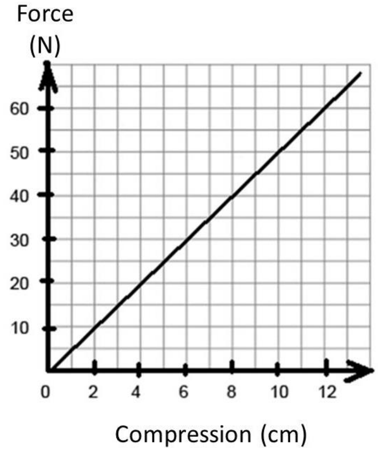
$v = ?$
$\Delta E_k = \Delta E_s$
$\quad = 36\ J$
$m = 0.5\ kg$

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ 36 &= \frac{1}{2} \times 0.5 \times v^2 \\ 36 &= 0.25 \times v^2 \\ \therefore v^2 &= \frac{36}{0.25} \\ \therefore v &= \sqrt{\frac{36}{0.25}} \\ \therefore v &= 12\ ms^{-1} \end{aligned}$$

Scenario 2: Strain Potential Energy ↔ Kinetic Energy Conversion

A 200 g ball is moving towards an ideal spring which it collides with and then compresses. The ball compressed the ideal spring by 10.0 cm from its natural position before coming to rest. The force-displacement graph is shown below. Ignore the effects of friction.





Question. 1
What is the spring constant (k) of the spring?

$$k = ?$$
$$k = gradient$$
$$= \frac{50\ N}{0.10\ m}$$
$$= 500\ Nm^{-1}$$

Question. 2
What is the strain potential energy stored in the compressed spring at a compression of 10.0 cm?

$E_s = ?$
$k = 500\ Nm^{-1}$
$\Delta x = 0.10\ m$

$$E_s = \frac{1}{2}k(\Delta x)^2$$
$$= \frac{1}{2} \times 500 \times (0.1)^2$$
$$= 2.5\ J$$

Question. 3
What is the speed of the ball upon impact with the spring?

$v = ?$
$\Delta E_k = \Delta E_s$
$= 2.5\ J$
$m = 0.200\ kg$

$$E_k = \frac{1}{2}mv^2$$
$$2.5 = \frac{1}{2} \times 0.200 \times v^2$$
$$2.5 = 0.100 \times v^2$$
$$\therefore v^2 = \frac{2.5}{0.100}$$
$$\therefore v = \sqrt{\frac{2.5}{0.100}}$$
$$\therefore v = 5\ ms^{-1}$$

Power

The term power describes the **rate** at which energy is transferred or converted.

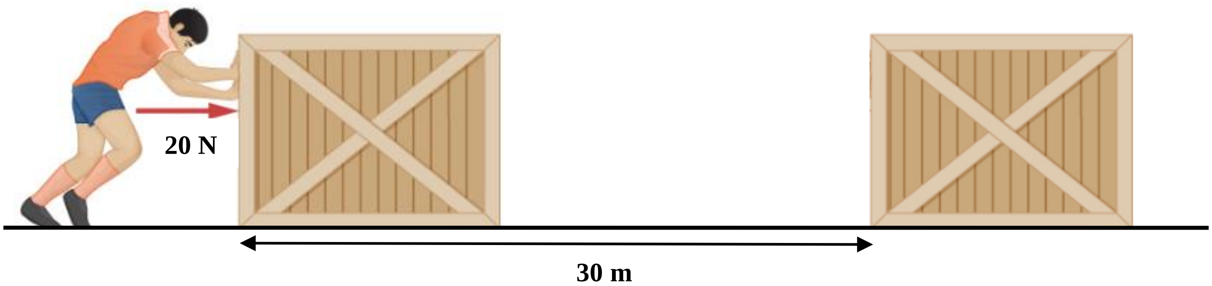
$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

Where P represents power ($Watts$)
W represents the work done (J)
ΔE represents the change in energy (J)
Δt represents the change in time (sec)

NB: $1\,Watt = 1\,Js^{-1}$

Example

Luke takes 1 minute to push a crate with a force of 20 N a distance of 30 m.
Ignore friction and air resistance.



Question. 1
What work has Luke done on the box?

$W = ?$
$F = 20\,N$
$s = 30\,m$

$$W = Fs = 20 \times 30 = 600\,J$$

Question. 2
What is Luke's power rating?

$P = ?$
$W = 600\,J$
$\Delta t = 1\,min = 60\,sec$

$$P = \frac{W}{\Delta t} = \frac{600}{60} = 10\,Watts$$

Efficiency

The efficiency of an energy system can be calculated using the following equation.

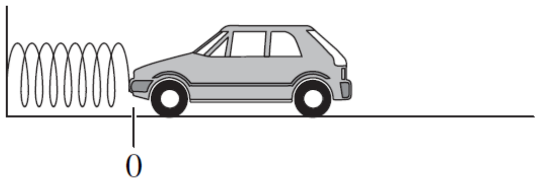
$$\eta = \frac{usefull\ energy\ out}{usefull\ energy\ in}$$

Where η represents efficiency ($no\ units$)

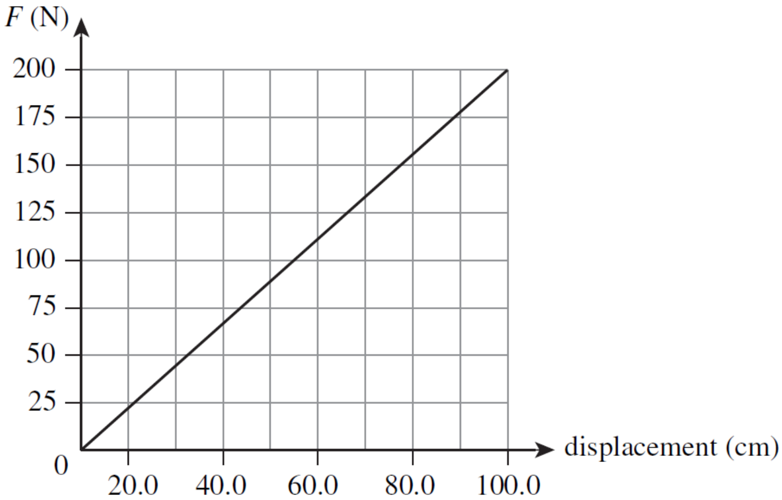
Exam Styled Questions

Questions 1 & 2 refer to the following information

The figure below shows a toy car on a frictionless track. The car collides with and compresses an ideal spring by 100.0 cm when the car comes to rest.



The force-versus-displacement graph for this process is shown in below. Assume that friction is negligible.



Question 1
What is the spring constant, k, of the spring? Include a unit in your answer.

$$k = ?$$
$$k = gradient$$
$$= \frac{200\ N}{1\ m}$$
$$= 200\ Nm^{-1}$$

200 Nm^{-1}

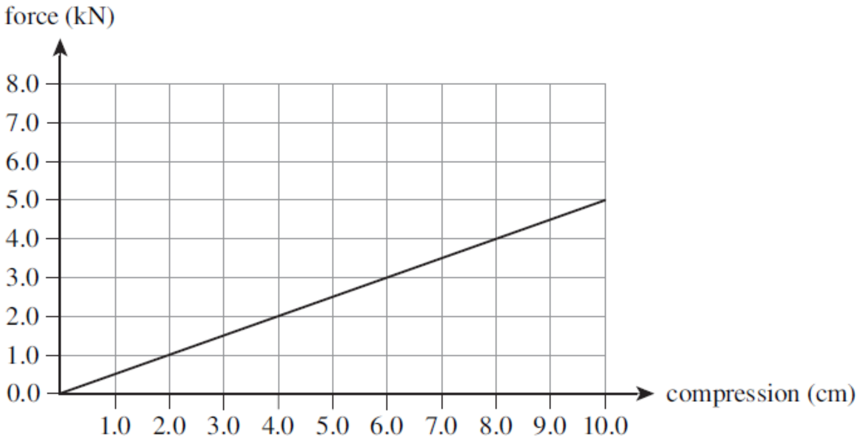
Question 2
What is the initial kinetic energy of the toy car?

$$E_k = ?$$
$$\Delta E_k = \Delta E_s = Area\ under\ the\ curve$$
$$= \frac{1}{2}BH$$
$$= \frac{1}{2} \times 1 \times 200$$
$$= 100\ J$$

100 J

Questions 3 & 4 refer to the following information

'Sprung floors' are used in sport halls. They use simple coil springs to absorb shock and reduce injuries. The force (F) versus compression graph of a sprung floor is shown below.



Question 3
Show that the spring constant is 50 000 Nm^{-1}.

$$k = ?$$
$$k = gradient$$
$$= \frac{5 \times 10^3\ N}{0.1\ m}$$
$$= 5.0 \times 10^4\ Nm^{-1}$$
$$= 50\ 000\ Nm^{-1}$$

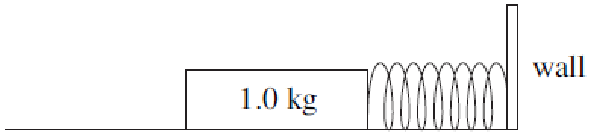
Question 4
Calculate the elastic potential energy stored in the sprung floor when it is compressed by 5.0 cm.

$$\Delta E_s = ?$$
$\Delta E_s =$ $Area\ under\ the\ curve$ (<u>up to 5.0 cm</u>)
$$= \frac{1}{2} BH$$
$$= \frac{1}{2} \times .05 \times 2.5 \times 10^3$$
$$= 62.5\ J$$

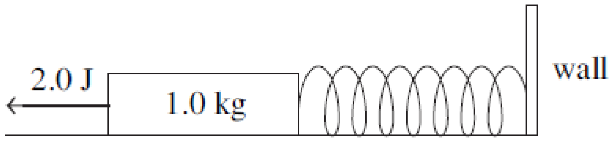
62.5 J

Questions 5 - 7 refer to the following information

A spring rests horizontally against a wall. Elle holds a block of mass 1.0 kg stationary against the spring, compressing it by 10.0 cm, as shown below.



Elle releases the block and it leaves the spring with a kinetic energy of 2.0 J, as shown below.



Question 5
Calculate the speed of the block as it leaves the spring. Ignore the effects of friction.
Show your working.

$v = ?$
$E_k = 20\,J$
$m = 1.0\,kg$

$$E_k = \frac{1}{2}mv^2$$
$$2.0 = \frac{1}{2} \times 1.0 \times v^2$$
$$2.0 = 0.5 \times v^2$$
$$\therefore v = \sqrt{\frac{2}{.5}} = 2.0\,ms^{-1}$$

2.0 ms^{-1}

Question 6
Calculate the work done on the block by the spring. Ignore the effects of friction.
Show your working.

$W = ?$
$W = \Delta E_k$
$= 2.0\,J$

2.0 J

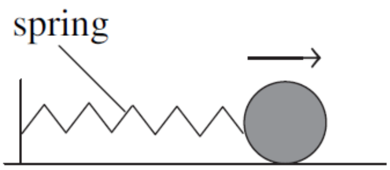
Question 7
Calculate the spring constant, k, of the spring. Assume that the spring obeys Hooke's law.
Show your working.

$k = ?$
$\Delta E_s = W$
$\frac{1}{2}kx^2 = 2.0$
$\frac{1}{2} \times k \times (0.1)^2 = 2.0$

$$\therefore k = \frac{2.0 \times 2}{(0.1)^2}$$
$$= 400\,Nm^{-1}$$

400 Nm^{-1}

Question 8
A ball of mass 100.0 g compresses a spring by 10.0 cm. When the spring is released, the ball is projected forward, as shown below.



The spring constant is 100.0 Nm^{-1}.
Determine the projection speed of the ball. Ignore friction. Show your working.

$v = ?$
$\Delta E_s = \Delta E_k$
$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$
$k(\Delta x)^2 = mv^2$
$\therefore v = \sqrt{\frac{k(\Delta x)^2}{m}}$

$v = \sqrt{\frac{100 \times (0.10)^2}{0.1}}$
$= 3.16\ ms^{-1}$

$\boxed{\mathbf{3.16\ ms^{-1}}}$