Section 3.1.3 - Log(base 10) Scale

VCAA "Dot Points"
Investigating data distributions, including:

- *review of representation, display and description of the distributions of numerical variables: the use of a log (base 10) scale to display data ranging over several orders of magnitude and their interpretation in powers of ten*

The Logarithmic Scale

A **logarithmic scale** is a nonlinear scale used to **compress** a large range of quantities. This scale is used where the quantities being measure increase **exponentially**.

Example 1

Value	1	10	100	1000	10000	100000	1000000	10000000
Exponential	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7
$\log_{10}$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**

NB: There is an obvious relationship between a value, its exponential form and its $\log_{10}$ form.

Example 2

Consider the following populations of towns and cities in Victoria.

[Source: 2011 population census]

Melbourne	3,707,530
Geelong	143,921
Ballarat	85,000
Horsham	15,292
Churchill	4,750
Wilcannia	688
Yanac	84

Express these as $\log_{10}$ values.

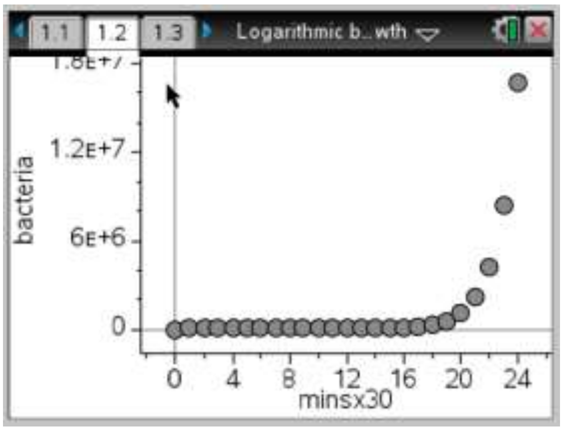
Melbourne	$\log_{10}(3,707,530)$	=	6.57
Geelong	$\log_{10}(143,921)$	=	5.16
Ballarat	$\log_{10}(85,000)$	=	4.93
Horsham	$\log_{10}(15,292)$	=	4.18
Churchill	$\log_{10}(4,750)$	=	3.68
Wilcannia	$\log_{10}(688)$	=	2.84
Yanac	$\log_{10}(84)$	=	1.92

NB: Melbourne's population is between 1000000 (10^6) & 10000000 (10^7).
So when the $\log_{10}$ is calculated you would expect the answer to be between 6 & 7.
Whereas Yanac's population is between 10 (10^1) & 100 (10^2).
So taking the $\log_{10}$ of Yanac's population will provide an answer between 1 & 2.

Example 3

Raw chicken when left out at room temperature will grow bacteria which doubles in number every half hour. This is an example of an **exponential growth**.

Task.1: Assuming you start with a single bacterium cell. Graph the bacterial growth over a 12 hour period using a linear scale.



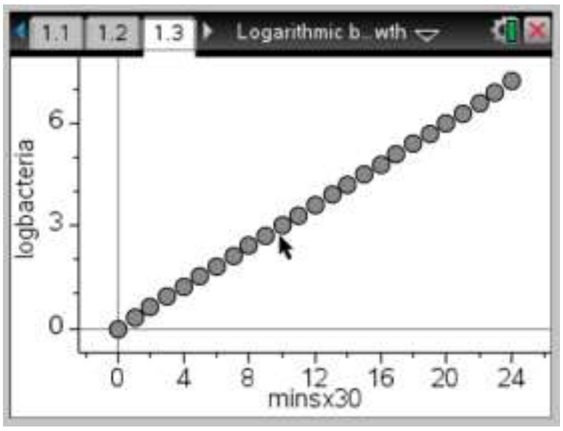
NB: 6E+6 as shown on the vertical axis of the TI-Nspire graph represents 6 "Exponential" to the power of 6.

ie. 6E+6 = 6×10^6 (or rather 6 million bacteria)

The orders of magnitude shown on this graph are massively different.

Values vary from 1 to 16.8 million bacteria

Task.2: Repeat the above graph. However, this time use a logarithmic to the base 10 scale for the bacteria number.



NB: the logarithmic scale provides a simple 0 to 6 scale for the population. Where each increase actually represents an increase in the order of magnitude.

ie. $\log_{10}(1) = 0$
$\log_{10}(10^1) = 1$
$\log_{10}(10^2) = 2$
$\log_{10}(10^3) = 3$
$\log_{10}(10^4) = 4$
$\log_{10}(10^5) = 5$

The bacteria values have been **compressed** by the logarithmic scale.

YouTube Video: https://www.youtube.com/watch?v=uWmZmr31K8I

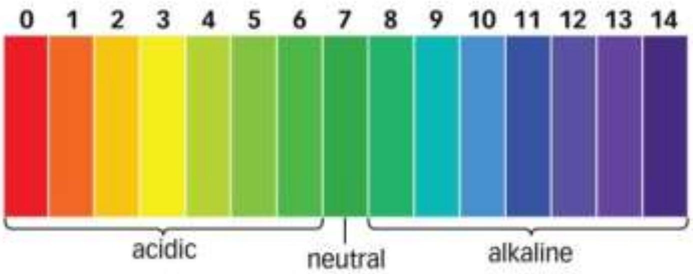
Expanding from a $\log_{10}$ value

Whilst it is clear to see that large orders of magnitudes can be compressed using a logarithmic scale. How can you reverse the procedure and expand from a compressed log value back to an original value?

Log10(x) = 5	Log10(x) = 3.25	Log10(x) = 2.5
$\therefore x = 10^5$	$\therefore x = 10^{3.25}$	$\therefore x = 10^{2.5}$
= 10,0000	= 1778	= 316

Example 4

The pH scale for chemical acidity and ranges from 0-14 and uses a logarithmic scale.

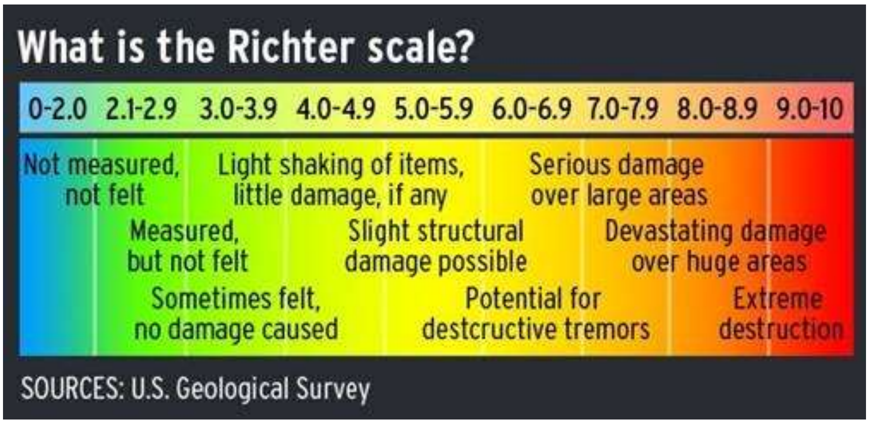


How many more times acidic is a solution of pH 5 than a solution of pH 3.

(ie. $\frac{10^5}{10^3} = 10^{(5-3)} = 10^2 = 100$)

Example 5

The Richter scale assigns a magnitude number to quantify the size of an earth quake from 1-10 and uses a logarithmic scale.

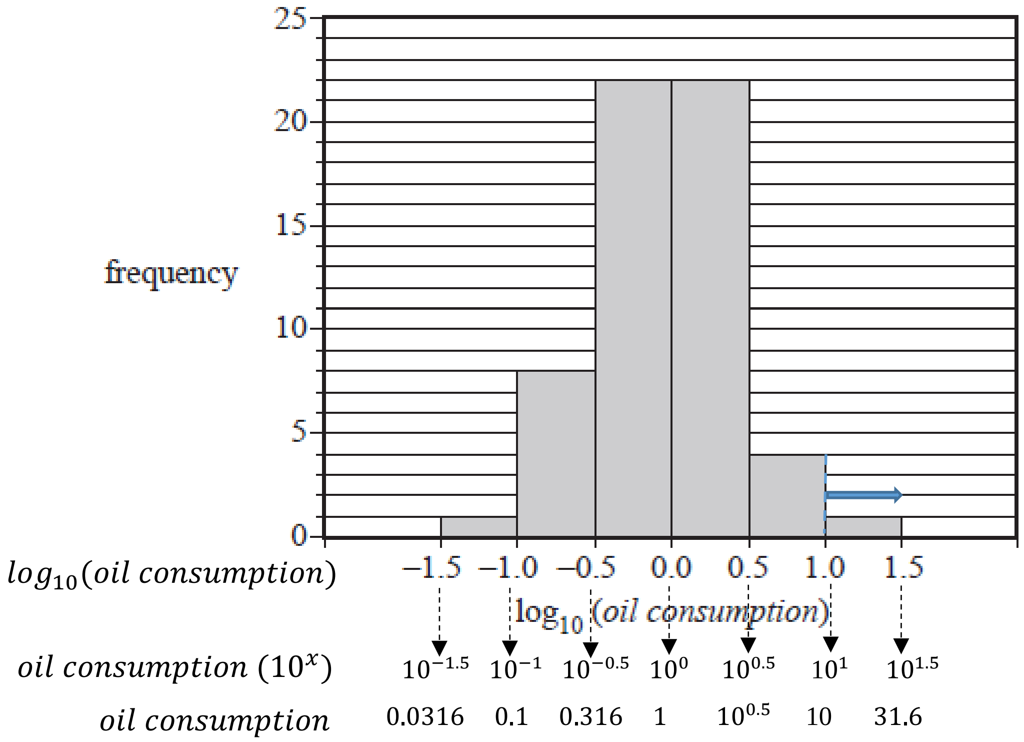


How many times greater is the intensity of a Richter scale reading of 8 than a Richter scale reading of 5?

(ie. $\frac{10^8}{10^5} = 10^{(8-5)} = 10^3 = 1000$)

Exam Styled Questions (current study design) – Multiple Choice

Question 1
(2016 Sample Exam 1 Section A – Qn 7)



The histogram above displays the distribution of the annual per capita oil consumption (tonnes) for 58 countries plotted on a log scale. The percentage of countries with an annual per capita oil consumption of more than 10 tonnes is closest to

A. 1%
B. 2%
C. 27%
D. 57%
E. 98%

$\boxed{\textbf{B}}$

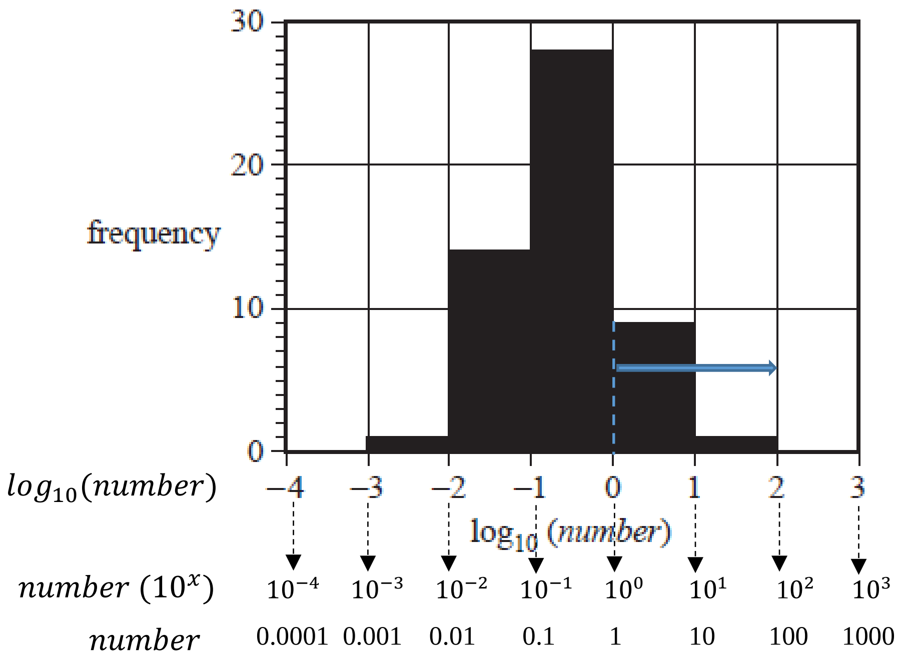
$\text{solve}\left(\log_{10}(x)=1, x\right) \qquad x=10$

$\text{Log}_{10}(10) = 1$
∴the column representing an annual per capita of 10 or above is the far right column (1.0 – 1.5)
Frequency (1.0→1.5) = 1

However, this represents a percentage of $\frac{1}{58} \times 100 = 1.72\%$. This is approximately 2%.
∴ **Option B**

Question 2
(2016 Exam 1 Section A – Qn 7)

The histogram below shows the distribution of the number of billionaires per million people for 53 countries plotted on a log10 scale.



Based on this histogram, the number of countries with one or more billionaires per million people is

A. 1
B. 3
C. 8
D. 9
E. 10

$\text{solve}\left(\log_{10}(x)=0,x\right) \qquad x=1$

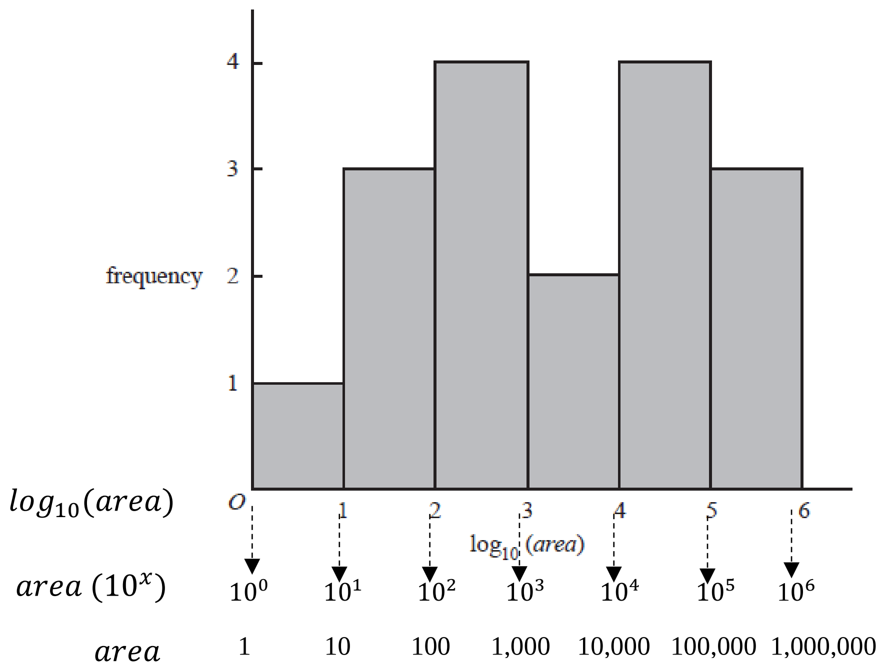
Recall $\log_{10}(x) = 0, \therefore x = 10^0 = 1$
$\log_{10}(x) = 1, \therefore x = 10^1 = 10$
$\log_{10}(x) = 2, \therefore x = 10^2 = 100$

E

Columns 0-1 & 1-2 represent one or more billionaires per million people
$\therefore$ Number of countries with one or more billionaires per million people = 9 + 1 = 10
$\therefore$ **Option E**

Question 3
(2017 Exam 1 Section A – Qn 4)

The histogram below shows the distribution of the log10 (area), with area in square kilometres, of 17 islands.



The median area of these islands, in square kilometres, is between

A. 2 and 3
B. 3 and 4
C. 10 and 100
D. 1000 and 10 000
E. 10 000 and 100 000

D

The histogram consist of 17 islands.
$\therefore$ the median score = $\frac{17+1}{2}$ = 9th score
The 90th score (or frequency) occurs in the interval of $3 < \log_{10}$ (area) < 4
$\therefore$ $10^3 <$ area $< 10^4$
$1000 <$ area < 10000
$\therefore$ **Option D**