Section 3.3.3 - Collisions

Conservation of Momentum and Energy

During a collision, the total amount of **momentum** and **energy** are both **conserved.**

ie. ΣE and ΣP are constant before and after interaction

The form of energy can transform and momentum transferred, but the total of both remains constant.

Recall Newton's 2nd Law of Motion:

$F = ma$
$F = \Delta P/\Delta t$ or;

$F \Delta t = \Delta P$
Impulse delivered = Δ Momentum

Eg. A 500 kg car travelling at 100 kmh^{-1} collides with a tree and is brought to rest.

1. Calculate the change in the cars momentum
2. What impulse is delivered to the tree?
3. If the impact lasted 0.1 of a second, what force does the tree experience?

1. $m = 500$ kg
   $u = 100\ kmh^{-1} = 27.8\ ms^{-1}$
   $v = 0\ ms^{-1}$
   $\Delta v = 27.8\ ms^{-1}$
   $\Delta p = ?$

   $\Delta p = m\Delta v$
   $= 500 \times 27.8$
   $= 13889$
   $= \underline{\mathbf{1.4 \times 10^4\ kgms^{-1}}}$

2. $I = ?$
   $\Delta p = 1.4 \times 10^4\ kgms^{-1}$

   $I = \Delta p$
   $= \underline{\mathbf{1.4 \times 10^4\ Ns}}$

3. $I = 1.4 \times 10^4$ Ns
   $\Delta t = 0.1$ s
   $F = ?$

   $I = F\Delta t$
   $\therefore F = I/\Delta t$
   $= 1.4 \times 10^4/0.1$
   $= \underline{\mathbf{1.4 \times 10^5\ N}}$

NB. A passenger within the vehicle will also lose momentum (as s/he too comes to rest). If s/he can be slowed down over a long period of time, via seat belts/air bags/crumple zones etc., then the force they experience will be much reduced.

Elastic & Inelastic Collisions

Nearly all collisions on Earth are **inelastic**, very few even approach being truly **elastic.**
In an **elastic collision,** kinetic energy **(E_k) is conserved.**

$$E_k \text{ (before collision)} = E_k \text{ (after collision)}$$

In an **inelastic collision**, kinetic energy **(E_k) is not conserved**

$$E_k \text{ (before collision)} \neq E_k \text{ (after collision)}$$

The total amount of energy is still conserved. However, some initial E_k is converted into another form such as heat, sound, light etc.

Energy & Work

Both **energy** and **work** are scalar quantities (ie. not direction specific) and measured in **Joules**.

Recall the basic forms and equations for energy:

Form	Description	Equation
Kinetic Energy	Energy associated with movement	$Ek = ½mv^2$
Gravitational Potential	Energy associated with elevation above ground level	$Ep = mgh$
Elastic Potential	Energy associated with change of shape	$Ee = ½k\Delta x^2$

Work is done when a **force** acts upon a body, causing it to **move**.
Work always results in a change in the energy of this body.

$\boxed{W = \Delta E = Fx}$

Where:
W represents Work (Joules)
ΔE represents Change in energy (Joules)
F represents Force (Newtons)
x represents Distance (Meters)

NB. If the applied force is not parallel to the direction of movement then the cosine of the angle produced is included in the equation. ie. $W = Fx\cos\theta$

Power

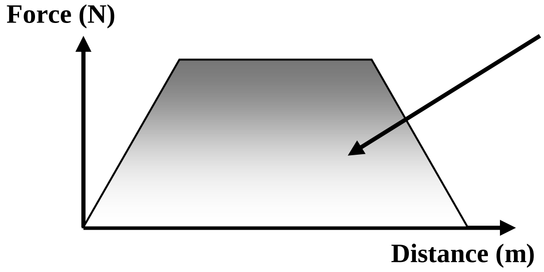
The term Power describes the **rate** at which energy is consumed or transferred.

$\boxed{P = W/\Delta t}$

Where:
P represents Power (Watts)
W represents Work done (Joules)
Δt represents Change in time (Seconds)

NB: 1 Watt = 1 Joule/sec

Force – Distance Graph



The area under a Force v Distance graph represents the work done and the change of energy (Joules).

To calculate the area under the graph, either use area equations (ie. squares, triangles or trapeziums) or the "counting squares" technique.

NB: 1 Joule = 1 Nm

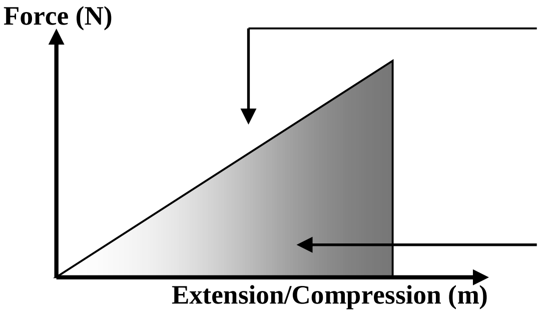
Applications of Energy: Hooke's Law

Robert Hooke
1635 – 1703

"Robert Hooke and Isaac Newton never went to the same bar to drink. They were always accusing each other of plagiarism" Dr Arnu Varshneya

Robert Hooke was the inventor of, amongst other things, the iris diaphragm in cameras, the universal joint used in motor vehicles, the balance wheel in a watch, the originator of the word 'cell' in biology – yet is known mostly for Hooke's Law.

Hooke's Law relates to the relationship between the extension of an "elastic" spring and the force applied to it. That is: "The more force applied to a spring, the more it will extend or compress".



The gradient (rise/run) represents the **spring constant (k)**.
It provides a measure of the amount of force required to extend or compress a spring a unit length (ie. meter).

The area represents the elastic potential energy stored in the spring.

$\boxed{F = k\Delta x}$

Where:
F represents Force (Newtons)
Δx represents Extension/Compression (Meters)
k represents Spring Constant (N/m)

$\boxed{E_e = ½F\Delta x}$

Where:
E_e represents Elastic Potential Energy (Joules)

NB. Substituting F = kx into the equation for Elastic potential energy, a second version of this equation can be derived:

$$\boxed{E_e = ½k\Delta x^2}$$

Eg. What kinetic energy and release velocity does a 50 g pinball acquire when it is fired by a spring that has been compressed by 5 cm, given its spring constant is $400Nm^{-1}$?

1. E_k = ?
   m = 0.05 kg
   Δx = .05 m
   k = 400 Nm^{-1}

   E_k (gained) = E_e (transferred)
   $\therefore E_k = ½k\Delta x^2$
   $= ½ \times 400 \times (0.05)^2$
   = **<u>0.5 J</u>**

2. v = ?
   E_k = 0.5 J
   m = 0.05 kg

   $E_k = ½mv^2$
   $v^2 = 2E_k/m$
   $\therefore v = \sqrt{2E_k / m}$
   $= \sqrt{2x(0.5)/0.05}$
   = **<u>4.5 ms⁻¹</u>**

NB. If a spring is overstretched it becomes permanently deformed. It will no longer return to its original length and will not follow Hooke's Law.

Exam Style Questions

The following information relates to questions 1 to 3

In a game of billiards, a ball (ball 1) travelling at 3.2 ms^{-1} collides head on with a stationary ball (ball 2). Immediately after this collision, ball 1 is stationary and ball 2 is in motion. Both balls are identical, each with a mass of 140 g.

Question 1
In this collision, what quantity is conserved?
Momentum

Question 2
What is the kinetic energy of ball 2 immediately after the collision?

$E_k = ?$
$m = 0.140$ kg
$v = 3.2$ ms^{-1} *[Due to conservation of momentum and identical masses $v_2 = u_1$]*

$E_k = \frac{1}{2}mv^2$
$= \frac{1}{2} \times 0.140 \times 3.2^2$
$= \underline{\mathbf{0.717\ J}}$

0.717 J

Question 3
Ball 2 travels 2.8 m along the table before coming to a rest. Draw a free body diagram of all the forces acting on ball 2 immediately after the collision, indicating direction and labelling the magnitude of each vector.

Step.1 find the horizontal force by finding the horizontal acceleration:

$u = 3.2$ ms^{-1}
$v = 0$ ms^{-1}
$x = 2.8$ m
$a = ?$

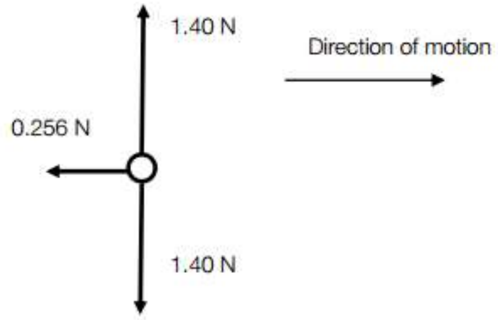
$v^2 = u^2 + 2ax$
$0^2 = 3.2^2 + 2 \times 2.8 \times a$
$0 = 10.24 + 5.6a$
$\therefore a = \frac{10.24}{-5.6}$
$= -1.82$ ms^{-2}

$F = ma$
$= 0.14 \times -1.82$
$= -0.255$ N *(or 0.255N in the direction opposing motion)*

Step.2 Calculate the weight force:

$W = mg$
$= 0.14 \times 10$
$= 1.4\ N$

Step.3 Draw the free body diagram as requested



The following information relates to questions 4 to 6

Casey's Comet is on a collision course with a planet of the same mass. It is travelling at 1000 m/s when it collides with the planet, which was travelling in the opposite direction at 720 m/s. After the collision the comet comes to a rest and the planet travels in the opposite direction to its original trajectory. Both the comet and planet retained their masses.
[mass of Casey's Comet is 3.7×10^{22} kg]

Question 4
In this collision, what quantity is conserved?
Momentum

Question 5
What is the total kinetic energy of planet immediately after the collision?

Step.1 Use conservation of momentum to find the velocity of the planet immediately after the collision.

NB: Take initial direction of comet as positive

$m_c u_c + m_p u_p = m_c v_c + m_p v_p$
$(3.7 \times 10^{22} \times 1000) + (3.7 \times 10^{22} \times -720) = (3.7 \times 10^{22} \times 0) + (3.7 \times 10^{22} \times v)$
$3.7 \times 10^{25} - 2.664 \times 10^{25} = 0 + 3.7 \times 10^{22} \times v$
$1.036 \times 10^{25} = 3.7 \times 10^{22} \times v$
$\therefore v = \frac{1.036 \times 10^{25}}{3.7 \times 10^{22}}$
$= 280\ ms^{-1}$ *(in the same direction as the comet was originally travelling)*

$E_k = \frac{1}{2}mv^2$
$= \frac{1}{2} \times 3.7 \times 10^{22} \times 280^2$
$= \underline{\mathbf{1.45 \times 10^{27}\ J}}$

$\boxed{\mathbf{1.45 \times 10^{27}\ J}}$

Question 6

Calculate the kinetic energy before the collision and the kinetic energy after the collision and show that it does not satisfy the conservation of energy. Explain where this energy has gone.

Step.1 calculate the total kinetic energy of the system before collision

Before collision

$E_k(total) = E_k(comet) + E_k(planet)$

$= ½m_cv_c^2 + ½m_pv_p^2$

$= ½ \times 3.7 \times 10^{22} \times 1000^2 + ½ \times 3.7 \times 10^{22} \times (-720)^2$

$= 1.85 \times 10^{28} + 9.59 \times 10^{27}$

$= 2.81 \times 10^{28}$ J

After collision

$E_k(total) = E_k(comet) + E_k(planet)$

$= ½m_cv_c^2 + ½m_pv_p^2$

$= ½ \times 3.7 \times 10^{22} \times 0^2 + ½ \times 3.7 \times 10^{22} \times (280)^2$

$= 0 + 1.45 \times 10^{27}$

$= 1.45 \times 10^{27}$ J

Step.2 Compare the energy of the system, both before and after

E_k *(after collision)* $\neq$ *Ek (before collision)*

Therefore, energy is not conserved and energy has been converted to sound or heat.

The following information relates to questions 7 to 10

A spring (that can be modelled as an ideal spring) is 10 cm long when no mass is attached. When a small 50 g block is attached to one end, the spring's length is 16 cm. When the object bounces up and down, its oscillations have an amplitude of 4 cm.

Question 7
What is the value of the spring constant, k?

$k = ?$
$F = W = mg$
$= 50 \times 10^{-3} \times 10$
$= 0.5\ N$
$x = -0.06\ m$

$F = -kx$ ***NB: The negative sign means the spring's restoring force is opposite the stretch direction***

$\therefore k = \frac{F}{x}$
$= \frac{0.5}{0.06}$
$= \underline{\mathbf{8.33\ Nm^{-1}}}$ ***NB: The spring constant (k) is always a positive value.***

8.33 Nm^{-1}

Question 8
What is the total energy in this system of the oscillating spring and block?

When the spring is totally extended, all energy is found in the form of elastic potential energy.
$E_S = ?$
$k = 8.33\ Nm^{-1}$
$x = 0.06\ m$

$E_S = ½kx^2$
$= ½ \times 8.33 \times (0.04)^2$
$= \underline{\mathbf{0.00666\ J\ (6.67 \times 10^{-3}\ J)}}$

6.67 x 10^{-3} J

Question 9
With reference to the conservation of energy, describe the energy transformations involved for one oscillation of the block.

- *Conservation of energy- energy cannot be created or destroyed, but it can change from one form to another*
- *When the block is at its maximum displacement from its equilibrium position (i.e. the spring is at 12 cm or 20 cm), all the energy of the system is potential energy in the spring*
- *As the block moves towards its equilibrium position, this is converted to kinetic energy, and at the equilibrium position all the energy in the system is kinetic energy. (or words to that effect)*

Question 10
Over time, it is observed that the amplitude of the oscillations gradually decreases. Explain how this can happen, while still obeying conservation of energy.

- *No energy is lost, but in a real world situation some energy is converted to sound or thermal energy as the block and spring oscillate*
- *Since some energy is lost to the surroundings, this results in a gradual decrease in the amplitude of these oscillations*

The following information relates to questions 11 to 14

A 50 g mass is attached to the end of the spring (which can be modelled as an ideal spring) with a length of 16 cm. A force of 14 N is applied to the spring and the spring is compressed by 4 cm.

Question 11
What is the value of the spring constant, k?

$F = -kx$
$\therefore k = F/x$
$= 14/0.04$
$= \underline{\mathbf{350\ Nm^{-1}}}$

350 Nm^{-1}

Question 12
What is the potential energy in this system as the spring is compressed to this length?

$E_S = ?$
$k = 350\ Nm^{-1}$
$x = 0.04\ m$

$E_S = ½kx^2$
$= ½ \times 350 \times (0.04)^2$
$= \underline{\mathbf{0.28\ J}}$

0.28 J

Question 13
What is the maximum velocity of the mass in this system as the spring is oscillating?

$\Delta E_S = \Delta E_k$
$½k\Delta x^2 = ½mv^2$
$\therefore v^2 = \frac{k\Delta x^2}{m}$
$\therefore v = \sqrt{\frac{k\Delta x^2}{m}}$
$= \sqrt{\frac{350 \times (0.04)^2}{50 \times 10^{-3}}}$
$= \underline{\mathbf{3.35\ ms^{-1}}}$

NB: As the mass started from rest its final velocity is the same as its change in velocity (Δv)

3.35 ms^{-1}

Question 14

If the spring had a resistance, describe the motion of the spring over time with reference to the principle of conservation of energy.

- *No energy is lost, but in a real world situation some energy is converted to sound or thermal energy as the block and spring oscillate*
- *Since some energy is lost to the surroundings, this results in a gradual decrease in the amplitude of these oscillations*
- *Mention of conservation of energy- energy cannot be created or destroyed, but it can change from one form to another*