Section 5 – Newton's Laws of Motion

Newton's 1st Law of Motion

Newton's first law of motion states that:

"An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force."

Newton's first law of motion is also referred to as the law of **inertia**, where inertia is the **resistance to change in motion.**

Example.1

Whilst the bike has been brought to rest by the force of the stacked tires upon the bike, the bike rider maintains their velocity until acted upon by an unbalanced force such as the fence!

Newton's 2nd Law of Motion

Newton's second law of motion states that:

"The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object."

Mathematically this can be written as:

$$\boldsymbol{a = \frac{\sum F}{m}}$$

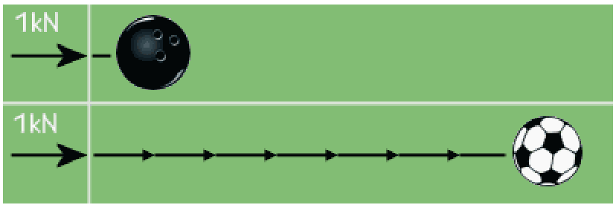
Or more commonly stated as:

$$\boldsymbol{\Sigma F = ma}$$

Where ΣF is the net force (Newtons)
m is the mass (kg)
a is the acceleration (ms^{-2})

Example.2

If a bowling ball and soccer ball are both accelerated by a net force of 1 kN, then the soccer ball will accelerate at a higher rate than the bowling ball due to its less mass.



Example.3

A net force of 20 N is exerted upon a billy cart. Given the billy cart accelerates at a rate of 5ms^{-2}, what is the mass of the billy cart?

$\Sigma F = 200$ N

$a = 5$ ms^{-2}

$m = ?$

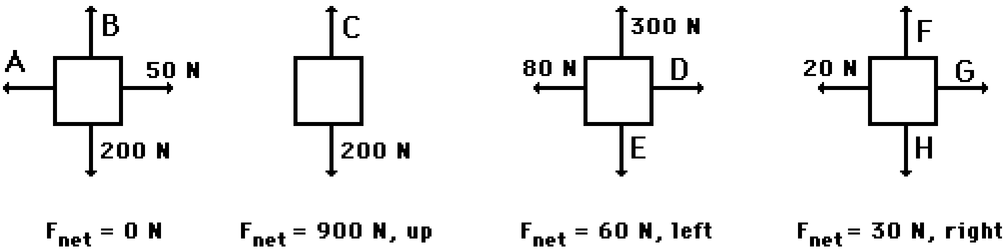
$\Sigma F = ma$

$$m = \frac{\Sigma F}{a}$$

$$m = \frac{200}{5} = 40 \text{ kg}$$

Example.4

Free-body diagrams for four situations are shown below. The net force is known for each situation. However, the magnitudes of a few of the individual forces are not known. Analyse each situation individually and determine the magnitude of the unknown forces.



<u>Diagram 1</u>

$\Sigma F = 0$ N

$\therefore F_{up} = F_{down}$

$\therefore B = 200$ N

$\Sigma F = 0$ N

$\therefore F_{\leftarrow} = F_{\rightarrow}$

$\therefore A = 50$ N $(\leftarrow)$

<u>Diagram 2</u>

$\Sigma F = 900$ N $(\uparrow)$

$\therefore\ 900 = C - 200$

$\therefore\ C = 900 + 200$

$= 1100$ N $(\uparrow)$

<u>Diagram 3</u>

Vertically

$\Sigma F = 0$

$\therefore F\downarrow = F\uparrow$

$\therefore E = 300$ N $(\uparrow)$

$\Sigma F = 60$ N $(\leftarrow)$

$\therefore\ 60 = 80 - D$

$\therefore\ D = 80 - 60$

$= 20$ N $(\rightarrow)$

<u>Diagram 4</u>

Vertically

$\Sigma F = 0$

$\therefore F\downarrow = F\uparrow$

$\therefore F = H$ (any values)

$\Sigma F = 30$ N $(\rightarrow)$

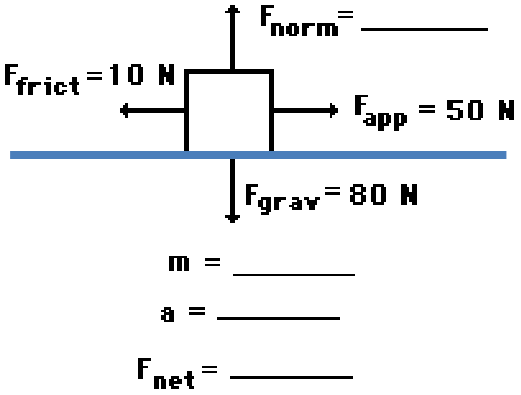
$\therefore\ 30 = G - 20$

$\therefore\ G = 30 + 20$

$= 50$ N $(\rightarrow)$

Example.5

An applied force of 50 N is used to accelerate an object to the right across a frictional surface. The object encounters 10 N of friction. Use the diagram below to determine the normal force, the net force, the mass, and the acceleration of the object. (Neglect air resistance.)



Consider vertically

$F_{up} = F_{down}$
$\therefore F_{normal} = F_g$
$= 80$ N

Consider horizontally

$\Sigma F = F_{app} + F_{fric}$ Take $\rightarrow$ +ve direction
$\therefore \Sigma F = 50 - 10$
$= 40$ N ($\rightarrow$)

Mass calculation

$F_g = 80$ N
$g = 9.8$ Nkg^{-1}
m = ?

$F_g = mg$
$\therefore m = \frac{F_g}{g}$
$= \frac{80}{9.8} = 8.16$ kg

Acceleration calculation

$\Sigma F = 40$ N ($\rightarrow$)
m = 8.16 kg
a = ?

$\Sigma F = ma$
$\therefore a = \frac{\sum F}{m}$
$= \frac{40}{8.16} = 4.90$ ms^{-2}

Newton's 2nd Law of Motion – Inclined Planes

Up until this point in the course forces have been examined in flat planes. Effectively we have been able to examine force experienced left, right, up and down.

Let us now examine an inclined plane.

When considering an inclined plane it is best to examine forces in terms of:

1. Forces parallel (||) to the inclined plane; &
2. Forces perpendicular ($\perp$) to the inclined plane

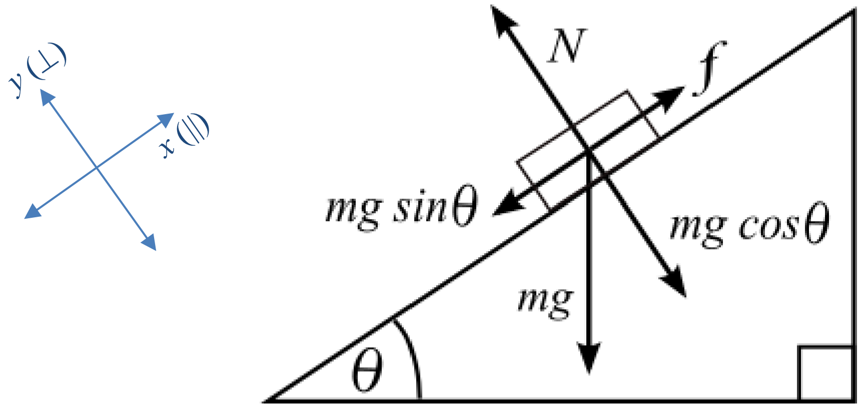


Figure 1 *- An inclined plane*

As can be seen in Figure 1 above, the gravitational force of the object on the plane (mg) can be broken into a component acting parallel and down the plane ($mgsin\theta$) and another perpendicular to the plane ($mgcos\theta$).

$\boxed{F_g = mg}$

$\boxed{F_{gx} = F_{||} \text{ down the plane} = mgsin\theta}$

$\boxed{F_{gy} = F_{\perp} = mgcos\theta}$

$\boxed{a = gsin\theta}$

Where F_g is the force due to gravity (N)
F_{gx} is the || component of F_g down the plane
F_{gy} is the $\perp$ component of F_g to plane
a is the rate of acceleration down the incline (for a frictionless surface)

NB: N = - $mgcos\theta$

Example.5

The object shown in Figure 1 has a mass of 100 kg and the plane is inclined at an angle of 30°.

F_g = ?
m = 100 kg
g = 9.8 Nkg^{-1}
θ = 30°

Question.1

Calculate the objects force due to gravity (ie. weight force).

$F_g = mg$
$= 100 \times 9.8$
$= 980$ N

Question.2

Calculate the component of the object's weight that is <u>down the slope</u> of the inclined plane.

$F_{gx} = F_{||}$ down the plane =
$= mg\sin\theta$
$= 100 \times 9.8 \times \sin(30)$
$= 490$ N

Question.3

Calculate the component of the object's weight that is <u>perpendicular to the slope</u> of the inclined plane.

$F_{gy} = F_{\perp}$
$= mg\cos\theta$
$= 100 \times 9.8 \times \cos(30)$
$= 849$ N

Given the object slides down the inclined plane at a constant speed.

Question 4.

Calculate the magnitude of the friction force experienced by the object.

As the object is travelling at a constant speed, acceleration is zero ($a = 0\ ms^{-2}$).
$\therefore F_{net} = 0$ N

$\therefore F_{\text{up the plane}} = F_{\text{down the plane}}$
$\therefore F_{fric} = F_{gx}$
$= 490$ N

Example.6

The object shown in Figure 2 below represents a 40 kg object that is sliding down a 25°.

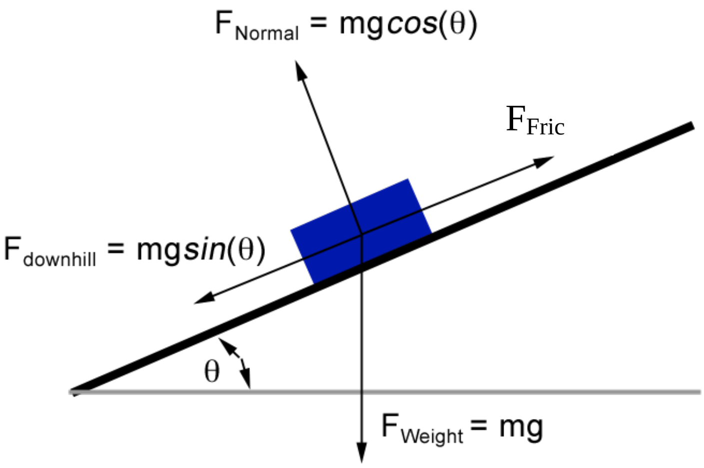


Figure 2 *– Object sliding down an inclined plane*

Question.1

If there were <u>no friction</u>, what would be the object's rate of acceleration down the plane?

$a = g sin\theta$
$a = 9.8 \times \sin(25)$
$= 4.1 \text{ ms}^{-2}$

Question.2

Given that friction is now considered and the object is now accelerating at a rate of 4 ms^{-2}, calculate the magnitude of the Frictional Force (F_{Fric}).

<u>Step.1</u> Construct an equation:

$F_{net} = F_{downhill} - F_{Fric}$
$ma = mg\sin(\theta) - F_{Fric}$

<u>Step.2</u> Substitute values into equation and solve:

$(40 \times 4) = (40 \times 9.8 \times \sin(25°)) - F_{Fric}$
$160 = 166 - F_{Fric}$
$\therefore F_{Fric} = 6 \text{ N}$

Question.3

If the object was initially stationary and maintained its rate of acceleration of 4 ms^{-2} for 5 seconds, what distance would it travel down the inclined plane?

$u = 0 \text{ ms}^{-1}$
$a = 4 \text{ ms}^{-2}$
$t = 5 \text{ sec}$
$s = ?$

$$s = ut + \frac{1}{2}at^2$$
$$= (0 \times 5) + \frac{1}{2} \times 4 \times (5)^2$$
$$= 50 \text{ m}$$

Newton's 2nd Law of Motion – Connected Multiple Body Systems

The next scenario that we need to consider is one involving connected bodies. Typical scenarios include ship and tug boat joined by cable, train and carriages joined by a connection, or a car and trailer joined via a tow bar.

Example.7

A truck of mass 1500 kg towing a boat of mass 500 kg accelerates at a constant rate on a horizontal road. A thrust of 5000 N is provided by the truck's engine. The Road friction of the truck is 1500 N, while that on the boat is 500 N. The air resistance on both the truck and the boat is negligible. The configuration is shown below in Figure 3.

Calculate:

1. The acceleration of both the truck and boat
2. The tension force in the tow bar between the truck and boat

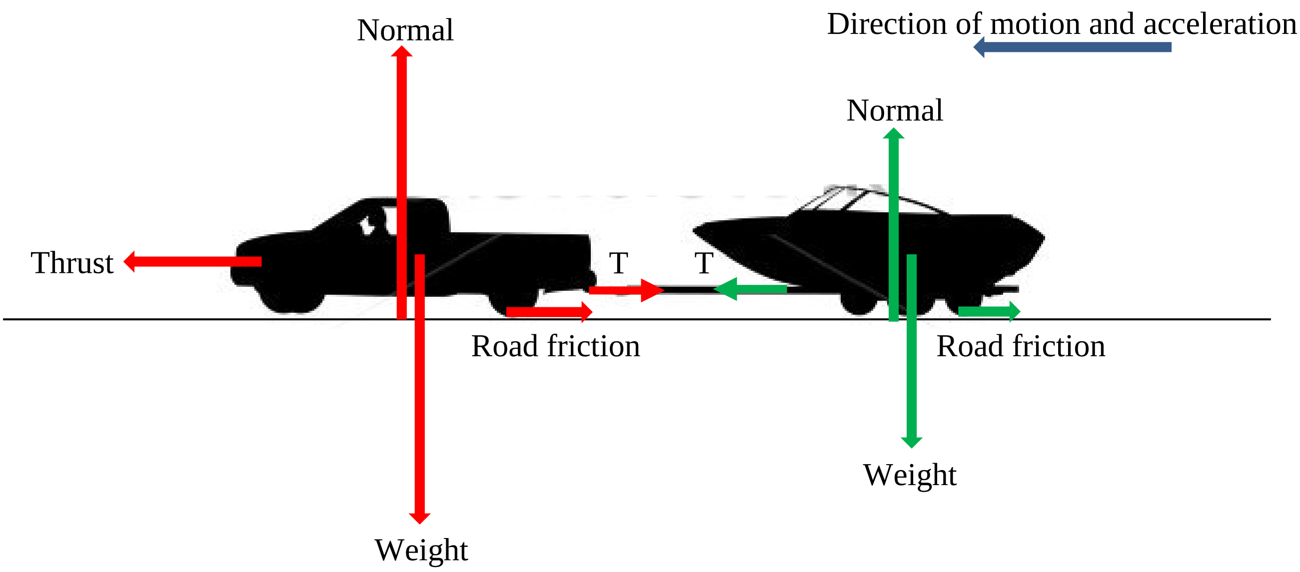
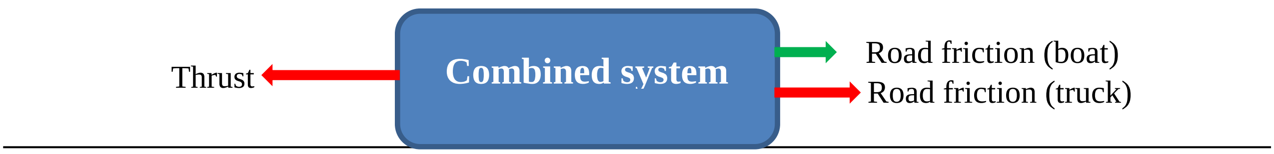


***Figure 3** – Truck towing boat*

Task.1 Calculate the acceleration of both the truck and boat

<u>Analyse the truck and boat as a single system.</u>

NB: vertical forces are balanced, examine the horizontal forces on the system, excluding the tension force (T) between the two objects.



Construct an equation (truck & boat combination):

F_{net} = Thrust – Road Friction (truck) – Road Friction (boat)

ma = Thrust - Road Friction (truck) – Road Friction (boat)

$(1500 + 500) \times a = 5000 - 1500 - 500$

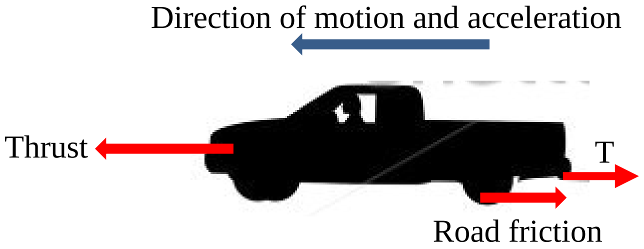
$2000 \times a = 3000$

$\therefore a = \frac{3000}{2000} = 1.5 \text{ ms}^{-2}$

Task.2 Calculate the tension force in the tow bar between the truck and boat

Option 1. Analyse the truck

NB: vertical forces are balanced, examine the horizontal forces on the system, including the tension force (T).



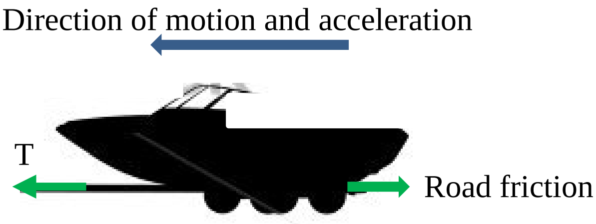
Construct an equation (truck alone):

F_{net} = Thrust – Road Friction – T
ma = Thrust - Road Friction - T
$1500 \times 1.5 = 5000 – 1500 - T$
$2250 = 3500 – T$
$\therefore$ $T = 3500 – 2250$
$= 1250$ N (→)

OR

Option 2. Analyse the boat

NB: vertical forces are balanced, examine the horizontal forces on the system, including the tension force (T).



Construct an equation (boat alone):

F_{net} = T – Road Friction
ma = T - Road Friction
$500 \times 1.5 = T – 500$
$750 = T - 500$
$\therefore$ $T = 750 + 500$
$= 1250$ N (←)

NB: By analysis of either the truck or boat the same value for the tension (T) is found.
The truck experiences a tension of 1250 N to the right.
The boat experiences a tension of 1250 N to the left.

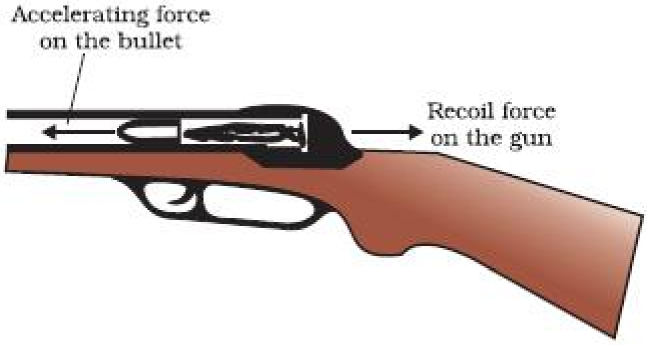
Newton's 3rd Law of Motion

Newton's third law of motion states that:

"In every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs - equal and opposite action-reaction force pairs."

Example.8

The firing of a gun is an example of Newton's 3rd law. When you fire a gun and the bullet comes hurtling out of the front, an equal and opposite reaction means the gun recoils into your shoulder.



NB: Due to the masses of both the gun and the bullet, the bullet has a much higher rate of acceleration and therefore velocity (Newton's 2nd Law).

Example.9

Lilly, the cheeky kitten, is sitting on top of her scratching post. Her weight force acts directly down and there is a normal reaction force pushing straight back up. These two forces are equal in size and opposite in direction.

Do they constitute an action reaction pair?

<u>Answer:</u> No.

<u>Reason:</u>
Action-reaction pairs always act on different objects. The two forces shown both act upon Lilly.