Section 4.2.4 – Quantum Physics

Quantised States

Recall from Notes 4.2.4 that one of Niels Bohr's features of his electron energy level model was that each atom consists of a number of possible states. These states have fixed or quantised energy levels.

To explain the quantised energy levels which electrons occupy, Louis de Broglie applied his matter wave model. He suggested that:

- Electrons have a **de Broglie wavelength** and they exist as **circular waves** around a nucleus. Refer to Figure 1a below.

- An electron will only be stable if it forms a **standing wave** in its orbit. This means the circumference of its orbit must be an integer multiple of the electron's de Broglie wavelength.

- Hence only specific wavelengths are allowed.

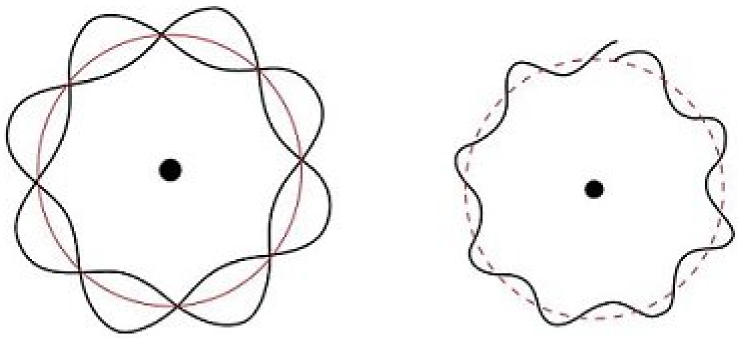


Figure 1a ***Figure 1b***

Figure 1a – Multiples of the de Broglie wavelength for an electron fit exactly (ie. constructively) within the electron energy level. This creates a standing wave within the energy level. So an electron can exist in **this quantised state.**

Figure 1b – Multiples of the de Broglie wavelength for an electron do not fit exactly (ie. destructively) within the electron energy level. A standing wave is not created and the vibration collapses. So an electron cannot exist in this state.

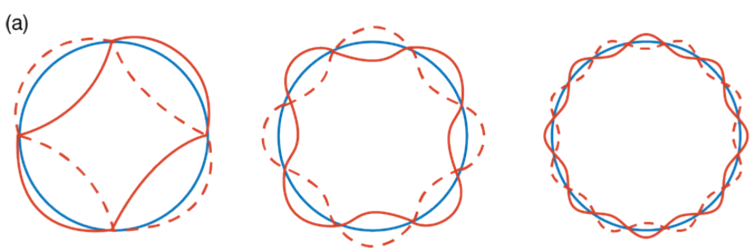


Figure 2 *– Possible electron energy levels within an atom*

Another more useful way of visualising these standing waves is to draw them on a strip of paper and join the ends of the strips to form a circle. Each circle represent a possible energy levels within an atom. This is shown below in Figure 3.

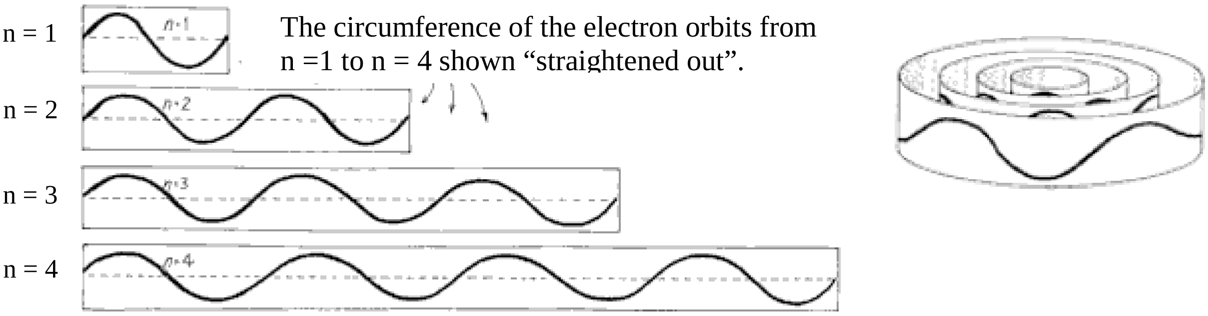


Figure 3 *– Alternative visualisation of quantised energy levels*

Recall: The circumference of a circle is equal to: C = $2\pi r$ and constructive interference only occurs when an exact multiple of the wavelength ($n\lambda$) fits within the circumference. Therefore the following equation can be used to describe the relationship between wavelength (λ) and energy level radii:

$$\boxed{n\lambda = 2\pi r}$$

Where n is the energy level number (ie. n=1 for the 1st energy level)

λ is the de Broglie wavelength of the electron (m)

r is the radius of the energy level

The quantisation of energy levels of atoms is evidence for the dual nature of matter. Electrons whilst matter, are located is specific energy levels as a result of their wave properties.

"Determination of the stable motion of electrons in the atom introduces integers, and up to this point the only phenomena involving integers in physics were those of interference and of normal modes of vibration. This fact suggested to me the idea that electrons too could not be considered simply as particles, but that frequency (wave properties) must be assigned to them also. "

*(**Louis de Broglie,** 1929, Nobel Prize Speech)*

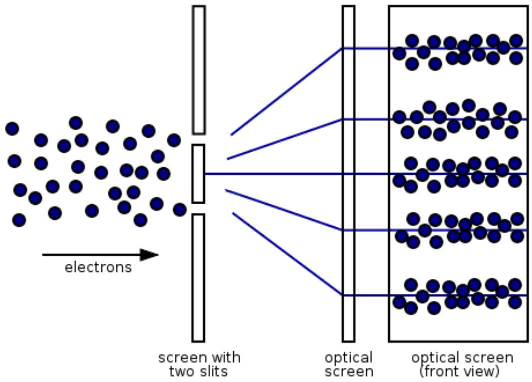
Is light a wave or particle?

Our timeline to this point provides the following evidence:

1801 Young's Double Slit Experiment
1887 The Photoelectric Effect
1927 Diffraction pattern of electrons
1929 deBroglie matter waves explain Niels Bohr's electron energy levels

Evidence existed to support light as both a particle and a wave and matter too as a particle with wavelike properties.

So Physicist returned back to Young's double slit experiment to re-evaluate the findings. Both electrons and photons were used and interference patterns were produced in both cases.



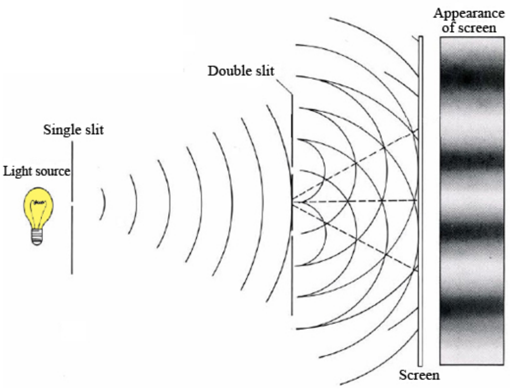


Figure 4a *– Interference pattern with electrons* ***Figure 4b*** *– Interference pattern with photons*

Scientists suggested that the interference patterns observed in the double slit experiment with light and electrons might be due to interactions between particles once they pass through the slits.

To test this hypothesis the experiment was repeated with one particle at a time (ie. a single photon or electrons at a time) and an interference pattern allowed to form over an extended period of time. This exact experiment was carried out by Geoffrey Taylor in 1909, while he was a University of Cambridge student. His experimental equipment is shown below in Figure 5.

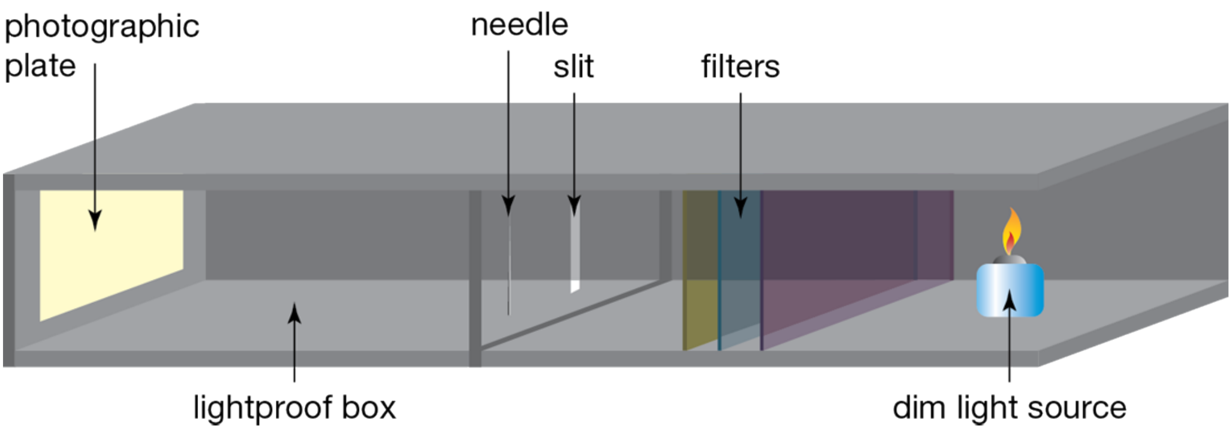


Figure 5 *– Taylor's single photon apparatus*

Despite this incredibly low intensity, after a long enough time the same interference pattern was observed. This can be seen in Figure 6 and that of single electron experiment in Figure 7.

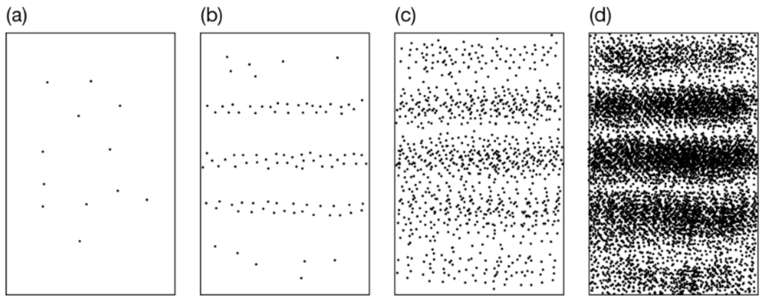


Figure 6 *– Single photon build-up interference pattern.*

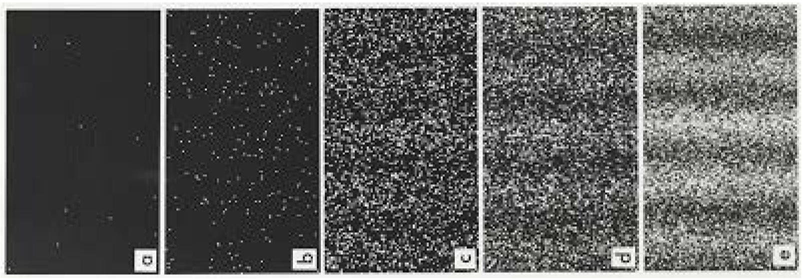


Figure 7 *– Single electron build-up interference pattern.*

The Interpretation of the Single Particle Double Slit Experiment

The object (ie. photon or electron) leaves the source with particle-like properties, that is one discrete object at a time. Yet at the same time both objects also display wave-like properties (ie. wavelength and momentum).

The immediate measurement on the screen consists of small discrete spots, which suggests the object is still behaving as a particle.

However, over an extended period of time the pattern forms a clearly defined interference pattern which suggests the object is still behaving as a wave.

Example 1

Explain how a single-photon Young's double-slit experiment can be used as evidence of the dual nature of light.

- *The intensity of light is reduce so that only one photon passes through the apparatus at any time. A discrete spot of light is observed on the screen each time a photon hits. This is particle-like behaviour.*
- *As the spots accumulate they form an interference pattern, identical to what we would expect from a wave.*
- *Individual photons have wave-like properties. Light is demonstrating wave and particle behaviour at the same time.*

Physicists concluded that:

*"The single photon/electron double slit experiment is evidence for the **dual nature of light and matter**."*

*"Light is considered as neither a particle nor a wave, rather it is described as a **quantum object**"*

Diffraction for a Single Slit Experiment

In addition to the traditional Young's double slit experiment, a single slit experiment can also be used to demonstrate the wave-like properties of light. A single slit arrangement also generates an interference pattern with a central and more intense fringe, with subsequent alternate dark and bright fringes, representing regions of subsequent destructive and constructive interference. Refer to Figure 8.

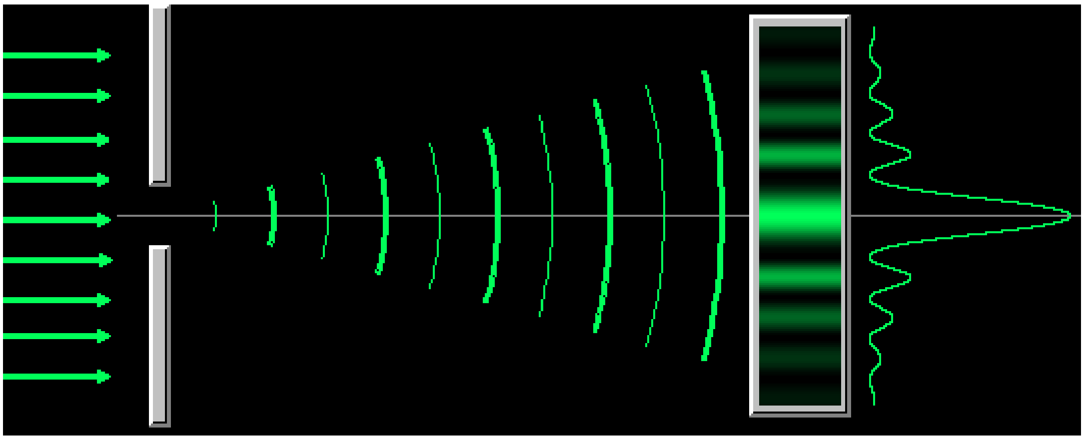


Figure 8 *– Single slit experiment*

We understand from our basic wave model that a small gap size (↓w) we get a greater extent of diffraction (as extent of diffraction $\alpha \frac{\lambda}{w}$), that is the interference pattern will spread more. But let's now consider this result in terms of the famous Heisenberg's uncertainty principle.

Heisenberg's Uncertainty principle

Consider the single slit interference experiment for two different arrangements. For the purpose of this discussion, let's consider the space between the slit as Δx.

Arrangement.1 - A wide slit width (↑Δx).
Figure 9 shows the single slit experiment and the interference pattern produced for a relatively wide slit gap.

For a wider slit: The uncertainty in the photon's x-axis position is larger AND diffraction pattern is less spread out which means uncertainty in the photon's x-axis momentum is smaller.

NB: Momentum is another way of describing the velocity of the photon once it has passed though the slit. A low uncertainty in the momentum means the interference pattern has little diffraction and bunched together. Whereas a high uncertainty means the interference pattern has diffracted significantly and has a wide spread.

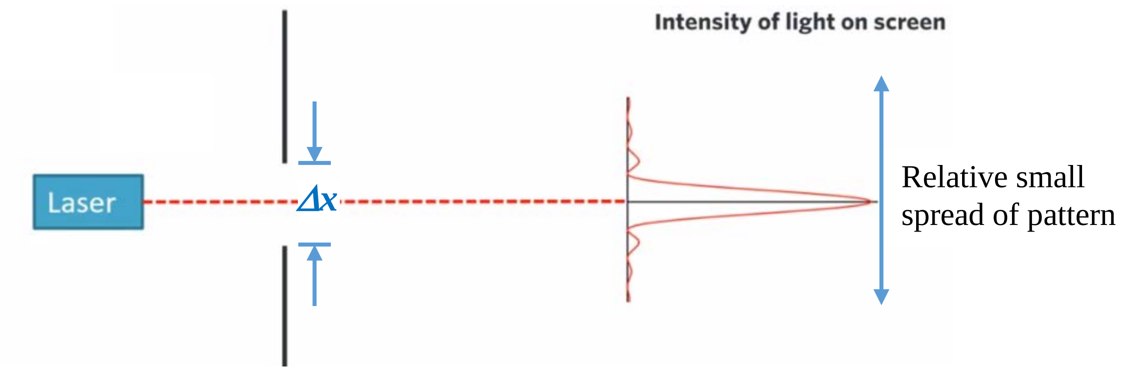


Figure 9 *– Single slit experiment with relative large gap size*

Arrangement.2 - A narrow slit width (↓Δx).
Figure 10 shows the single slit experiment and the interference pattern produced for a relatively narrow slit gap.

For a narrower slit: The uncertainty in the photon's x-axis position is smaller AND diffraction pattern is more spread out which means uncertainty in the photon's x-axis momentum is larger.

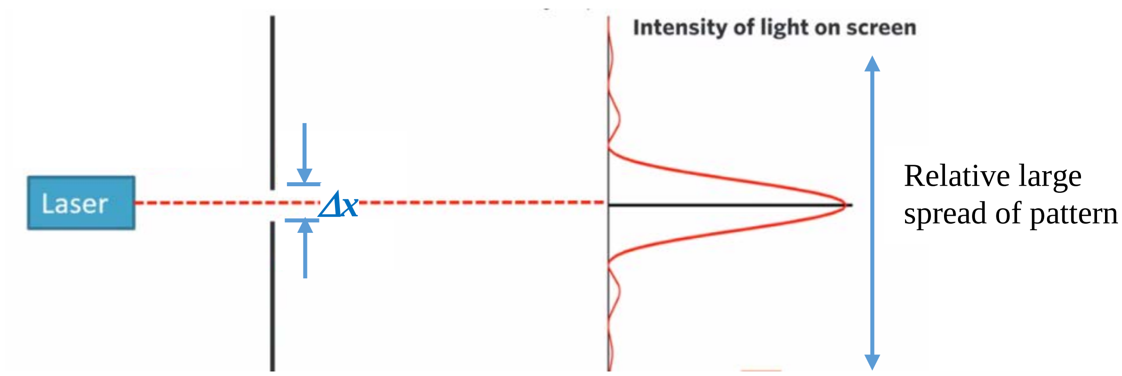


Figure 10 *– Single slit experiment with relative small gap size*

Heisenberg's uncertainty principle states:

It is impossible to exactly define a particle's position and momentum at the same time.
The more exactly a particle's position is determined, the more uncertainty there is in the particle's momentum.
The more exactly a particle's momentum is determined, the more uncertainty there is in the particle's position.

$$\boxed{\Delta x \Delta p_x \geq \frac{h}{4\pi}}$$

Where Δx is the uncertainty in x-position
Δp_x is the uncertainty in x-momentum

As the right hand side of this expression is a constant value (ie. $\frac{h}{4\pi}$), the right hand side of this expression must also be a constant value (ie. $\Delta x \Delta p_x$).

In order for the left hand side of this expression to remain constant, a change in a particles position (Δx), must result in the particles change as the momentum (Δp_x).

That is.

- If $\Delta x \uparrow$, then $\Delta p_x \downarrow$
- If $\Delta x \downarrow$, then $\Delta p_x \uparrow$

Example 2

Explain how the effect of increasing the slit width in a single slit diffraction experiment can be interpreted using Heisenberg's uncertainty principle?

- Increasing slit width results in a decrease in the extent of diffraction
- The uncertainty in position (Δx) is proportional to the slit width. The uncertainty in momentum is proportional to the extent of diffraction.
- Accordingly to Heisenberg's uncertainty principle, an increase in positional uncertainty should cause a decrease in the uncertainty of momentum – which agrees with our results.

Classical laws fail

The term "Classical laws of physics" refer to the laws of physics which were understood before quantum mechanics.

Classical laws assume that physical quantities can be determined to unlimited certainty. That is, it assumes an object's position and motion can be exactly defined.

This assumption is appropriate for larger objects because the quantum uncertainty is so small. But for small enough objects the uncertainty is too significant and so classical laws fail to adequately describe their behaviour.

Example 3

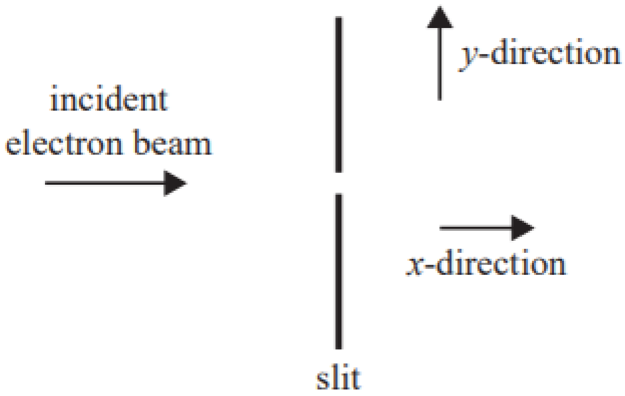
Explain why the position and momentum of a football can be defined to a sufficient level of uncertainty but the position and momentum of an electron cannot be.

- A football is a large enough object that the quantum uncertainty describes by Heisenberg's Uncertainty Principle (HUP) is negligible (since h is so small)
- An electron is a small enough object that the quantum uncertainty described by HUP is significant
- That is, to meaningfully describe the position of an electron we need to make Δx very small, which means Δp_x becomes significantly large due to $\Delta x \Delta p_x \geq \frac{h}{4\pi}$.

Exam Style Questions

Question 1.

A diffraction pattern is produced by a stream of electrons passing through a narrow slit, as shown in the diagram below.



This electron diffraction pattern can be used to illustrate Heisenberg's uncertainty principle. This is because knowing the uncertainty in the

A. electron's speed is large leads to the uncertainty in its kinetic energy being small.
B. slit width is small leads to a large uncertainty in the electron's momentum in the y-direction.
C. electron's momentum in the y-direction is small leads to a large uncertainty in the slit's width.
D. electron's angle of approach to the slit leads to a large uncertainty in the electron's momentum in the y-direction

B

A: does not make sense, C: Properties of slit width don't depend on the electron, D: Approach is head on

Question 2.

Quantised energy levels within atoms can best be explained by

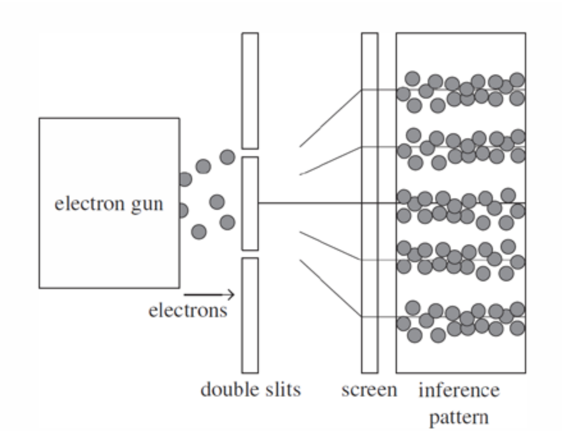
A. electrons behaving as individual particles with varying energies.
B. atoms having specific energy requirements that can only be satisfied by electrons.
C. electrons behaving as waves, with each energy level representing a diffraction pattern.
D. electrons behaving as waves, with only standing waves at particular wavelengths allowed.

D

A; wrong, electrons have wave properties, B: energy requirements determined by interaction between electrons and the nucleus, C: Energy levels are not diffraction patterns

Question 3.

The diagram below shows the firing of electrons one at a time toward a pair of slits that are very close together.



Which of the following is the best interpretation of the results of the experiment?

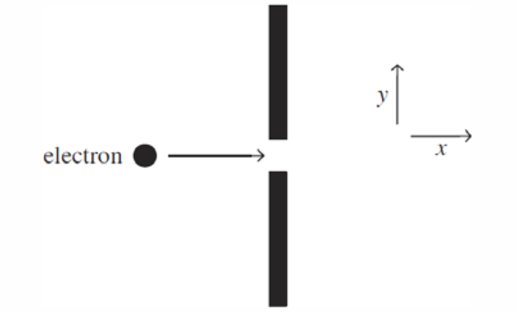
A. The results represent evidence that the electrons are behaving as particles.
B. The results represent evidence that the electrons are behaving as waves.
C. The results represent evidence that the electrons behave as waves during their passage through the double slit, but behave as particles when they strike the screen.
D. The results represent evidence that the electrons behave simultaneously as particles and waves throughout their motion.

C

The electrons are individually passing through and striking a particular point on the screen as particles do.
The final pattern, however, is typical of constructive and destructive interference, resulting in bright and dark bands as though the electrons were of a wave nature.
During its passage through the double slit, an electron interferes with itself as a wave would.

Question 4.

An electron is passed through a single slit as shown below. The x and y directions are shown.



The electron's characteristics are such that Heisenberg's uncertainty principle applies to it once it has passed through the slit.
Explain how the position and momentum of the electron are subject to Heisenberg's uncertainty principle. Make reference to either of the directions x or y as shown.

As the electron passes through the slit, it has a probability of diffracting and therefore having a vertical velocity and momentum. ***1 mark***

Its uncertainty in position in the y plane becomes approximately half the slit width. ***1 mark***

If the slit width is reduced to define the particle's position, the electron has a greater probability of diffracting further up the y plane. Thus the uncertainty in its momentum increases. ***1 mark***

The product of the position and momentum uncertainties is of a minimum value given by the Heisenberg uncertainty principle. ***1 mark***

Question 5.

De Broglie suggested that the quantised energy states of atoms could be explained in terms of electrons forming standing waves.
Describe how the concept of standing waves can help explain the quantised energy states of an atom. You should include a diagram.

Electrons exhibit a wave behaviour.
Electrons form standing waves in orbits where the circumference is a whole multiple of the electron wavelength.
This means that only certain discrete energy states can exist.