Section 2.1.3 – Equations of Constant Acceleration (SUVAT)

SUVAT Equations

A series of motion equations have been constructed from mathematical relationships derived from the various motion graphs. Each is built upon the following motion variables:

u = initial velocity (ms^{-1})
v = final velocity (ms^{-1})
a = acceleration (ms^{-2})
t = time (sec)
s = displacement (m)

NB: All units of measurements are SI (metres and seconds)

The following equations can only be applied to a situation where an object is undergoing a **constant rate of acceleration** (which includes a = 0 ms^{-2}):

$$\boldsymbol{v = u + at}$$
$$\boldsymbol{s = ½(u + v)t}$$
$$\boldsymbol{s = ut + ½at^2}$$
$$\boldsymbol{s = vt - ½at^2}$$
$$\boldsymbol{v^2 = u^2 + 2as}$$

When solving problems using the motion equations, use the following steps & layout.

1. State the variables
2. Select the equation
3. Substitute & solve

Example.1
A car is travelling at 10 ms^{-1} accelerates at a rate of 2 ms^{-2} for 10 seconds. What is its final velocity after this period of acceleration?

u = 10 ms^{-1}	$v = u + at$
a = 2 ms^{-2}	$v = 10 + (2 \times 10)$
t = 10 sec	$v = 10 + 20$
v = ?	$\boldsymbol{v = 30\,ms^{-1}}$

Example.2
Chris trows a stone down a well with an initial speed of 15 ms^{-1}. If it takes 8 seconds to reach the bottom how deep is the well?

u = 15 ms^{-1}	$s = ut + ½at^2$
a = 9.8 ms^{-2} (gravity)	$s = (15 \times 8) + (½ \times 9.8 \times 8^2)$
t = 8 sec	$s = 120 + 313.6$
s = ?	$\boldsymbol{s = 433.6\,m}$

Example.3

What final speed would a model rocket reach if from "lift off" it accelerated at a constant rate of 15 ms^{-2} over a distance of 1 km?

$v = ?$
$u = 0\ ms^{-1}$
$a = 15\ ms^{-2}$
$s = 1000\ m$

$$v^2 = u^2 + 2as$$
$$\therefore v = \sqrt{u^2 + 2as}$$
$$= \sqrt{0^2 + 2 \times 15 \times 1000}$$
$$= \sqrt{30000}$$
$$= \mathbf{173\ ms^{-1}}$$

Example.4

James nervously steps off the edge of the 10 m high diving platform and falls into the pool below.

i. Calculate the time it takes him to fall the 10 m distance
ii. Calculate the impact velocity with which he hits the water

$u = 0\ ms^{-1}$
$a = 9.8\ ms^{-2}$
$s = 10\ m$
$t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$10 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$
$$10 = 4.9 \times t^2$$
$$\therefore t^2 = \frac{10}{4.9}$$
$$\therefore t = \sqrt{\frac{10}{4.9}}$$
$$= \mathbf{1.43\ s}$$

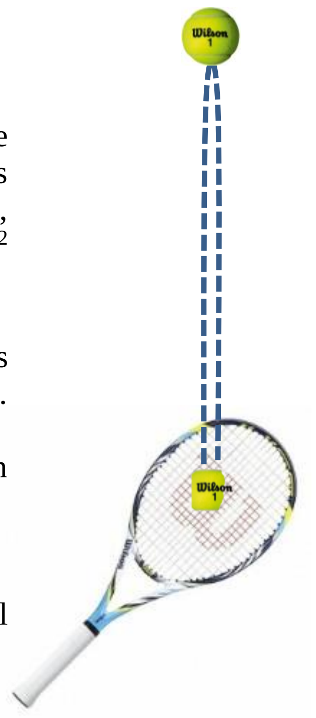
$u = 0\ ms^{-1}$
$a = 9.8\ ms^{-2}$
$s = 10\ m$
$v = ?$

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2 \times 9.8 \times 10$$
$$v^2 = 196$$
$$v = \sqrt{196}$$
$$= \mathbf{14.0\ ms^{-1}}$$

NB: In all constant acceleration questions it is important to correctly allocate a given direction as positive. For simplicity it is easiest to make the direction of the initial motion as a **positive direction.**

In example.1 the initial motion is forward, so the forward direction is used as a positive value.
In example.2 the initial motion is downward, so the downward direction is used as a positive value.
In example.3 the initial motion is upward, so the upward direction is used as a positive value.
In example.4 the initial motion is downward, so the downward direction is used as a positive value.

Vertical Motion



In the case of vertical motion, an object is projected into the air with an initial velocity (u). The instance it leaves its accelerating force, say a racquet in the example shown right, the ball experiences an acceleration due to gravity of 9.8 ms^{-2} downwards.

Accordingly, on the way to its maximum height the ball is slowing down (decelerating) relative to its initial velocity (u).

Upon reaching its maximum height, the ball has an instantaneous velocity of 0 ms^{-1}

As the ball falls from its maximum height, it is speeds up (accelerates) in a negative direction relative to its initial velocity (u).

Facts:

- The time for the ball to rise to its maximum height = time for the ball to fall back to the racquet
- The magnitude of the balls initial velocity = the magnitude of the balls final velocity

Example.1

Consider a tennis ball that is hit with a racquet straight up with an initial velocity of 40 ms^{-1}.

i. How long does it take to reach its maximum height?
ii. What is the maximum height reached by the ball?
iii. What is the velocity of the ball when it returns to the racquet?

i. ***Consider the <u>first half</u> of the journey only (taking ↑ as positive)***

$u = 40\,ms^{-1}$
$a = -9.8\,ms^{-2}$
$v = 0\,ms^{-1}$
$t = ?$

$$v = u + at$$
$$0 = 40 + (-9.8 \times t)$$
$$0 = 40 - 9.8t$$
$$\therefore 9.8t = 40$$
$$\therefore t = \frac{40}{9.8}$$
$$= 4.1\,s$$

ii. ***Consider the <u>first half</u> of the journey only (taking ↑ as positive)***

$u = 40\,ms^{-1}$
$a = -9.8\,ms^{-2}$
$v = 0\,ms^{-1}$
$s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 40^2 + 2 \times (-9.8) \times s$$
$$0 = 1600 - 19.6s$$
$$\therefore 19.6s = 1600$$
$$\therefore s = \frac{1600}{19.6}$$
$$= 81.6\,m$$

iii. ***Consider the <u>entire journey</u> (taking ↑ as positive)***

$v = ?$
$u = 40\ ms^{-1}$

$v(down) = -u(up)$
$v = -40\ ms^{-1}$ (up)

Or

$v = 40\ ms^{-1}$ (down)

Example.2

While celebrating his 18th birthday Mathew pops a cork off a bottle of champagne. The cork travels vertically into the air. He notices that the cork takes 4.0 s to return to its starting position.

i. How long does the cork take to reach its maximum height?
ii. What was the maximum height reached by the cork?
iii. How fast was the cork travelling initially?
iv. What was the speed of the cork as it returned to its starting point?
v. What is the corks acceleration at 1.0 sec, 2.0 sec and 3.0 sec after launch?

i. ***Consider the <u>first half</u> of the journey only (taking ↑ as positive)***

$t_{\max height} = ?$
$t_{total} = 4.0\ s$

$$t_{\max\ height} = \frac{1}{2} \times t_{total}$$
$$= \frac{1}{2} \times 4.0$$
$$= 2.0\ s$$

ii. ***Consider the <u>first half</u> of the journey only (taking ↑ as positive)***

$s = ?$
$a = -9.8\ ms^{-2}$
$v = 0\ ms^{-1}$
$t = 2.0$ s

$$s = vt - \frac{1}{2}at^2$$
$$= 0 \times 2.0 - \frac{1}{2} \times (-9.8) \times 2^2$$
$$= 0 - (-19.6)$$
$$= 19.6\ m$$

iii. ***Consider the <u>first half</u> of the journey only (taking ↑ as positive)***

$u = ?$
$a = -9.8\ ms^{-2}$
$v = 0\ ms^{-1}$
$t = 2.0$ s

$$v = u + at$$
$$0 = u + (-9.8) \times 2$$
$$0 = u - 19.6$$
$$\therefore u = 19.6\ ms^{-1}\ (upwards)$$

iv. ***Consider the <u>entire journey</u> (taking ↑ as positive)***

$v = ?$
$u = 19.6\ ms^{-1}$

$v(down) = -u(up)$
$v = -19.6\ ms^{-1}$ (up)

Or

$v = 19.6\ ms^{-1}$ (down)

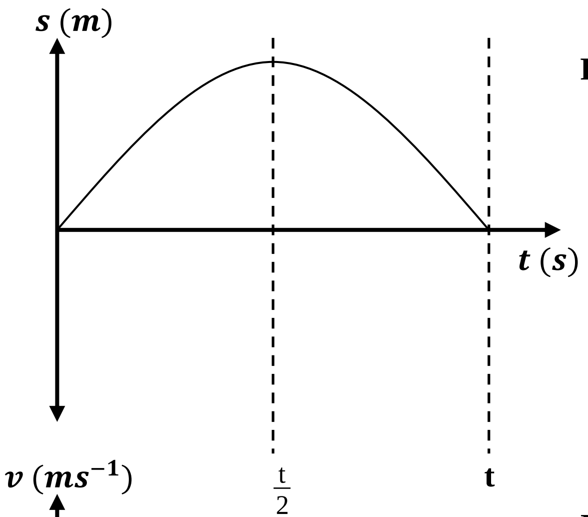
v. ***Consider the entire journey***

Acceleration is <u>**9.8 ms^{-2}**</u> towards the ground at all times!

Graphical Analysis of Vertical Motion

Scenario: An object is projected vertically up into the air, where it is first decelerated by gravity to a momentary velocity of 0 ms^{-1}, and then accelerated by gravity as the projectile returns to ground.

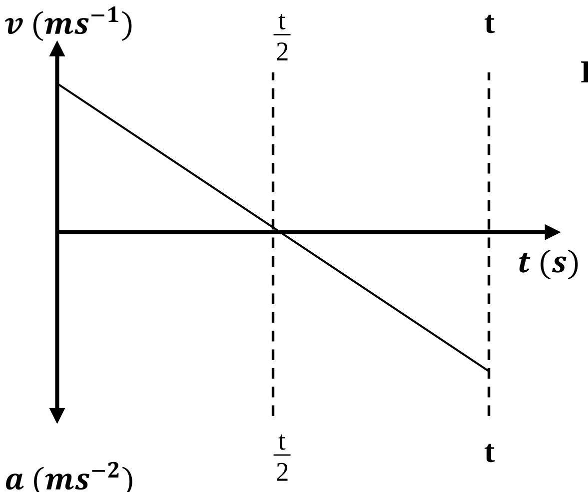
NB: For all below graphs, consider the upwards direction as a positive value



Recall: Displacement is the comparison between a final and initial position.

The projectile moves a positive distance away from the starting position in the first half of motion.

The projectile moves back towards its starting position in the second half of motion.



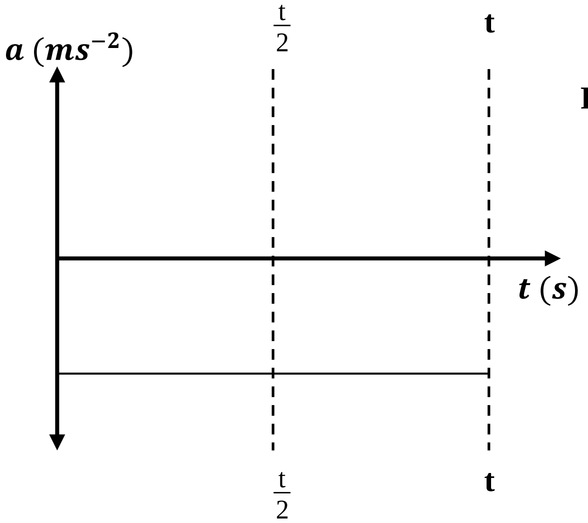
Recall: Velocity is the rate of change of distance.

$$v = \frac{\Delta s}{\Delta t} = gradient\ of\ the\ s - t\ graph$$

The projectile starts with a high positive velocity and decelerates.

It reaches a velocity of 0 ms^{-1} at the mid-point.

The projectile now accelerates to a high negative velocity.



Recall: Acceleration is the rate of change of velocity.

$$a = \frac{\Delta v}{\Delta t} = gradient\ of\ the\ v - t\ graph$$

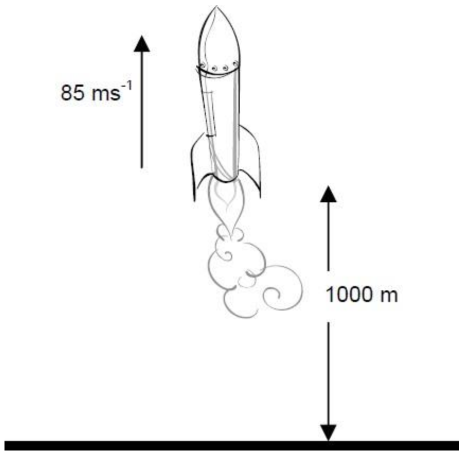
Regardless of the projectiles velocity, the gradient of the V – t graph is a constant negative value.

∴ Acceleration is a constant negative value
Acceleration is 9.8 ms^{-2} (downwards)

Exam Style Question

Questions 1 & 2 refer to the following information

A model rocket is fired vertically into the air. At a height of +1000 m above the ground level the engine runs out of fuel as the rocket is travelling at a speed of +85 ms^{-1}.



Question 1
What is the acceleration of the rocket immediately after the fuel runs out?

A. $-9.8\ ms^{-2}$
B. $9.8\ ms^{-2}$
C. $85\ ms^{-1}$
D. Zero

From the duration of the question, take upwards as positive,
$\therefore$ acceleration after the fuel runs out is downwards at 9.8 ms^{-2}.
$\therefore$ answer is A.

A

Question 2
What is the maximum height, above the ground level, reached by the rocket?

$u = 85\ ms^{-1}$
$a = -9.8\ ms^{-2}$
$v = 0\ ms^{-1}$
$s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 85^2 + 2 \times (-9.8) \times s$$
$$0 = 7225 - 19.6s$$
$$\therefore 19.6s = 7225$$
$$\therefore s = \frac{7225}{19.6}$$
$$= 368.6\ m$$

Plus the 1000m already above ground level.
$\therefore height$ (max) $= \mathbf{1368.6\ m}$

1368.6 m

Questions 3 & 4 refer to the following information

A motorcyclist, travelling at 12 ms^{-1}, is going to overtake a truck so he accelerates at 2.0 ms^{-2} for 6.0 seconds until he is in front of the truck at which point he stops accelerating and continues at a constant speed.

Question 3

What is the final constant speed of the motorcyclist?

$v = ?$
$a = 2\,ms^{-2}$
$u = 12\,ms^{-1}$
$t = 6.0$ s

$$\begin{aligned} v &= u + at \\ &= 12 + 2 \times 6 \\ &= 12 + 12 \\ &= 24\,ms^{-1} \end{aligned}$$

24 ms^{-1}

Question 4

How far does the motorcyclist travel while he is overtaking the truck?

$s = ?$
$a = 2\,ms^{-2}$
$u = 12\,ms^{-1}$
$t = 6\,s$

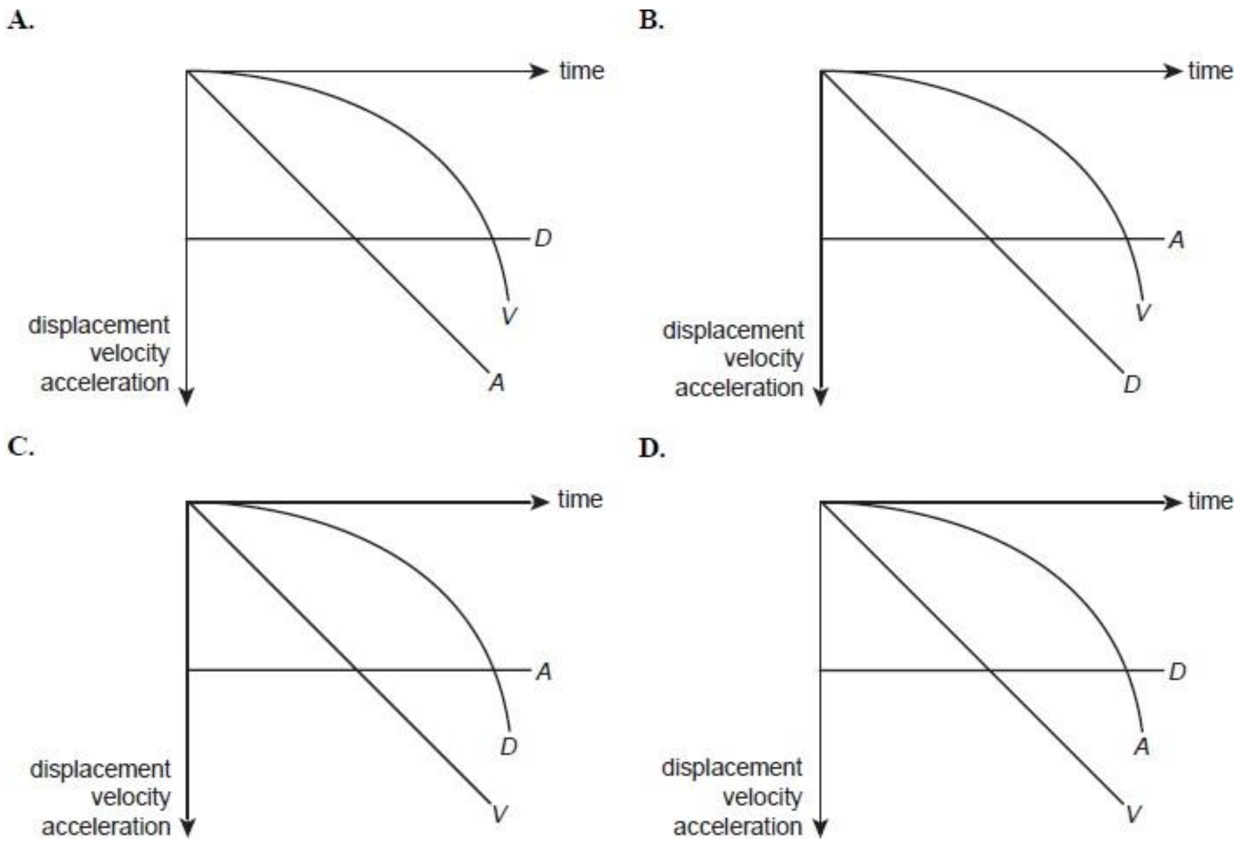
$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= 12 \times 6 + \frac{1}{2} \times 2 \times 6^2 \\ &= 72 + 36 \\ &= 108\,m \end{aligned}$$

108 m

Question 5

A ball is dropped vertically from the top of a 250 m tower. Which of the following graphs best represents displacement (*D*), velocity (*V*) and acceleration (*A*)?

<u>Note: Take down as negative from the top of the tower and ignore air resistance.</u>



The acceleration remains constant (9.8 ms^{-2}), the velocity increases at a constant rate and the displacement increases at a quadratic rate.

C