WHAT IS THE OPTIMUM FORCE AND ANGLE TO WIN BEER PONG EVERY TIME?

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INTRODUCTION:

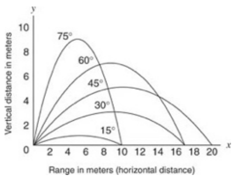
Aim:

The aim of this investigation is to determine the most appropriate force and angle needed in order to successfully shoot a ping pong ball in a cup a specific distance away.

Hypothesis:

It was predicted that, by graphing the range against the angle for different values that a graph with a maximum range for a specified angle will be seen. This shape is expected to be parabolic and from here an equation linking the two could be derived. For a force vs. range a linear proportionality would be expected (as the force on the ball increases, so too does the range) as the amount of spring elastic potential energy increases, so will the kinetic energy of the ball as it's released.

Background Physics:



Projectile motion is defined by the motion of an object whereby the only force being applied is the force due to gravity (it's own weight force).

Physics suggests that the angle necessary for the maximum range of the projectile is 45° (where $0 \leq \theta \leq 90°$)

Link to mathematical proof of this:
https://www.khanacademy.org/science/physics/two-dimensional-motion/optimal-projectile-angle/v/optimal-angle-for-a-projectile-part-4-finding-the-optimal-angle-and-distance-with-a-bit-of-calculus

Variables:

The variables to be analysed are the range of the ping pong ball which is dependent on the angle of projection of the ball and the compression of the spring ejecting the ball. The variables kept constant in the experiment are the distance between the cups and the spring, as well as the spring constant used and the mass and size of the ping pong ball.

METHOD:

Spring Constant:

- The spring constant was determined by attaching the spring to a clamp on a retort stand hanging off the edge of the table.
- A meter-long ruler was used to measure the extension of the spring as 0.5 kg was added in increments.

Force and Angle on Range:

- First we measured the range by changing the angle of the projectile on the maximum extension of the spring. Angles measured: 10°-80° at 10° increments using protractor.
- By attaching the spring projector to the edge of a table and shooting it upwards, the range was able to be recorded.
- The same was done for the extension of the spring (and thus the force) except the angle was kept at a constant 45° and the extension was changed from 0.1-0.3 m at 0.05m increments.
- The constant variables were maintained by using the same ping pong ball and keeping the measured distance between the cups at 2m.

RESULTS:

Raw results tables:

The spring constant:

Spring Constant of spring (N/m)		
force on spring (N)	extension (m)	weight (kg)
0	0	0
4.9	0.001	0.5
9.8	0.056	1
14.7	0.154	1.5
19.6	0.253	2
24.5	0.344	2.5

The range of the projectile as the angle is changed:

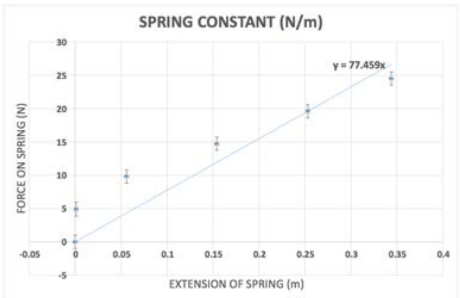
extension (m)	Force (N)
0.3	23.24
Angle (degrees)	Range (m)
10	1.1
20	1.58
30	1.8
40	2.02
50	2.07
60	1.82
70	1.3
80	0.867

The range of the projectile as extension (thus force) is changed (at constant 45°):

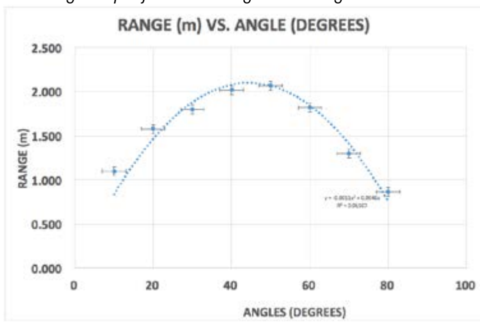
Angle (constant)	45°	
distance between cups (constant)	2 m	
Extension (m)	Force (N)	Range (m)
0.1	7.7459	0.617
0.15	11.61885	0.943
0.2	15.4918	1.337
0.25	19.36475	1.876
0.3	23.2377	2.019

Graphing results:

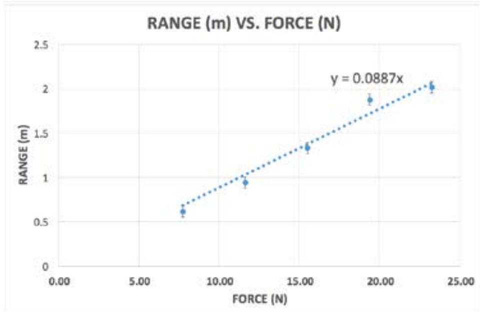
Spring Constant:



The range of projectile as angle is changed:



The range of the projectile as the force is changed:



DISCUSSION:

Analysis and Evaluation of primary data:

By analyzing the graphs that were produced as a result of the primary data, two equations were able to be produced:

- $R = 2.2sin(2\theta)$
- $F = 11.255 \times R$

where R is the range of the projectile(m), θ is the angle of projection(°) and F is the force(N).

The first can be transposed to make θ the subject:

$$\theta = \frac{1}{2}\sin^{-1}(\frac{R}{2.2})$$

and for the second you can substitute in $F = k\Delta x$ to get:

$$\Delta x = \frac{11.255R}{k} = 0.14532R$$

From these two equations, if you are a given a range between the projectile and the table, you will be able to calculate the angle at which to shoot the ball (given a spring extension of 0.3m) or the spring extension at which you shoot the ball (given the ball is at 45°).

Links to relevant physics concepts:

The data from the range (m) vs. force (N) graph indicated a linear proportionality between the two variables. This is relevant to the physics concept that energy cannot be created nor destroyed as the spring elastic potential energy (E_{sp}) is converted to kinetic energy (E_k) in the ball when released. So it makes sense that as the elastic potential energy is increased, through a greater extension of the spring, so too will the kinetic energy of the object as it is projected. Greater kinetic energy will allow the object to travel further before stopping.

The primary data taken from the range vs. angle graph gave a trend line that suggested that the angle at which the range is a maximum is 45°. This supports the physics and mathematical proofs that were researched beforehand.

Outliers:

There were outliers when attempting to measure the spring constant as the first weight did nothing to the extension of the spring, creating two points at o along the x-axis (although when averaged this worked out). This value was still included as part of the line of best fit and thus contributed to the value of the spring constant because it was not a methodical error but just the function of the spring.

Limitations and improvements:

There were sources of error in this investigation, both systematic and random. The masses used for measuring the spring constant weren't exactly 0.5 kg so this provided a margin of error when calculating the spring constant.

This investigation could be extended by attempting to find equations for different objects of different projectiles, for example shooting a basketball.

CONCLUSION:

Through this investigation, the optimum angle and extension of the projectile is able to be calculated. This investigation concluded that it is possible to use an equation to determine the range of the projectile when the angle and force is known or vise versa. Our results were also consistent with relevant physics that states that the maximum range of a projectile at a constant force will be when the projectile is angled at 45° compared to any other angle.

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