Mutually exclusive events

Mutually Exclusive Event

Mutually exclusive events <u>cannot</u> occur as the same time.
For example, it is impossible to toss a coin and get a "head" and "tails", it is one or the other.

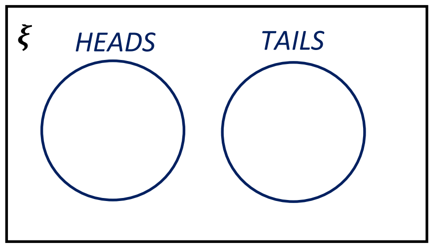
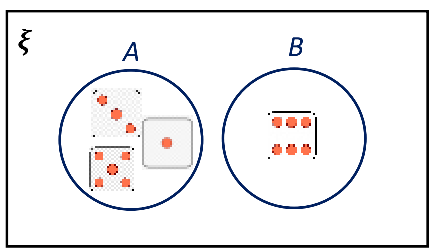


Figure 1

The Venn diagram for mutually exclusive events is shown above in figure 1.

Example 1

A fair six-sided die is rolled. Consider the following events
ξ = {1, 2, 3, 4, 5, 6}
A = {spinning an odd number}
B = {spinning a 6}



$$\Pr(A) = \frac{n(A)}{n(\xi)} = \frac{3}{6} = \frac{1}{2}$$

$$\Pr(B) = \frac{n(B)}{n(\xi)} = \frac{1}{6}$$

$$\Pr(A\ or\ B) = \Pr(A) + \Pr(B) = \frac{1}{2} + \frac{1}{6}$$
$$= \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\Pr(A\ and\ B) = 0$$

<u>Addition rule for mutually exclusive events</u>

$$\boldsymbol{Pr(A\ or\ B) = Pr(A) + Pr(B)}$$
$$\boldsymbol{Pr(A \cup B) = Pr(A) + Pr(B)}$$

$$\mathbf{Pr}(\boldsymbol{A \cap B}) = \mathbf{0}$$

NB: When two events are mutually exclusive, it is impossible for both of them to occur at the same time.

Not Mutually Exclusive Event

Events that are **not mutually exclusive** can occur at the same time.

For example, it is possible to roll a fair six-sided die and get an "even number" and "the number six".

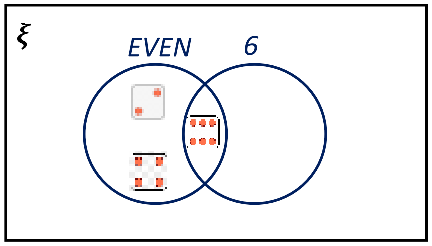


Figure 2

The Venn diagram for an event that is not mutually exclusive is shown above in figure 2.

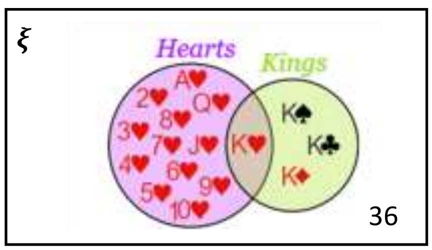
Example 2

A deck of cards has 52 cards in total. Consider the following events

ξ = {52 cards}

A = {drawing a King}

B = {drawing a Heart}



$$\Pr(A) = \frac{n(A)}{n(\xi)} = \frac{4}{52} = \frac{1}{13}$$

$$\Pr(B) = \frac{n(B)}{n(\xi)} = \frac{13}{52} = \frac{1}{4}$$

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$$
$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$= \frac{1}{13} + \frac{1}{4} - \frac{1}{52}$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$$

General addition rule for events

$$\boldsymbol{Pr(A \text{ or } B) = Pr(A) + Pr(B) - \mathrm{Pr}(\mathrm{A\ and\ B})}$$
$$\boldsymbol{Pr(A \cup B) = Pr(A) + Pr(B) - \mathrm{Pr}(\mathrm{A} \cap \mathrm{B})}$$