## Venn diagrams and sample space

## Venn diagrams

A Venn diagram consists of a rectangle containing one or more circular areas.


The rectangle represents the universal set ( $\xi$ ).
The universe set is not "everything in existence", but "everything that we're working with right now". The universal set represents every element for a particular situation being examined.

The circles represent particular outcomes or groups of the sample space
A represents Group A
B represents Group B

## Example 1

In a class of 20 students, 10 study Art, 5 study Biology and 3 study both.
Construct a Venn diagram, where;
$\xi=\{$ students in the class $\}$
A $=\{$ student who study Art $\}$
$B=\{$ students who study Biology\}


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If 1 student were selected from the class what would be the probability that the student is:
i. An Art student
ii. A Biology student
iii. An Art student but not a Biology student
iv. A Biology student but not an Art student
v. Neither an Art or Biology student
vi. Both an Art and Biology student
i. An Art student

$$
\begin{gathered}
n(A)=10 \\
\operatorname{Pr}(A)=\frac{n(A)}{n(\xi)}=\frac{10}{20}=\frac{1}{2}
\end{gathered}
$$


ii. A Biology student
$n(B)=5$
$\operatorname{Pr}(B)=\frac{n(B)}{n(\xi)}=\frac{5}{20}=\frac{1}{4}$

iii. An Art student but not a Biology student
$n(A$ and not $B)=7$
$n\left(A \cap B^{\prime}\right)=7$
$\operatorname{Pr}\left(A \cap B^{\prime}\right)=\frac{n\left(A \cap B^{\prime}\right)}{n(\xi)}=\frac{7}{20}$

iv. A Biology student but not an Art student
$n(B$ and not $A)=2$
$n\left(B \cap A^{\prime}\right)=2$
$\operatorname{Pr}\left(B \cap A^{\prime}\right)=\frac{n\left(A \cap B^{\prime}\right)}{n(\xi)}=\frac{2}{20}=\frac{1}{10}$

v. Neither an Art or Biology student $n(\boldsymbol{n o t} A$ and not $B)=8$
$n\left(A^{\prime} \cap B^{\prime}\right)=8$
$\operatorname{Pr}\left(B \cap A^{\prime}\right)=\frac{n\left(A \cap B^{\prime}\right)}{n(\xi)}=\frac{8}{20}=\frac{2}{5}$

vi. Both an Art and Biology student
$n(A$ and $B)=3$
$n(A \cap B)=3$
$\operatorname{Pr}(A \cap B)=\frac{n(A \cap B)}{n(\xi)}=\frac{3}{20}$


Tables
In a similar way, a table can be used to show similar information. Consider the following two -way table.

|  | $\boldsymbol{B}$ | $\boldsymbol{n o t} \boldsymbol{B}\left(\boldsymbol{B}^{\prime}\right)$ |
| :---: | :---: | :---: |
| $\boldsymbol{A}$ | $A$ and $B(A \cap B)$ | $A$ and not $B\left(A \cap B^{\prime}\right)$ |
| $\operatorname{not} \boldsymbol{A}\left(\boldsymbol{A}^{\prime}\right)$ | not $A$ and $B\left(A^{\prime} \cap B\right)$ | not $A$ and not $B\left(A^{\prime} \cap B^{\prime}\right)$ |

Example 2
In a class of 20 students, 10 study Art, 5 study Biology and 3 study both. Construct a two-way table.

The number of each group is as follows:

|  | Biology $(B)$ | not Biology $\left(B^{\prime}\right)$ | Total |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{\operatorname { A r t }}(\boldsymbol{A})$ | 3 | 7 | 10 |
| $\boldsymbol{n o t} \boldsymbol{A r t}\left(\boldsymbol{A}^{\prime}\right)$ | 2 | 8 | 10 |
| Total | 5 | 15 | 20 |

The probability of each group is as follows:

|  | Biology $(B)$ | not Biology $\left(\boldsymbol{B}^{\prime}\right)$ |
| :---: | :---: | :---: |
| Art $(\boldsymbol{A})$ | $3 / 20$ | $7 / 20$ |
| not $\boldsymbol{A r t}\left(\boldsymbol{A}^{\prime}\right)$ | $2 / 20$ | $8 / 20$ |

