Section 3.2.4 – Effective Interest Rate

VCAA "Dot Points"

Compound interest investments and loans, including:

- difference between nominal and effective interest rates and the use of effective interest rates to compare investment returns and the cost of loans when interest is paid or charged, for example, daily, monthly, quarterly

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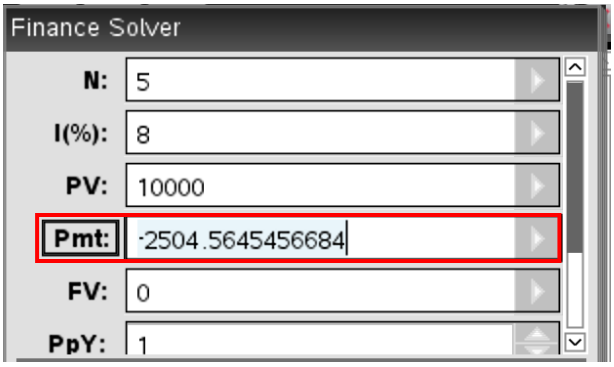
Nominal Interest Rate → Effective Interest Rate

Previously we have investigated the effect of using different compounding periods upon the amount of interest either paid or earned across an investment or loan.

Consider a personal loan for $10,000 at 8%p.a. paid in full over 5 years if **compounded**:

1. Annually
2. Monthly
3. Weekly

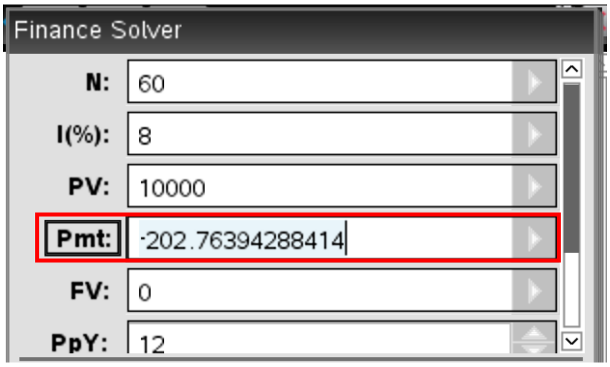
Option 1 – Annual payments



5 annual payments @ $2504.56 = 12522.80

Interest = payments – loan amount
= 12522.80 – 10000
= **<u>$2522.80</u>**

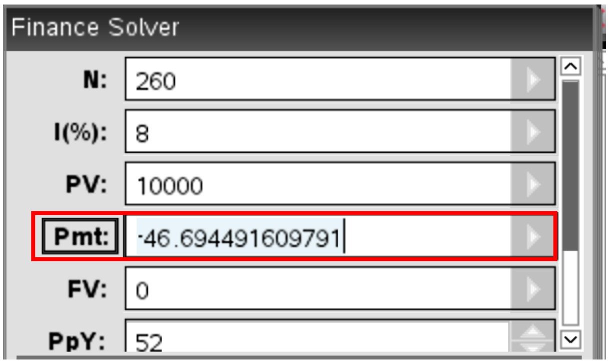
Option 2 – Monthly payments



60 monthly payments @ $202.76 = 12165.60

Interest = payments – loan amount
= 12165.60 – 10000
= **<u>$2165.60</u>**

Option 3 – Weekly payments



260 weekly payments @ \$46.69 = 12139.40

Interest = payments – loan amount
= 12139.40 – 10000
= **<u>\$2139.40</u>**

Each of the above options involves an interest rate of 8% p.a. However, the **compounding terms** (ie. Annual, monthly, weekly etc.) result in you paying **more or less interest** over the term of your loan.

In order to compare each possible terms you need to consider the **Effective Annual Interest Rate.**

$$r_e = \left(\left(1 + \frac{\frac{r}{n}}{100}\right)^n - 1\right) \times 100$$

where:

r_e = the effective annual interest rate
r = the nominal rate (as a decimal)
n = the number of compounding periods per year

Task:
Calculate the effective annual interest rate if you were to invest \$10,000 at 8% p.a. over 5 years via three different investment strategies; annually, monthly & weekly.

<u>Option.1</u> **8% p.a. compounding annually**

r_e = ?
n = 1
r = 8% p.a.

$$r_e = \left(\left(1 + \frac{\frac{r}{n}}{100}\right)^n - 1\right) \times 100$$
$$= \left(\left(1 + \frac{\frac{8}{1}}{100}\right)^1 - 1\right) \times 100$$
= 8.0 % p.a.

$$\left(\left(1+\frac{\frac{8}{1}}{100}\right)^1 - 1\right)\cdot 100 \qquad 8.$$

Or alternatively, you can use the effective interest rate feature on your TI-nspire CX CAS calculator to find the effective rate of interest:

Nominal interest rate (% p.a.)

Effective interest rate function → eff(8,1) 8.

Compounding periods per year

Effective interest rate function

<u>Option.2</u> **8% p.a. compounding monthly**

r_e = ?
n = 12
r = 8% p.a.

$$r_e = ((1+\frac{\frac{r}{n}}{100})^n - 1) \times 100$$
$$= ((1+\frac{\frac{8}{12}}{100})^{12} - 1) \times 100$$
= 8.30 % p.a.

$$\left(\left(1+\frac{\frac{8}{12}}{100}\right)^{12}-1\right)\cdot 100 \quad 8.29995$$

Or alternatively using your TI-nspire CX CAS calculator:

Nominal interest rate (% p.a.)

Effective interest rate function → eff(8,12) 8.29995

Compounding periods per year

Effective interest rate function

<u>Option.3</u> **8% p.a. compounding weekly**

r_e = ?
n = 52
r = 8% p.a.

$$r_e = ((1+\frac{\frac{r}{n}}{100})^n - 1) \times 100$$
$$= ((1+\frac{\frac{8}{52}}{100})^{52} - 1) \times 100$$
= 8.32 % p.a.

$$\left(\left(1+\frac{\frac{8}{52}}{100}\right)^{52}-1\right)\cdot 100 \quad 8.32205$$

Or alternatively using your TI-nspire CX CAS calculator:

Nominal interest rate (% p.a.)

Effective interest rate function → eff(8,52) 8.32205

Compounding periods per year

Effective interest rate function

Example.1
Calculate the effective annual interest rate for a compounding interest rate of 5.75% p.a. charged on a loan with monthly repayments over 2 years.

$r_e = ?$
$n = 12$
$r = 5.75\%$ p.a.

$$r_e = \left(\left(1 + \frac{\frac{r}{n}}{100}\right)^n - 1\right) \times 100$$
$$= \left(\left(1 + \frac{\frac{5.75}{12}}{100}\right)^{12} - 1\right) \times 100$$
$= 5.9$ % p.a.

$$\left(\left(1+\frac{\frac{5.75}{12}}{100}\right)^{12}-1\right)\cdot 100 \qquad 5.90398$$

NB: the full term period of 2 years <u>does not</u> impact upon the effective annual interest rate.

The effective annual interest rate is 5.9% p.a.

Or alternatively using your TI-nspire CX CAS calculator:

Nominal interest rate (% p.a.)

Effective interest rate function → eff(5.75,12) 5.90398

Compounding periods per year

Effective interest rate function

Effective Interest Rate → Nominal Interest Rate
In some scenarios you will be given the effective interest rate and be asked to "work back" to find the original nominal interest rate. Consider the following example.

Example.2
If the effective annual interest rate is 8.2% p.a. on a loan with monthly repayments for 4 years, then what would be the nominal compounding interest rate?

$r_e = 8.2$ % p.a.
$n = 12$
$r = ?$

$$r_e = \left(\left(1 + \frac{\frac{r}{n}}{100}\right)^n - 1\right) \times 100$$
$$8.2 = \left(\left(1 + \frac{\frac{r}{12}}{100}\right)^{12} - 1\right) \times 100$$

$$\text{solve}\left(8.2=\left(\left(1+\frac{\frac{r}{12}}{100}\right)^{12}-1\right)\cdot 100, r\right)$$
$$r=-2407.91 \text{ or } r=7.90705$$

NB: the full term period of 4 years <u>does not</u> impact upon the effective annual interest rate.

The answer must be a positive percentage.

∴ compounding interest rate = 7.91% p.a.

The compounding interest rate is 7.91% p.a. compounded monthly.

Or alternatively you can use the nominal interest rate feature on your TI-nspire CX CAS calculator to find the nominal rate of interest:

Effective interest rate (% p.a.)

Nominal interest rate function → nom(8.2,12) 7.90705

Compounding periods per year

Nominal interest rate (% p.a.)

Exam Styled Questions – Multiple Choice

Question 1
(2016 VCAA Sample Exam 1 Section A - Qn 19)

Eva has $1200 that she plans to invest for one year. One company offers to pay her interest at the rate of 6.75% per annum compounding daily. The effective annual interest rate for this investment would be closest to

A. 6.75%
B. 6.92%
C. 6.96%
D. 6.98%
E. 6.99%

eff(6.75,365) 6.98236

∴ Option D

$\boxed{\textbf{D}}$