Section 3.3.2 – Newton's Laws of Motion

Sir Isaac Newton
1643 – 1727

Newton was an English physicist, mathematician, astronomer, alchemist, and natural philosopher, regarded by many as the greatest figure in the history of science.

His treatise Philosophiae Naturalis Principia Mathematica, published in 1687, described universal gravitation and the three laws of motion, laying the groundwork for classical mechanics.

By deriving Kepler's laws of planetary motion from this system, he was the first to show that the motion of objects on Earth and of celestial bodies are governed by the same set of natural laws.

English poet Alexander Pope wrote the famous epitaph:

Nature and nature's laws laid hid in night;
God said "Let Newton be" and all was light

Newton's 3 Laws of Motion

Law #1 "A body will travel at a constant speed in a constant direction, unless acted upon by an external force" [*Inertia*]

Eg. Object in space

Law #2 "An objects acceleration is proportional to the applied force and inversely proportional to its mass."

$\boldsymbol{F} = \boldsymbol{ma}$, or more accurately; $\boldsymbol{a} = \frac{\boldsymbol{F}}{\boldsymbol{m}}$

The acceleration of an object is *proportional* to the applied force; $\boldsymbol{a} \propto \boldsymbol{F}$

The acceleration of an object is inversely proportional to the objects mass; $\boldsymbol{a} \propto \frac{\boldsymbol{1}}{\boldsymbol{m}}$

Eg. Trajectory of a marble in a slingshot

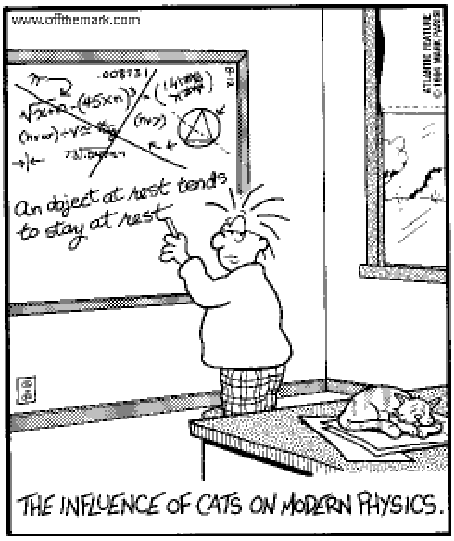
NB: F represents the net (F_{net}) or resultant force (ΣF)

Law #3 "Every force has an equal & opposite force"

Eg. A Person leaning upon a wall applies a force which is equal, but in the opposite direction, as the force applied by the wall upon the person.

$F_{\text{of person on wall}} = -F_{\text{of wall on person}}$

Newton's 1st Law



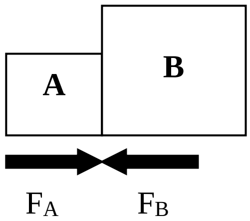
Newton's 2nd Law

Newton's 3rd Law

Applications of Newton's 3 Laws of Motion

1. Conservation of momentum

"The derivation of the conservation of momentum equation"
Consider Newton's 3rd Law:



The force of object A upon object B is **equal and opposite** to the force of object B upon object A.

$$F_{A\ on\ B} = -F_{B\ on\ A}$$
(expand using Newton's 2nd Law)

$$m_a a_a = -m_b a_b$$
(expand using $a = \frac{(v-u)}{t}$)

$$m_a \frac{(v_a - u_a)}{\Delta t} = m_b \frac{(v_b - u_b)}{\Delta t}$$
(as contact time is equal for both objects A & B, Δt is constant)

$$\therefore m_a(v_a - u_a) = -m_b(v_b - u_b)$$
$$m_a v_a - m_a u_a = -m_b v_b + m_b u_b$$
(group the final and initial speeds)

$$\underbrace{m_a v_a + m_b v_b}_{\text{Momentum (P) after collision}} = \underbrace{m_a u_a + m_b u_b}_{\text{Momentum (P) before collision}}$$

Momentum (P) after collision = Momentum (P) before collision

Example 1
Calculate the recoil velocity of a hand pistol of mass 1.4 kg, if a bullet of mass 75 g is fired at a muzzle velocity of 300 ms^{-1}.

m_p = 1.4 kg
m_b = 0.075 kg
u_p = 0 ms^{-1}
u_b = 0 ms^{-1}
v_b = 300 ms^{-1}
v_p = ?

NB: Consider in terms momentum before and after firing
Take direction of bullet projection as positive

momentum before firing = momentum after firing

$$m_b u_b + m_p u_p = m_b v_b + m_p v_p$$
$$\therefore m_p v_p = (m_b u_b + m_p u_p) - m_b v_b$$
$$\therefore v_p = [(m_b u_b + m_p u_p) - m_b v_b]/m_p$$
$$= [(0.075 \times 0) + (1.4 \times 0) - (0.075 \times 300)]/1.4$$
$$= -22.5/1.4$$
$$= -16.1\ ms^{-1}$$
$= 16.1\ ms^{-1}$ (in the opposite direction to the bullet)

Example 2

A sports car of mass $500\ kg$ is travelling at $80\ kmh^{-1}$, when it collides into a stationary $750\ kg$ car. Upon collision the two cars become joined. Calculate their combined velocity following the collision.

$\Sigma m = m_s + m_c$
$\Sigma m = 500\ kg + 750\ kg$
$= 1250\ kg$
$v = v_s = v_c = ?$

$m_c = 750\ kg$
$u_c = 0\ ms^{-1}$

$m_s = 500\ kg$
$u_s = 80\ kmh^{-1}$
$u_s = 22.2\ ms^{-1}$

NB: The velocity for both the sports car and car will be the same upon uniting at collision
Take initial direction of the sport car as positive

momentum before collision = momentum after collision

$$m_s u_s + m_c u_c = m_s v_s + m_c v_c$$

$$m_s u_s + m_c u_c = (m_s + m_c)v$$

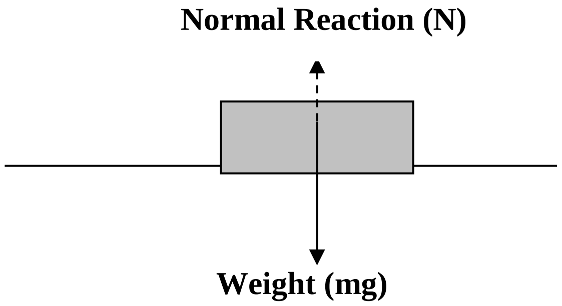
$$v = \frac{m_s u_s + m_c u_c}{m_s + m_c}$$

$$v = \frac{500 \times 22.2 + 750 \times 0}{1250}$$

$\therefore v = 8.88\ ms^{-1}$
(continuing in the same initial direction of the sports car)

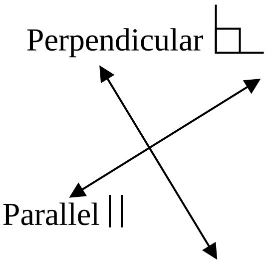
2. Inclined Planes

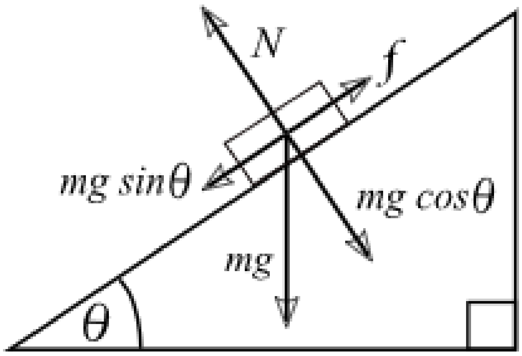
An object placed upon a **flat surface** will not move unless acted upon by an outside force.



$$\Sigma F = 0$$
$$\therefore Weight = -Normal\ Reaction$$

However, when this object is placed upon an **inclined plane** (frictionless), the weight and normal reaction forces are no longer equal and opposite and a net force exists.





NB: The angle of the plane and that of the right-angled triangle formed by the Weight components are identical.

Using trigonometry

Weight component perpendicular to plane is equal in magnitude to the Normal Reaction force

$$\boxed{N = mg \cos \theta}$$

Weight component parallel to plane is equal in magnitude to the force down the slope

$$\boxed{F_{down\ plane} = mg \sin \theta}$$

All object "sliding" down a **frictionless** inclined plane have an acceleration of:

$$\boxed{a = g \sin \theta}$$

Example 3

A truck of mass 2 tonnes begins to slide down a frictionless slope of angle 40°. Calculate:
(**NB:** the surface is frictionless, ∴ f = 0N)

1. the Normal Reaction force created
2. the force down the incline plane
3. Its speed after 5 seconds

1.

$$\theta = 40^\circ \qquad m = 2\ tonnes = 2 \times 10^3\ kg \qquad N = ?$$

$$N = W\cos\theta = mg\cos\theta = 2.0 \times 10^3 \times 9.8 \times \cos(40^\circ) = 15014.5\ N = \mathbf{1.5 \times 10^4}\ N$$ (perpendicular above the surface plane)

2.

$$\theta = 40^\circ \qquad m = 2\ tonnes = 2 \times 10^3\ kg \qquad F_{down\ plane} = ?$$

$$F_{down\ plane} = W\sin\theta = mg\sin\theta = 2.0 \times 10^3 \times 9.8 \times \sin(40^\circ) = 12599\ N = \mathbf{1.3 \times 10^4}\ N$$ (down the incline)

3.

$$u = 0\ ms^{-1} \qquad t = 5\ sec \qquad a = g\sin\theta \qquad \theta = 40^\circ \qquad v = ?$$

$$v = u + at$$
$$v = u + (g\sin\theta)t = 0 + (9.8 \times \sin(40^\circ)) \times 5 = \mathbf{32}\ ms^{-1}$$ (down the plane)

Example 4

A truck of mass 2 tonnes begins to slide down an inclined plane of angle 40 with an acceleration rate of 2 ms^{-2} Calculate:

1. the net force acting upon the truck
2. the component of the weight force acting down the incline plane
3. the force due to friction

1.

$$\Sigma F = ? \qquad m = 2 \times 10^3\ kg \qquad a = 2\ ms^{-2}$$

$$\Sigma F = ma = 2 \times 10^3 \times 2 = \mathbf{4 \times 10^3}\ N$$

2.

$$\theta = 40^\circ \qquad m = 2\ tonnes = 2 \times 10^3\ kg \qquad F_{down\ plane} = ?$$

$$F_{down\ plane} = mg\sin\theta = mg\sin\theta = 2.0 \times 10^3 \times 9.8 \times \sin(40^\circ) = \mathbf{1.3 \times 10^4\ N}$$ (down the incline)

3.

$$F_{fric} = ? \qquad \theta = 40^\circ \qquad m = 2\ tonnes = 2 \times 10^3\ kg$$

$$\Sigma F = F_{down\ plane} - F_{fric}$$
$$\therefore F_{fric} = F_{down\ plane} - \Sigma F = mg\sin\theta - ma = 1.3 \times 10^4 - 4 \times 10^3 = \mathbf{9 \times 10^3}\ N$$

Momentum and Impulse

Recall:

Momentum = mass x velocity $P = mv \; (kgms^{-1})$	Impulse = force x time $I = Ft \; (Ns)$

NB: Both quantities are **vectors**

Recall Newton's 2nd Law:

$$F = ma,$$
(substitute $a = \frac{\Delta v}{\Delta t}$)
$$F = m\frac{\Delta v}{\Delta t}$$

This equation is easily manipulated into the following equation:

$$\boxed{F\Delta t = m\Delta v}$$

[Change of impulse = Change of momentum]

The **impulse (I)** delivered to an object is equal to its **change of momentum (ΔP).**

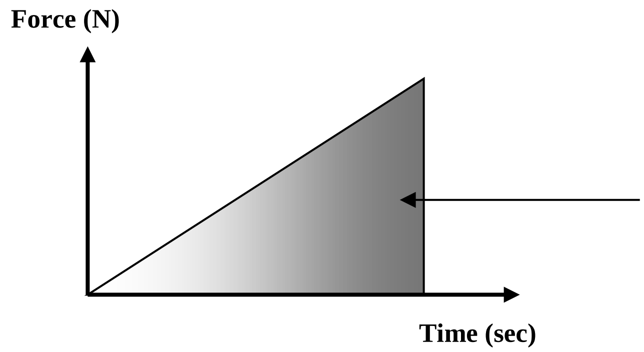
Example 5

A golf club delivers a force over a period of 5 milliseconds that causes the 100 g ball to accelerate from 0 ms^{-1} to 50 ms^{-1}. Calculate the magnitude of force exerted upon the ball by the club.

$F = ?$
$\Delta t = 5 \times 10^{-3} \; s$
$m_{ball} = 0.1 \; kg$
$$\left.\begin{array}{l} u = 0 \; ms^{-1} \\ v = 50 \; ms^{-1} \end{array}\right\} \Delta v = 50 \; ms^{-1}$$

$$F\Delta t = m\Delta v$$
$$\therefore F = m\frac{\Delta v}{\Delta t}$$
$$= 0.1 \times \frac{50}{5\times10^{-3}}$$
$$= \mathbf{1000} \; N$$

Force – Time Graphs



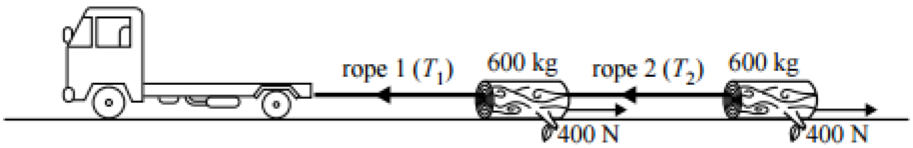
The area under a Force – Time graph represents the: **impulse delivered** & **change of momentum**

NB: Be sure to display the correct units of measurement when calculating the area under an F-t graph (ie. Ns or $kgms^{-1}$)

Exam Style Questions

The following information relates to questions 1 to 4

A truck is dragging two logs along level ground in a straight line. The mass of each log is 600 kg and each log experiences a constant retarding friction force of 400 N with the ground.
The connections between the truck and the logs are made with ropes that have a breaking force of 2400 N. T1 and T2 are the tensions in the ropes as shown in Figure 7. The truck and the logs are moving towards the left in the below figure.



Question 1
Calculate the magnitude of T_1 when the truck is driving at a constant speed.

<u>Step.1 Consider the second log</u>

At constant speed, accel'n = zero, so T_2 = 400 N

<u>Step.2 Consider the first log</u>

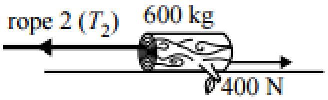
NB: In this scenario T_2 is actually pulling to the right (ie. opposing the direction of motion for log 1)

At a constant speed, forces acting upon the log to the left = forces acting upon the log to the right
$T_1 = T_2 + 400\ N$
$\therefore T_1 =$ ***<u>800 N.</u>***

800 N

Question 2

The truck then accelerates at a rate of 0.50 ms^{-2}. Calculate the magnitude of T_2.



F_{net} = ma (Using Newton's 2nd Law)
= 600 x 0.50
= 300 N

$F_{net} = T_2 - 400$ (Analysing forces upon the 2nd log)
$\therefore 300 = T_2 - 400$
*$\therefore T_2$ = **700N***

700 N

Question 3

At a point in time, the driver observes that the speed of the truck is 4.0 ms^{-1}. The truck then keeps accelerating at 0.50 ms^{-2} for another 20 m. Calculate the speed of the truck at the end of the 20 m.

v = ?
$u = 4.0\ ms^{-1}$
$a = 0.50\ ms^{-2}$
$x = 20\ m$

$v^2 = u^2 + 2ax$
$v^2 = 4^2 + 2 \times 0.5 \times 20$
$v^2 = 16 + 20$
$v^2 = 36$
$\therefore v = \mathbf{6.0\ ms^{-1}}$

6.0 ms^{-1}

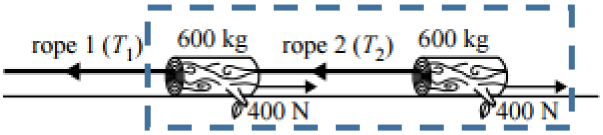
Question 4

The ropes have a breaking force of 2400 N. Rope 1 connects the truck to the front log and rope 2 connects the two logs. The truck, still on level ground, increases its acceleration until one of the ropes is about to break. Identify which rope is about to break, and calculate the magnitude of the acceleration of the truck and the logs at this instant.

Rope 1

Tension in rope 1 is always greater than that in rope 2.

Analyse the entire log system



Calculate F_{net} (Consider forces to the left versus forces to the right)
[NB: When analysing the entire system, T_2 is not included as it joins the two logs]

$F_{net} = T_1 - (400 + 400)$
$= 2400 - (800)$
$= 1600\ N$

$F_{net} = ma$ *(Using Newton's 2nd Law)*
$1600 = (600 + 600)a$
$1600 = 1200a$
$\therefore a = \frac{1600}{1200}$
$= \underline{\mathbf{1.33\ ms^{-2}}}$

1.33 ms^{-2}

The following information relates to questions 5 & 6

Students set up an experiment that consists of two masses, m_1, of 2.0 kg, and m_2, of 6.0 kg, connected by a string, as shown below. The mass of the string can be ignored. The surface is frictionless. The pulley is frictionless.

At the start of the experiment, the bottom of mass m_1 is 1.2 m above the floor and both masses are stationary.

Question 5

Calculate the gravitational force on m_1. Include the correct unit in your answer.

w = ?
m = 2.0 kg
g = 10 Nkg^{-1}

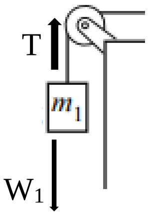
w = mg
= 2.0 x 10
*= **20 N***

20 N

Question 6

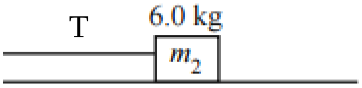
Calculate the acceleration of the system and the tension in the string as m_1 is falling.

Step.1 Calculate the acceleration of the system
Examine m_1



$F_{net} = m_1 a = W_1 - T$
$\therefore T = W_1 - m_1 a$ [Eqn 1]

<u>Examine m_2</u>



$F_{net} = m_2 a = T$
$\therefore$ $T = m_2 a$ [Eqn 2]

NB: The tension is the same in both Eqn 1 and Eqn 2 so we can equate the RHS from both equations

$\therefore$ $W_1 - m_1 a = m_2 a$
$W_1 = m_1 a + m_2 a$
$m_1 g = (m_1 + m_2)a$
$2 \times 10 = (2 + 6)a$
$20 = 8a$
$\therefore$ $a = \frac{20}{8}$
$= \underline{\mathit{2.5\ ms^{-2}}}$

2.5 ms^{-2}

Step.2 Calculate the tension of the string
Substitute $a = 2.5$ ms^{-2} into Eqn 2

$T = m_2 a$
$= 6.0 \times 2.5$
$= 15$ N

15 N