# Section 2.1.2 – Graphing Motion

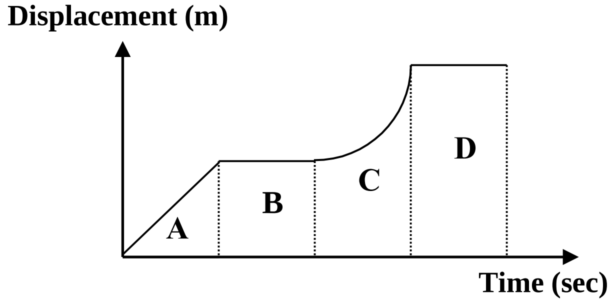
## Motion Graphs

Physicists use motion graphs in order to accurately describe an objects motion, relative to time. There are three main graphs analysed at the VCE level of study.

- ✓ Displacement - Time Graph
- ✓ Velocity - Time Graph
- ✓ Acceleration - Time Graph

**NB:** For all motion graphs consider what the **gradient** and **area** under graph represents!

## Displacement - Time Graph



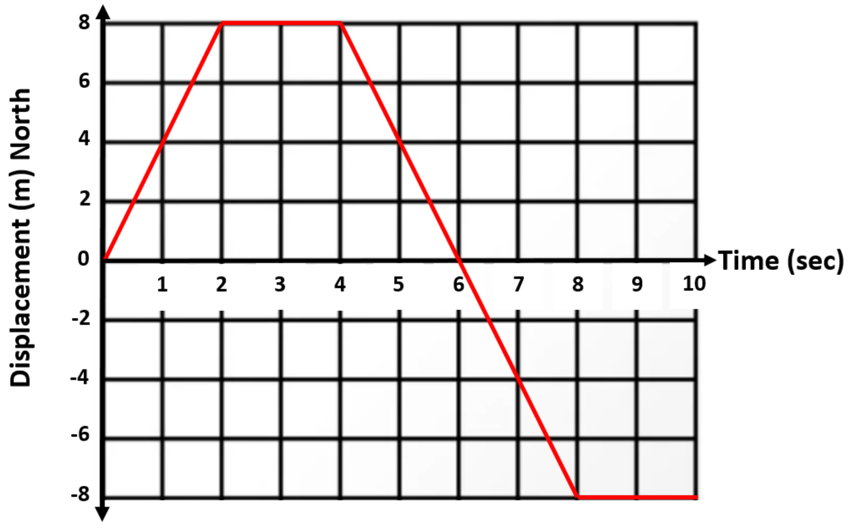

**Section A** Period of constant (uniform) velocity
**Section B** Stationary (constant velocity of 0 $ms^{-1}$)
**Section C** Period of non-uniform increasing velocity (acceleration)
**Section D** Stationary (constant velocity of 0 $ms^{-1}$)

## Summary Table: Displacement - Time Graph

| Feature | Quantity | How to find |
|---|---|---|
| **Individual point** | Displacement at a particular time | Draw a dotted line from a point to each axis and read off values |
| **Area under graph** | ✘ | ✘ |
| **Gradient between two points** | Average velocity | $v_{avg} = gradient = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$ |

## Example.1

Consider the below displacement v time graph.




**Qn.1** What was the objects displacement after 5 seconds?
Reading off the graph, @ a time of 5 seconds, the object's displacement = 4 m North

**Qn.2** What was the objects displacement after 10 seconds?
Reading off the graph, @ a time of 10 seconds, the object's displacement = -8 m North (or 8 m South)

**Qn.3** How far did the object travel over the 10 second journey?
8 m + 8m + 8 m = 24 m

**Qn.4** What was the object's average velocity between t = 0 s and t = 2 s?

$v_{avg} = ?$
$\Delta s = 8\,m$
$\Delta t = 2\,sec$

$$v_{avg} = gradient = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$
$$= \frac{8}{2}$$
$$= 4\,ms^{-1}\,North$$

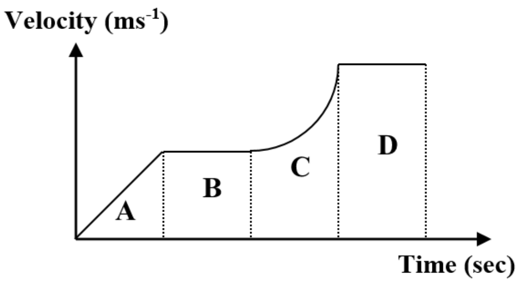
**Qn.5** What was the object's average velocity between t = 6 s and t = 8 s?

$v_{avg} = ?$
$\Delta s = 8\,m$
$\Delta t = 2\,sec$

$$v_{avg} = gradient = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$
$$= \frac{-8}{2}$$
$$= -4\,ms^{-1}\,North\;(4\,ms^{-1}\,South)$$

**Qn.6** At what time did the object first commence moving in a southerly direction?
Reading off the graph, the object first commenced moving southerly at t = 4 seconds.

## Velocity – Time Graph




**Section A** Period of constant (uniform) acceleration
**Section B** Period of constant (uniform) velocity
**Section C** Period of non-uniform increasing acceleration
**Section D** Period of constant (uniform) velocity

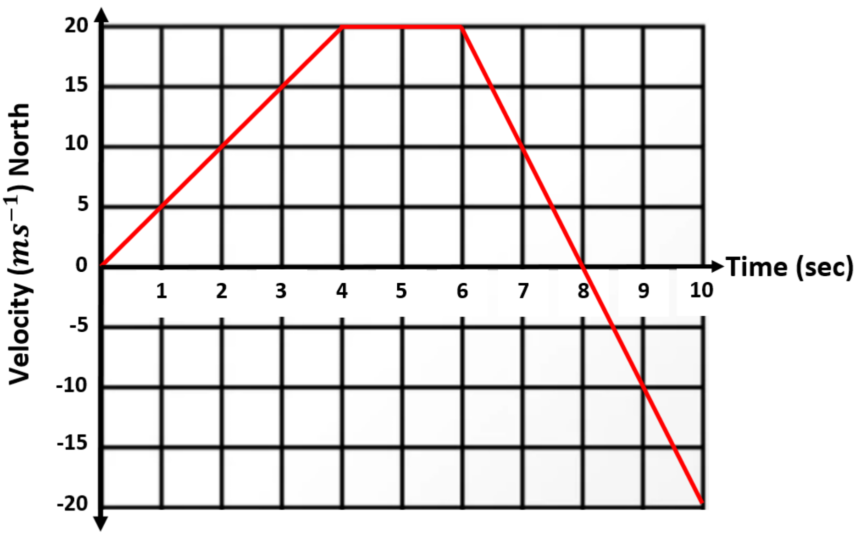
**NB:** At no point on the above graph is the object stationary

## Summary Table: Velocity - Time Graph

| Feature | Quantity | How to find |
|---|---|---|
| **Individual point** | Velocity at a particular time | Draw a dotted line from the point to each axis and read off values |
| **Signed area under the graph** | Displacement | Geometric method<br>Or counting squares method (curved shapes) |
| **Total area under the graph** | Distance travelled | Geometric method<br>Or counting squares method (curved shapes) |
| **Gradient between two points** | Average acceleration | $a_{avg} = gradient = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$ |

## Example.2

Consider the below velocity - time graph.



**Qn.1** What was the objects velocity at t = 4 seconds?
Reading off the graph, @ a time of 4 seconds, the object's velocity = $20\ ms^{-1}\ North$

**Qn.2** What was the objects velocity at t = 10 seconds?
Reading off the graph, @ a time of 10 seconds, the object's velocity = $-20\ ms^{-1}\ North$ or $20\ ms^{-1}\ South$

**Qn.3** How far did the object travel over the 10 second journey?

$Distance = ?$

$$
\begin{aligned}
Distance &= Total\ area \\
&= Trapezium\ (0-8\ sec) + Triangle(8-10\ sec) \\
&= \left[\frac{1}{2}(a+b)h\right] + \left[\frac{1}{2}bh\right] \\
&= \left[\frac{1}{2}(2+8)\times 20\right] + \left[\frac{1}{2}\times 2\times 20\right] \\
&= 100 + 20 \\
&= 120\ m
\end{aligned}
$$

**Qn.4** What was the objects displacement after the 10 second journey?

$Displacement\ (s) = ?$

$$
\begin{aligned}
s &= Area\ above\ the\ x\ axis -\ Area\ below\ the\ x\ axis \\
&= Trapezium\ (0-8\ sec) - Triangle(8-10\ sec) \\
&= \left[\frac{1}{2}(a+b)h\right] - \left[\frac{1}{2}bh\right] \\
&= \left[\frac{1}{2}(2+8)\times 20\right] - \left[\frac{1}{2}\times 2\times 20\right] \\
&= 100 - 20 \\
&= 80\ m\ North
\end{aligned}
$$

**Qn.5** What was the object's average acceleration between t = 0 s and t = 4 s?

$a_{avg} = ?$
$\Delta v = 20\ ms^{-1}$
$\Delta t = 4\ sec$

$$
\begin{aligned}
a_{avg} &= gradient = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \\
&= \frac{20}{4} \\
&= 5\ ms^{-2}\ North
\end{aligned}
$$

**Qn.6** What was the object's average acceleration between t = 8 s and t = 10 s?

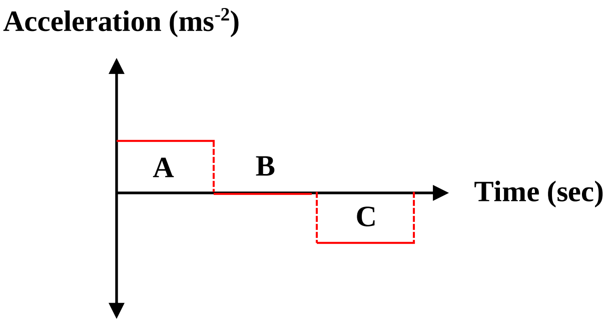
$a_{avg} = ?$
$\Delta v = -20\ ms^{-1}$
$\Delta t = 2\ sec$

$$
\begin{aligned}
a_{avg} &= gradient = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \\
&= \frac{-20}{2} \\
&= -10\ ms^{-2}\ North\ (10\ ms^{-2}\ South)
\end{aligned}
$$

**Qn.7** At what time did the object first commence moving in a southerly direction?
Reading off the graph, the object first commenced moving southerly at t = 8 seconds.

**NB:** The object is moving southerly when the graph line falls below the x axis.

## Acceleration – Time Graph



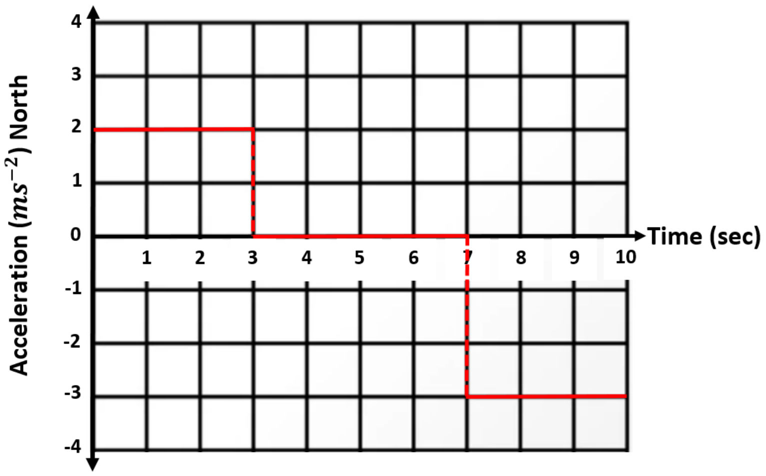

**Section A** Period of constant (uniform) positive acceleration
**Section B** Period of constant velocity (no acceleration)
**Section C** Period of constant (uniform) negative acceleration

## Summary Table: Acceleration - Time Graph

| Feature | Quantity | How to find |
|---|---|---|
| **Individual point** | Acceleration at a particular time | Draw a dotted line from the point to each axis and read off values |
| **Area under the graph** | Change in velocity | Geometric method<br>Or counting squares method (curved shapes) |
| **Gradient between two points** | ✘ | ✘ |

### Example.3

Consider the below acceleration - time graph.




**Qn.1** What was the objects acceleration at t = 2 seconds?
Reading off the graph, @ a time of 2 seconds, the object's acceleration = $2\ ms^{-2}\ North$

**Qn.2** What was the objects acceleration at t = 10 seconds?
Reading off the graph, @ a time of 10 seconds, the object's acceleration = $-3\ ms^{-2}\ North$ or $3\ ms^{-2}\ South$

**Qn.3** What was the object's change in velocity between t = 0 sec and t = 3 sec?

$\Delta v = ?$

$$\begin{aligned}\Delta v &= area \\ &= Rectangle\ (0 - 3\ sec) \\ &= b \times h \\ &= 3 \times 2 \\ &= 6\ ms^{-1}\ North\end{aligned}$$

**Qn.4** What was the object's change in velocity between t = 7 sec and t = 10 sec?

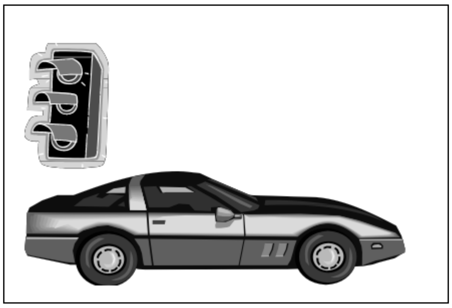
$\Delta v = ?$

$$\begin{aligned}\Delta v &= area \\ &= Rectangle\ (7 - 10\ sec) \\ &= b \times h \\ &= 3 \times -3 \\ &= -9\ ms^{-1}\ North\ or\ 9\ ms^{-1}\ South\end{aligned}$$

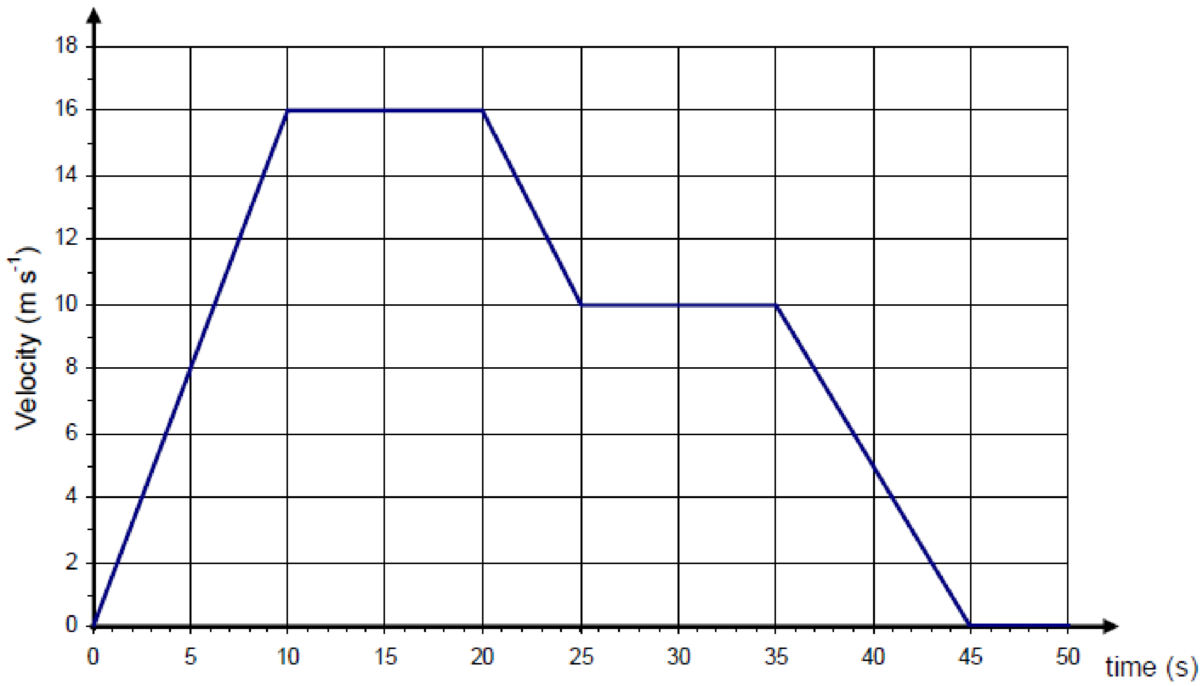
## Exam Style Questions

***Questions 1 to 5 refer to the following information***

The Doc is travelling to work in peak hour traffic. He is stopped at a set of traffic lights. When the light turns green he travels forward in the traffic until he comes to another red light and stops again.



A graph of the velocity of this part of his trip is given below




**Question 1.**
What is the Doc's speed at t = 5 seconds?

Reading off the graph, at a time of 5 seconds the corresponding velocity is 8 $ms^{-1}$

| |
|---|
| 8.0 $ms^{-1}$ |

**Question 2.**
What is the magnitude of Doc's acceleration at t = 5 seconds?
Acceleration at 5 seconds is measured by the gradient of the graph at that time

$$Acceleration = \frac{Rise}{Run} = \frac{\Delta V}{\Delta t} = \frac{16}{10} = 1.6\ ms^{-2}$$

| |
|---|
| 1.6 $ms^{-2}$ |

**Question 3.**
How far did the Doc travel in his journey between the two sets of traffic lights?

Area is found by calculating the area under the velocity – time graph
This can be done by breaking the graphs area into a combination of triangles and rectangles.
Alternatively the number of squares can be counted and multiplied by the area of one single square.

45.5 squares
@ 10 m per square

∴ total area = 45.5 x 10
= 455 m

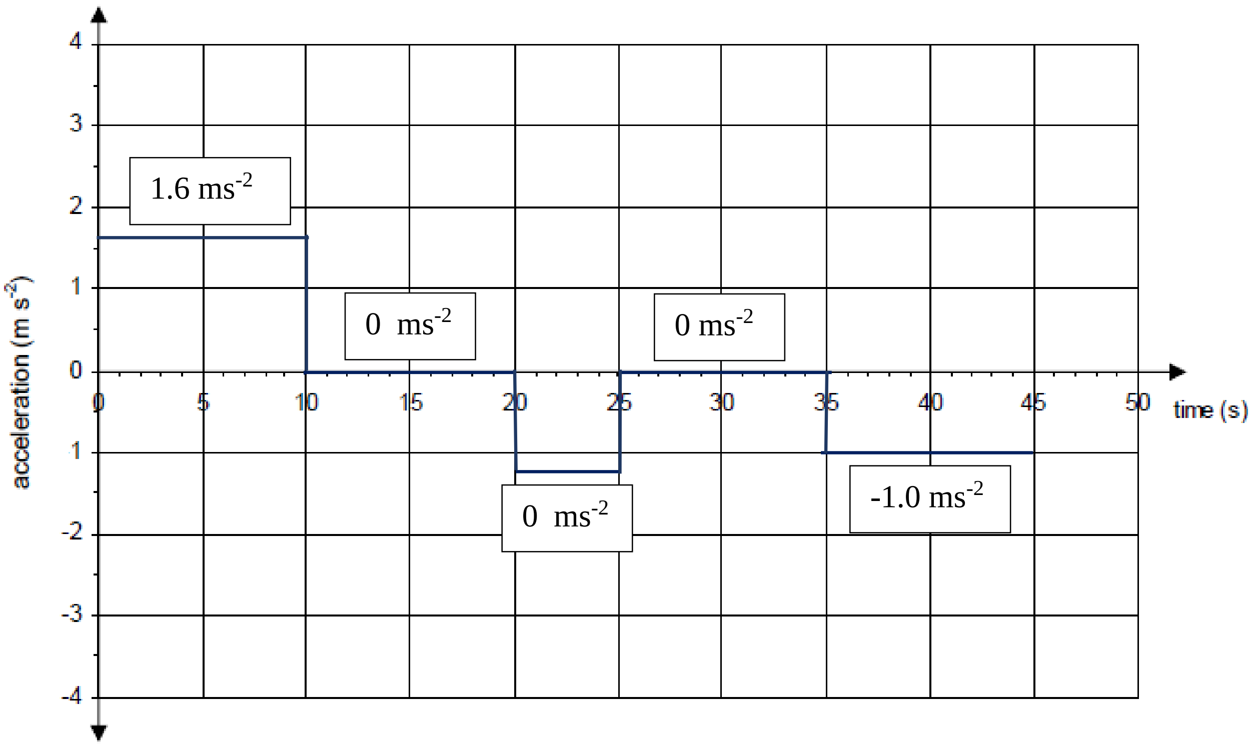
| |
|---|
| 455 m |

**Question 4.**
What was the Doc's average speed for his 45 second journey?

$$Average\ speed = \frac{distance}{time} = \frac{455}{45} = 10.1\ ms^{-1}$$

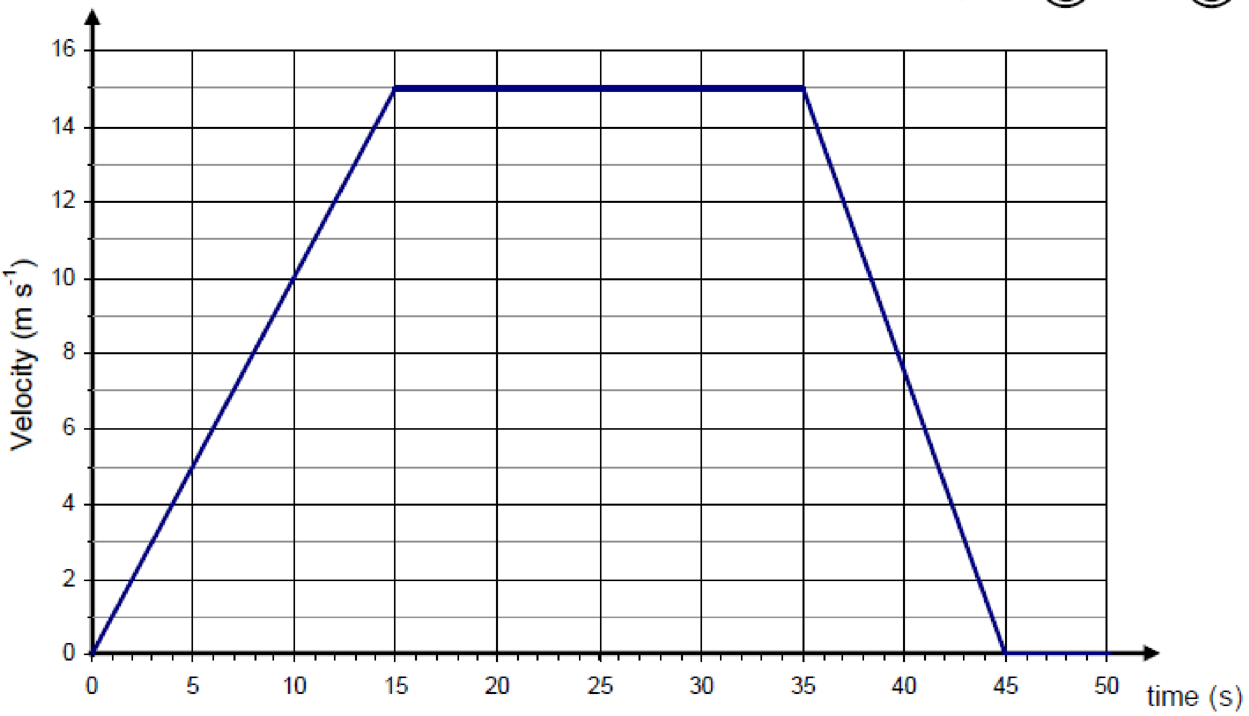
| |
|---|
| 10.1 $ms^{-1}$ |

**Question 5.**
On the following axes carefully draw the acceleration – time graph for the Doc's journey.



***Questions 6 to 10 refer to the following information***

Joffa is peddling like crazy as he is running late for school, and he knows he has Physics in period 1.
A graph of Joffa's speed for a 45 second section of his trip is given below.




Question 6.
What is Joffa's highest speed?

Reading off the graph, the highest speed is 15 $ms^{-1}$

| 15 $ms^{-1}$ |
|---|

**Question 7.**
What is the magnitude of Joffa's highest positive acceleration?

The highest positive acceleration (gradient) occurs between 0-15 seconds.

$$Acceleration = \frac{Rise}{Run} = \frac{\Delta V}{\Delta t} = \frac{15}{15} = 1.0\ ms^{-2}$$

| 1.0 $ms^{-2}$ |
|---|

**Question 8.**
How far did Joffa travel in this section of his journey to school?

Area is found by calculating the area under the velocity – time graph
In this case the shape under the graph is a simple trapezium.

$$\text{Area} = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(20 + 45)15$$

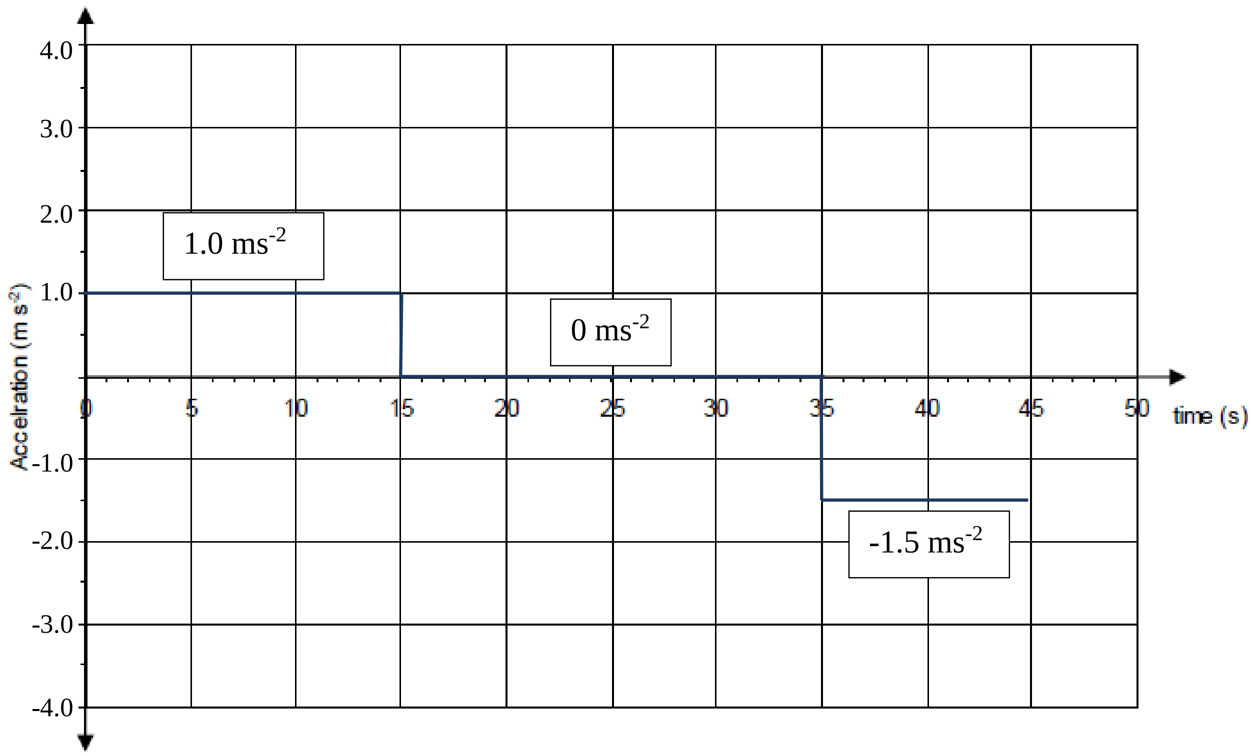
| |
|---|
| 487.5 m |

**Question 9.**
What was Joffa's average speed for this 45 second section of his journey?

$$Average\ speed = \frac{distance}{time} = \frac{487.5}{45} = 10.8\ ms^{-1}$$

| |
|---|
| 10.8 $ms^{-1}$ |

**Question 10.**
On the following axes carefully draw the acceleration – time graph for this 45 second section of Joffa's journey. Add a scale to the y-axis.



**Question 11.**

The Doc's old Kombi van is a bit of a snail and takes 21 seconds to reach a speed of 60 $kmh^{-1}$ from rest. Assuming it accelerates at a constant rate what is the magnitude of the Kombi's acceleration in $ms^{-2}$?

Acceleration = ?
$\Delta$velocity = 60 $kmh^{-1}$
= (60/3.6)
= 16.67 $ms^{-1}$
$\Delta$t = 21 sec

$$Acceleration = \frac{\Delta V}{\Delta t} = \frac{16.67}{21} = 0.79\ ms^{-2}$$

| 0.79 $ms^{-2}$ |
|---|