Section 3.3.9 – Einstein's Special Relativity (Time dilation)

Time dilation

$$t = t_o\gamma$$

Where t is the apparent or dilated time (sec)
t_0 is the proper time (sec)
γ is the Lorentz factor (unit less)

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where γ is the Lorentz factor (unit less)
v is the object speed (ms^{-1})
c is the speed of light (ms^{-1})

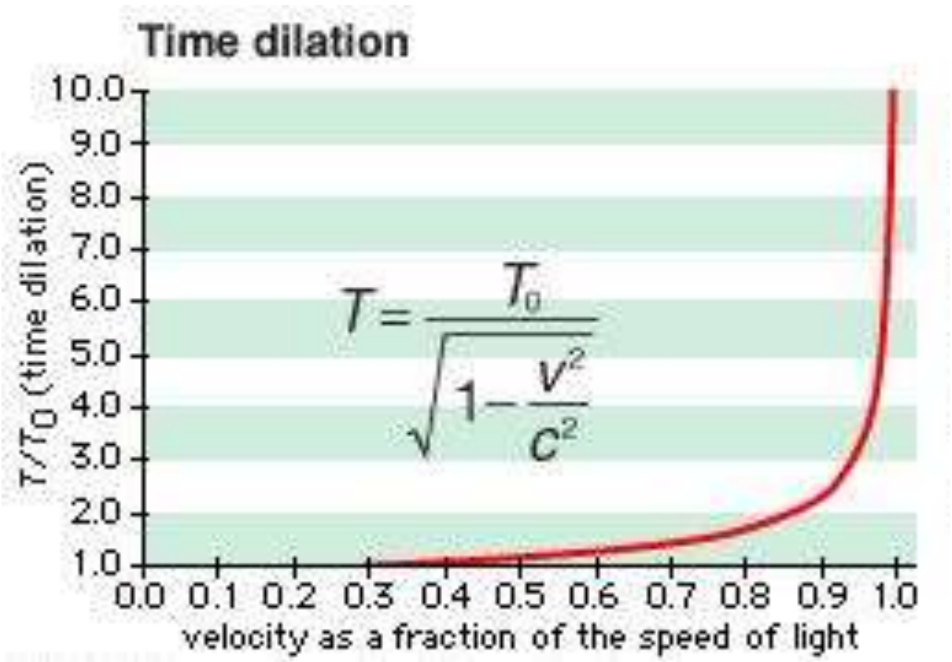
Observers in reference frames which are **moving relative to each other** will measure **different time intervals** between the same two events.

Proper time (t_0) refers to the time interval between two events measured by an observer for whom the two events occur at the same location in space. Proper time is measured in the **same inertial frame of reference** as the **moving object**.

An observer in any other reference frame will measure a **dilated (greater) time interval (t)** between the same two events.

Time moves slower upon a moving object relative to a stationary observer in their inertial frame of reference.

NB: At everyday speeds, you don't observe any noticeable time dilation.
As the object speed approaches c, γ increases and time dilation is observed
When the object speed is close to that of c , γ approaches ∞ and time effectively stops
As $\gamma \geq 1, t \geq t_0$



Exam Styled Questions

Question 1 (2018 VCAA Qn 16 SA)

Quasars are among the most distant and brightest objects in the universe. One quasar (3C446) has a brightness that changes rapidly with time.

Scientists observe the quasar's brightness over a 20-hour time interval in Earth's frame of reference. The quasar is moving away from Earth at a speed of 0.704c (γ = 1.41).

Calculate the time interval that would be observed in the quasar's frame of reference. Show your working.

$t = 20\ hours$ (external frame of reference - Earth)
$t_0 = ?\ (objects\ frame\ of\ reference - quasar)$
$\gamma = 1.41$

$$t = t_0\gamma$$
$$\therefore\ t_0 = \frac{t}{\gamma}$$
$$= \frac{20}{1.41}$$
$$= 14.2\ hours$$

14.2	***h***

Question 2 (2016 VCAA Qn 1 MC)

Anna and Barry have identical quartz clocks that use the precise period of vibration of quartz crystals to determine time. Barry and his clock are on Earth. Anna accompanies her clock on a rocket travelling at constant high velocity, v, past Earth and towards a space lab (which is stationary relative to Earth), as shown in Figure 1.

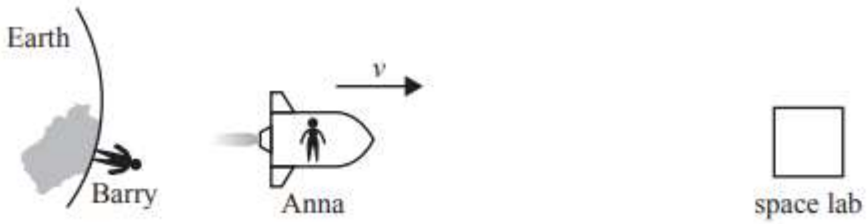


Figure 1

Which one of the following statements correctly describes the behaviour of these two clocks?

A. The period of vibration in Anna's clock (as observed by Anna) will be shorter than the period of vibration in Barry's clock (as observed by Barry).
B. The period of vibration in Anna's clock (as observed by Anna) will be longer than the period of vibration in Barry's clock (as observed by Barry).
C. The period of vibration in Anna's clock (as observed by Anna) will be the same as the period of vibration in Barry's clock (as observed by Barry).
D. Only the time on Barry's clock is reliable because it is in a frame that is not moving.

C

Although both observers are in different inertial frames each will observe time passing at the same rate in their own frames.

Use the following information to answer Questions 3–5.

Spacecraft S66 is travelling at high speed away from Earth carrying a highly accurate atomic clock. Another spacecraft, T50, is travelling in the opposite direction to S66, as shown in Figure 1.

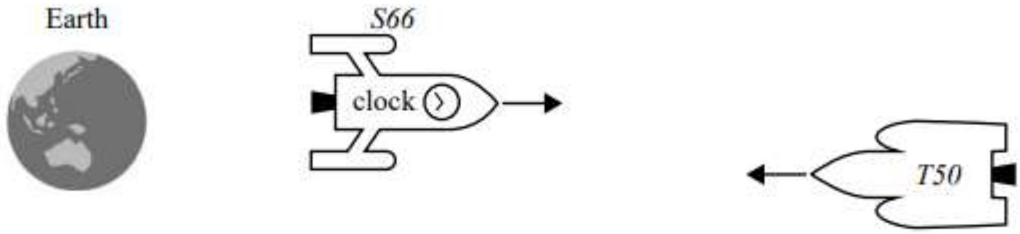


Figure 1

Question 3 (2015 VCAA Qn 4 MC)
Which one of the following observers will be able to measure proper time using this clock?

A. an astronaut seated on spacecraft S66 five metres behind the clock's position
B. a scientist on Earth at the clock's original position
C. no observer can measure proper time since light within the clock moves at the speed of light
D. the navigator of the other spacecraft, T50, travelling at the moment when that navigator is opposite the clock

A

An astronaut seated on spacecraft S66 is in the <u>same inertial frame of reference</u> as the clock and can therefore measure proper time using this clock.

Question 4 (2015 VCAA Qn 5 MC)
An observer, E, on Earth emits a short radio pulse to spacecraft S66, which reflects it directly back towards the observer. The time elapsed for E between sending and receiving the pulse is 20.0 ms. Which one of the following is true?

A. According to E, spacecraft S66 was more than 3000 km away when the pulse reached it.
B. According to E, the pulse took longer to reach spacecraft S66 than it did to return from spacecraft S66 to E.
C. The 20.0 ms interval measured by E is not a proper time because the radio pulse travelled away and back.
D. According to spacecraft S66, the time interval between the signal being sent and being received back by E is greater than 20.0 ms

D

A is wrong, E would calculate the distance at 3000 km,
B: wrong times will be the same,
C: The time interval occurred is E's frame so it is a proper time,
D: In the frame of S66 a longer time would be measured due to time dilation

Question 5 (2015 VCAA Qn 6 MC)

The clock on spacecraft S66 has an indicator showing time intervals of 0.100 s. The navigator of spacecraft T50 observes that the duration of those time intervals is 0.115 s.

What is the relative speed of spacecraft S66 to spacecraft T50?

A. 0.80c
B. 0.70c
C. 0.60c
D. 0.50c

$\boxed{\textbf{D}}$

Step.1 Find $\boldsymbol{\gamma}$

$t_0 = 0.100\ sec$ (clock's frame of reference S66)
$t = 0.115$ sec($observers\ frame\ of\ referefence\ \ T50$)
$\gamma = ?$

$$t = t_0\gamma$$
$$\therefore \gamma = \frac{t}{t_0}$$
$$= \frac{0.115}{0.100}$$
$$= 1.15$$

Step.2 Find $\boldsymbol{v}$

$\gamma = 1.15$
$c = 3.0 \times 10^8 ms^{-1}$
$v = ?$

$$v = c\sqrt{1 - \frac{1}{\gamma^2}}$$
$$v = c\sqrt{1 - \frac{1}{1.15^2}}$$
$$v = 0.49c$$

Use the following information to answer Questions 6–8.

Two spacecraft travel in opposite directions, with spacecraft Ajax travelling at a speed of 0.5c and spacecraft Hector travelling at a speed of 0.4c. Both are travelling relative to the inertial frame of the galaxy. The situation is shown in Figure 1.

Figure 1

A radio signal is emitted by Ajax towards Hector. The navigator of Hector uses the classical physics understanding of radio waves travelling at a speed relative to a medium fixed with respect to the galaxy.

Question 6 (2014 VCAA Qn 4 MC)

Using this <u>classical understanding</u>, the speed of the radio signal relative to Hector is expected to be

A. c
B. 0.6c
C. 0.5c
D. 0.1c

B

By comparison with sound in air, the speed of radio signal relative to galaxy = c towards Hector. Hector is moving away at 0.4c, so the radio signal would be gaining on Hector at 0.6c.

Question 7 (2014 VCAA Qn 5 MC)

Measured in the frame of Ajax, the radio signal reaches Hector 0.0100 s after it is emitted by Ajax. According to the navigator of Ajax, who is correctly using special relativity, how far did the radio signal travel between leaving Ajax and reaching Hector?

A. 3000 km
B. 300 km
C. 4200 km
D. 1500 km

A

Distance = speed x time = 3 x 108 x 0.0100 = 3 x 106 m = 3000 km.

Question 8 (2014 VCAA Qn 6 MC)
How can proper time be measured for the interval between the radio signal being emitted on Ajax and the signal reaching Hector?

A. Use measurements made by the crew on Ajax.
B. Use measurements made by the crew on Hector.
C. Use measurements made by an observer stationary at the point where the signal was emitted.
D. No single observer can measure proper time for this case.

D

Neither crew can measure the proper time. C is the same as A. D is best answer.