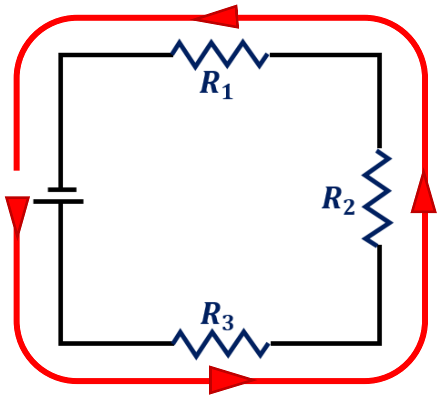
Section 1.2.2 – Circuit Analysis

Circuits can be classified as either ***series*** or ***parallel***;

Series Circuit

In a series circuit, coulombs of charge have only ***one possible path*** along which to travel. The circuit consists of one ***continuous loop***.



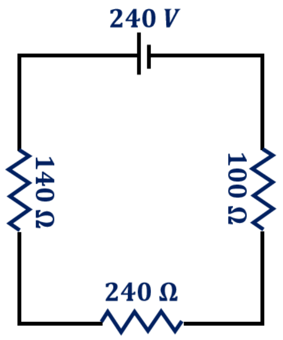
Within a ***series circuit***:

- The current remains constant:
  $\boldsymbol{I_T = I_1 = I_2 = I_3}$
- The sum (**Σ**) of the voltage drops of each component = Total (supply) voltage
  $\boldsymbol{V_T = V_1 + V_2 + V_3}$
- The Total (effective) resistance = the sum (Σ) of the individual resistances
  $\boldsymbol{R_T = R_1 + R_2 + R_3}$
- The Total power = the sum (Σ) of the power given off by the individual resistances
  $\boldsymbol{P_T = P_1 + P_2 + P_3}$

Task.1 A circuit consists of three resistors connected in series (100 Ω, 240 Ω & 140 Ω) to a 240 Volt DC supply

1. Draw a circuit diagram
2. Calculate the total resistance (R_T)
3. Calculate the current running through each resistor
4. Calculate the voltage drop across each resistor
5. Calculate the power given off (dissipated) by each resistor

1. Circuit diagram



2.

$R_T = ?$
$R_1 = 100\ \Omega$
$R_2 = 240\ \Omega$
$R_3 = 140\ \Omega$

$$R_T = R_1 + R_2 + R_3 = 100 + 240 + 140 = \mathbf{480\ \Omega}$$

3. Consider the <u>entire circuit</u> (as I is constant anywhere in the circuit)

$V_T = 240\ V$
$R_T = 480\ \Omega$
$I = ?$

$$V_T = IR_T$$
$$\therefore I = \frac{V_T}{R_T} = \frac{240}{480} = \mathbf{0.5\ A}$$

4. The voltage drop across each resistor

$\boldsymbol{V(100\ \Omega)}$		$\boldsymbol{V(240\ \Omega)}$		$\boldsymbol{V(140\ \Omega)}$	
$V = ?$ $R = 100\ \Omega$ $I = 0.5\ A$	$V = IR$ $= 0.5 \times 100$ $= 50\ V$	$V = ?$ $R = 240\ \Omega$ $I = 0.5\ A$	$V = IR$ $= 0.5 \times 240$ $= 120\ V$	$V = ?$ $R = 140\ \Omega$ $I = 0.5\ A$	$V = IR$ $= 0.5 \times 140$ $= 70\ V$

NB: $\boldsymbol{\Sigma}$ voltage drops $= 50\ V + 120\ V + 70\ V = 240\ V = V_T$

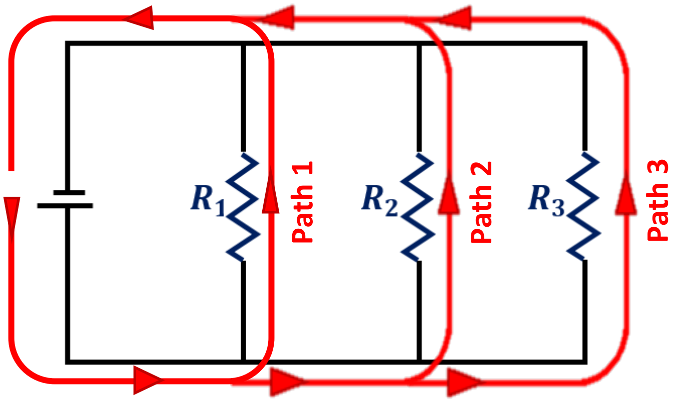
5. The power given off each resistor

$\boldsymbol{P(100\ \Omega)}$		$\boldsymbol{P(240\ \Omega)}$		$\boldsymbol{P(140\ \Omega)}$	
$P = ?$ $V = 50\ V$ $I = 0.5\ A$	$P = VI$ $= 50 \times 0.5$ $= 25\ W$	$P = ?$ $V = 120\ V$ $I = 0.5\ A$	$P = VI$ $= 120 \times 0.5$ $= 60\ W$	$P = ?$ $V = 70\ V$ $I = 0.5\ A$	$P = VI$ $= 70 \times 0.5$ $= 35\ W$

NB: $\boldsymbol{\Sigma}$ power dissipated $= 25\ W + 60\ W + 35\ W = 120\ W = P_T$

Parallel Circuit

In a parallel circuit, coulombs of charge have ***multiple pathways*** along which to travel. The circuit consists of ***one or more branches***.



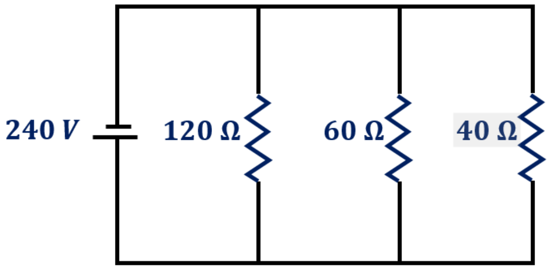
Within a ***parallel circuit***:

- The sum (Σ) of the current in each branch = Total (supply) current
  $$I_T = I_1 + I_2 + I_3$$
- The voltage drop across each branch = Total (supply) voltage
  $$V_T = V_1 = V_2 = V_3$$
- The Total (effective) resistance
  $$R_T = \left(R_1^{-1} + R_2^{-1} + R_3^{-1}\right)^{-1}$$
- The Total power = the sum (Σ) of the power given off by the individual resistances
  $$P_T = P_1 + P_2 + P_3$$

Task.2 A circuit consists of three resistors connected in parallel (120 Ω, 60 Ω & 40 Ω) to a 240 Volt DC supply

1. Draw a circuit diagram
2. Calculate the total resistance (R_T)
3. Calculate the voltage drop across each resistor
4. Calculate the current running through each resistor
5. Calculate the power given off (dissipated) by each resistor

1. Circuit diagram



2.
$R_T = ?$
$R_1 = 120\ \Omega$
$R_2 = 60\ \Omega$
$R_3 = 40\ \Omega$

$$R_T = \left(R_1^{-1} + R_2^{-1} + R_3^{-1}\right)^{-1}$$
$$= (120^{-1} + 60^{-1} + 40^{-1})^{-1}$$
$$= \mathbf{20\ \Omega}$$

3. By definition, the voltage in each branch will be equal to that of the total (supply) voltage.
$V_T = 240\ V$
$\therefore V(120\ \Omega) = V(60\ \Omega) = V(40\ \Omega) = V_T = 240\ V$

4. The current running through each resistor

$\boldsymbol{I(120\ \Omega)}$		$\boldsymbol{I(60\ \Omega)}$		$\boldsymbol{I(40\ \Omega)}$	
$I = ?$ $R = 120\ \Omega$ $V = 240\ V$	$V = IR$ $\therefore I = \frac{V}{R}$ $= \frac{240}{120}$ $= 2.0\ A$	$I = ?$ $R = 60\ \Omega$ $V = 240\ V$	$V = IR$ $\therefore I = \frac{V}{R}$ $= \frac{240}{60}$ $= 4.0\ A$	$I = ?$ $R = 40\ \Omega$ $V = 240\ V$	$V = IR$ $\therefore I = \frac{V}{R}$ $= \frac{240}{40}$ $= 6.0\ A$

NB: $\boldsymbol{\Sigma}$ **branch currents** $\mathbf{= 2.0\ A + 4.0\ A + 6.0\ A = 12.0\ A =}\ \boldsymbol{I_T}$

5. The power given off each resistor

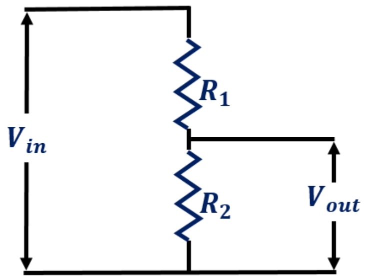
$\boldsymbol{P(120\ \Omega)}$		$\boldsymbol{P(60\ \Omega)}$		$\boldsymbol{P(40\ \Omega)}$	
$P = ?$ $V = 240\ V$ $I = 2.0\ A$	$P = VI$ $= 240 \times 2.0$ $= 480\ W$	$P = ?$ $V = 240\ \Omega$ $I = 4.0\ A$	$P = VI$ $= 240 \times 4.0$ $= 960\ W$	$P = ?$ $V = 240\ V$ $I = 6.0\ A$	$P = VI$ $= 240 \times 6.0$ $= 1440\ W$

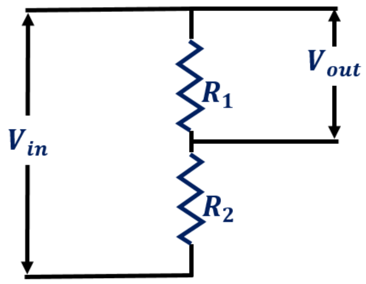
NB: $\boldsymbol{\Sigma}$ **power dissipated** $\mathbf{= 480\ W + 960\ W + 1440\ W = 2880\ W =}\ \boldsymbol{P_T}$

The Voltage Divider

A ***voltage divider*** is a relatively simple circuit comprising of **two (or more) resistors** ***($R_1, R_2 \ldots R_n$)*** placed in ***series***. An ***input voltage (V_{in})*** is applied and a reduced ***output voltage (V_{out})*** is produced.

The circuit diagrams are as follows:





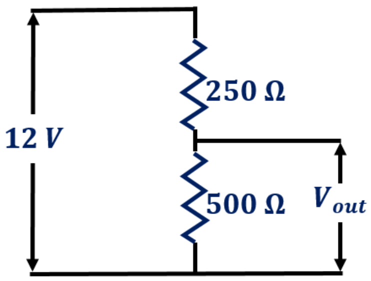
$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2} \qquad\qquad V_{out} = V_{in} \times \frac{R_1}{R_1 + R_2}$$

A general rule that can be applied for any voltage divider circuit is as follows:

$$\boldsymbol{V_{out} = V_{in} \times \frac{R_n}{R_T}}$$

Where V_{out} = Output voltage (V)
V_{in} = Input voltage (V)
R_n = the resistor across which the output voltage is measured (Ω)
R_T = the total resistance of the voltage divider (Ω)

Example.1 Calculate the output voltage ($\boldsymbol{V_{out}}$) for the following voltage divider circuit:



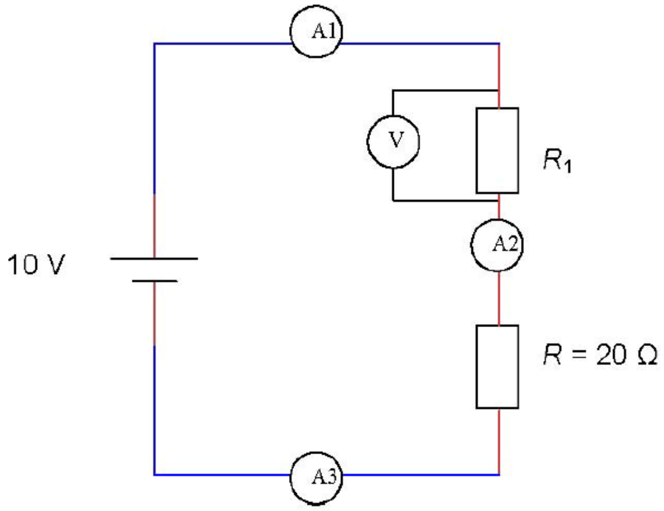
$V_{out} = ?$
$V_{in} = 12\ V$
$R_n = 500\ \Omega$
$R_T = 250\ \Omega + 500\ \Omega$
$\quad = 750\ \Omega$

$$V_{out} = V_{in} \times \frac{R_n}{R_T}$$
$$= 12 \times \frac{500}{750}$$
$$= \mathbf{8\ V}$$

Exam Styled Questions

The following information applies to Questions 1–3.

Jono sets up the circuit shown in the figure below. He knows that the total resistance for the whole circuit is 50 Ω.



Question 1
What is the value of R_1?

$R_1 = ?$
$R_2 = 20\ \Omega$
$R_T = 50\ \Omega$ (a series circuit)

$R_T = R_1 + R_2$
$50 = R_1 + 20$
$\therefore$ **$R_1 = 30\ \Omega$**

Question 2
What is the reading of the voltmeter?

Option.1 Using a voltage divider circuit

$V_{out} = ?$
$V_{in} = 10$ V
$R_n = 30\ \Omega$
$R_T = 20\ \Omega + 30\ \Omega$
$= 50\ \Omega$

$$V_{out} = V_{in} \times \frac{R_n}{R_T}$$
$$= 10 \times \frac{30}{50}$$
$$= 6\ V$$

Option.2 Using Ohms Law

Step.1 Calculate circuit current
$V_T = 10$ V
$R_T = 50\ \Omega$
$I_T = ?$

$I_T = \frac{V_S}{R_T}$
$I_T = \frac{10}{50}$
$= 0.2$ A

Step.2 Find V
V = ?
I = 0.2 A
$R_1 = 30\ \Omega$

$V = IR_1$
$= 0.2 \times 30$
= **6 Volts**

Question 3
Choose the correct answer.

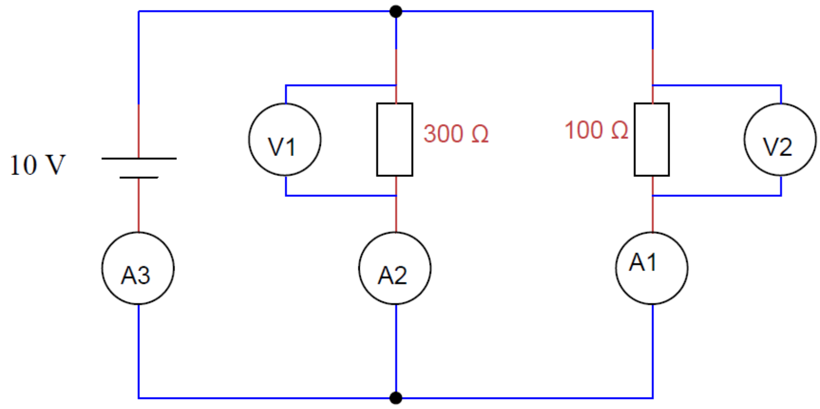
Which of the following is the relationship between the readings of the three ammeters?

A. A1 > A2 > A3
B. A1 < A2 < A3
C. A1 + A2 = A3
D. A1 = A2 = A3

D

The following information applies to Questions 4–6.

Jono then plays with his circuit a little, and comes up with the circuit shown below.



Question 4
What is the total resistance of the circuit?

R_T = ?
R_1 = 300 Ω
R_2 = 100 Ω

$$R_T = \left(R_1^{-1} + R_2^{-1}\right)^{-1}$$
$$= (300^{-1} + 100^{-1})^{-1}$$
$$= \mathbf{75\ \Omega}$$

Question 5
What is the reading on A1? Write your answer in milliamps.

Examine the second branch of the parallel circuit.

V_2 = 10 V
R_2 = 100 Ω
I_2 = ?

$$I_2 = \frac{V_2}{R_2}$$
$$= \frac{10}{100}$$
= 0.1 A
= **100 mA**

Question 6
Choose the correct answer.

Which of the following is the relationship of the readings of the three ammeters?

A. A1 > A2 > A3
B. A1 < A2 < A3
C. A1 + A2 = A3
D. A1 = A2 = A3

C

The following information applies to Questions 7 and 8.

Frankie is given three resistors: one 10 Ω, one 20 Ω, and one 30 Ω.

Question 7
What is the smallest resistance that Frankie can make with his three resistors? Explain your answer.

Smallest resistor = 5.45 Ω
Explanation: Place the three resistors in parallel to one another

R_T = ?
R_1 = 10 Ω
R_2 = 20 Ω
R_3 = 30 Ω

$$\begin{aligned} R_T &= \left(R_1{}^{-1} + R_2{}^{-1} + R_3{}^{-1}\right)^{-1} \\ &= (10^{-\mathbf{1}} + 20^{-\mathbf{1}} + 30^{-\mathbf{1}})^{-\mathbf{1}} \\ &= \mathbf{5.45\ \Omega} \end{aligned}$$

Question 8
What is the largest resistance that Frankie can make with his three resistors? Explain your answer.

Largest resistor = 60 Ω
Explanation: Place the three resistors in series to one another

R_T = ?
R_1 = 10 Ω
R_2 = 20 Ω
R_3 = 30 Ω

$$\begin{aligned} R_T &= R_1 + R_2 + R_3 \\ &= 10 + 20 + 30 \\ &= \underline{\mathbf{60\ \Omega}} \end{aligned}$$