Section 2.3.2 – Measuring Data

Types of Error

There are three (3) types of limitations to measurements:

1) Instrumental limitations

Any measuring device can only be used to measure to with a certain degree of fineness. Our measurements are no better than the instruments we use to make them.

2) Systematic errors and blunders

These are caused by a mistake which does not change during the measurement. For example, if a digital balance was not correctly "tared" to zero with no weight on the pan, all your subsequent measurements of mass would be too large.

Systematic errors do not enter into the uncertainty. They are either identified and eliminated or lurk in the background producing a shift from the true value.

3) Random errors

These arise from unnoticed variations in measurement technique, tiny changes in the experimental environment, etc. Random variations affect precision. Truly random effects average out if the results of a large number of trials are combined.

Precision and accuracy

The terms "precision" and "accuracy" are often incorrectly interchanged. The two terms have very specific and quite different meanings.

A **precise measurement** is one where independent measurements of the same quantity closely cluster about a single value that may or may not be the correct value.

Example 1

Three measurements of the top speed of a toy car are: 30.2 ms^{-1}, 30.1 ms^{-1} & 30.1 ms^{-1}. These three separate measurements are considered precise.

An **accurate measurement** is one where independent measurements cluster about the true value of the measured quantity.

Example 2

A class uses radii and circumferences of a circle and find the value of π to be: 3.14

Example 3

"Your wrist watch may measure time with a precision of one second. A stop watch may time your race with a precision of one hundredth of a second.

However, if the clocks change and you forget to reset your wrist watch, then you have a very precise time but it is not very accurate – you will be an hour early or late for all of your meetings."

Precise readings will have high repeatability (reliability). Be aware that you can have very precise measurements, without having considerable accuracy. Refer to Figure 1.

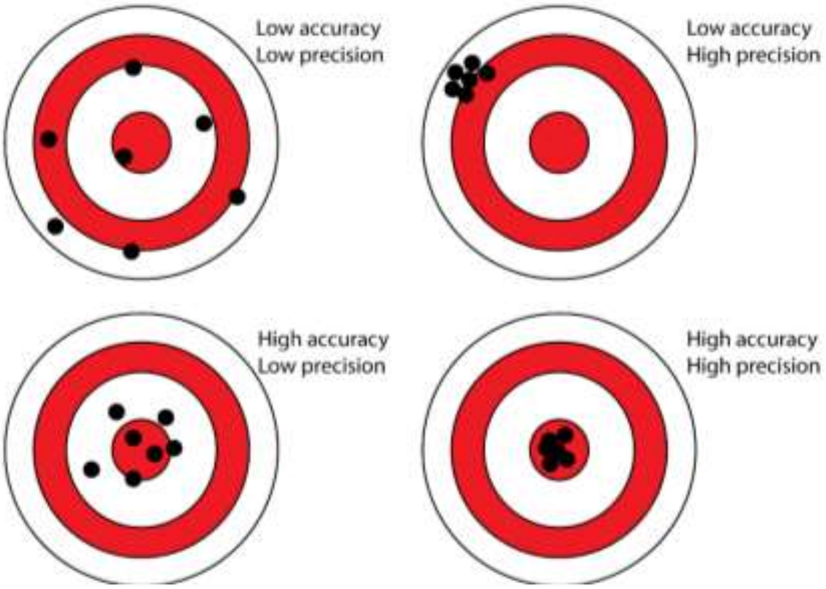


Figure 1 *– Visual representation of accuracy and precision*

Repeatability & Reliability

Repeatability

Completion of experiment more than once to obtain closely agreeing results.
Good practice in improving experimental precision.

Reliability:

Likelihood that another experimenter with different/similar equipment will obtain the same results

Where personal judgement/technique/parallax is involved this impacts more on reliability than repeatability.

Significant Figures

Students are often confused by numerical answers in the back of a Physics and/or Chemistry textbook. Students regularly say to their teachers "my answer was close, but different to the back of the book". Often this is a result of not understanding significant figures.

Rules for Significant Figures

1. All non-zero digits are significant
2. Zeros between two significant digits are significant
3. A zero after the decimal point that is at the end of a number is significant
4. Zeros that are used only to space a decimal point are not considered significant

Example.4

How many significant figures are there in the following number?

406.30

Let's consider the above rules:

1. All **non-zero digits** are significant 406.30
2. **Zeros between two significant digits** are significant 406.30
3. A **zero after the decimal point that is at the end** of a number is significant 406.30

The number 406.30 has **<u>5 significant figures</u>**.

Example.5

How many significant figures are there in the following number?

0.004

Let's consider the above rules:

1. All **non-zero digits** are significant 0.004
2. **Zeros that are used only to space a decimal point** are not considered significant 0.004

The number 0.004 has only **<u>1 significant figure</u>**.

Correct Rounding of Data

To keep the correct number of significant figures, numbers must be rounded off. The **discarded digit** is called the **remainder**. There are three rules for rounding:

Rule 1: If the remainder is less than 5, drop the last digit.
Rounding to one decimal place: 5.346 → 5.3

Rule 2: If the remainder is greater than 5, increase the final digit by 1.
Rounding to one decimal place: 5.798 → 5.8

Rule 3: If the remainder is exactly 5 then round the last digit to the closest even number. This is to prevent rounding bias. Remainders from 1 to 5 are rounded down half the time and remainders from 6 to 10 are rounded up the other half.
Rounding to one decimal place: 3.55 → 3.6, also 3.65 → 3.6

Calculations Combining Significant Figures

Example.6

Consider an investigation where you are required to calculate the volume of a sphere based upon its diameter measured using a set of Vernier callipers

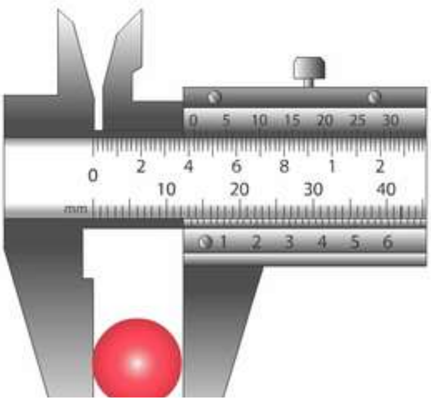


Figure 2 – *Vernier callipers*

Measured diameter = 12.25 mm

Volume = ?

$$\boldsymbol{V} = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$$
$$= \frac{4\pi}{3}\left(\frac{12.25}{2}\right)^3$$
$$= 926.51363047\ldots$$

NB: You cannot express a calculated answer to a higher number of significant figures than the measurement from which it is calculated.

Therefore the volume of the sphere is expressed as <u>926.5 mm^3</u> (expressed to 4 sig figs)

Example.7

Students have measured the current running through, and the voltage across, a component. They then use these measurements to calculate the power rating of the device using P = VI
Their measurements and power calculations are as follows:

V = 12.4 Volts (3 sig figs)
I = 3.012 Amps (4 sig figs)

P = VI
= 12.4 × 3.012
= 37.3488 Watts

NB: You cannot express a calculated answer to a higher number of significant figures than the smallest number of significant figures expressed in the measured values. In this example 3 sig figs is the smallest measured value.

∴ the Power rating of the component is 37.3 Watts (expressed to 3 sig figs)

Measurement Uncertainties

All physical measurements have an associated error or uncertainty. This less the uncertainty the higher the degree of confidence that the reading is accurate.

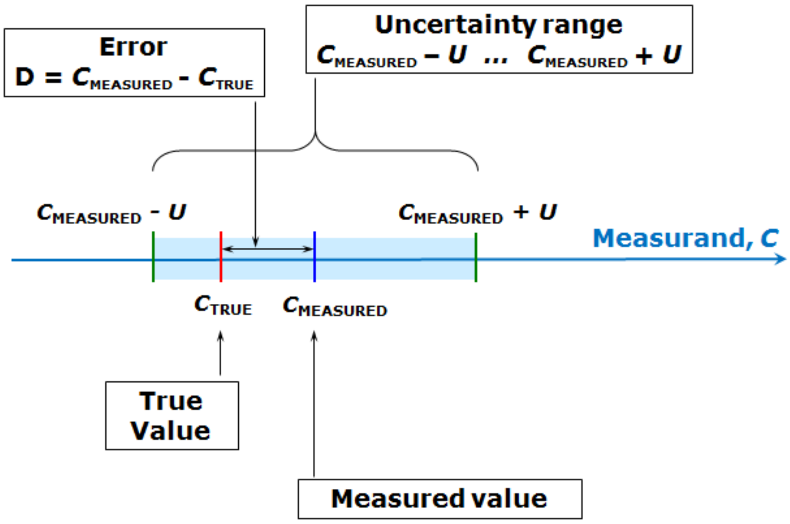


***Figure 3** – Comparison between error and uncertainty*

Figure 3 shows a measured value for some quantity C ($C_{measured}$). Unbeknown to the investigator this is not the true value of c (C_{true})

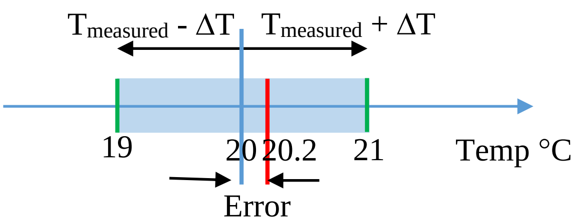
The Error is the difference between the measured and true value for c
Error = $C_{measured} - C_{true}$

The uncertainty ranges from $C_{measured}$ - Uncertainty to $C_{measured}$ + Uncertainty

Example.8

Consider a measured Temperature using a basic "homemade" thermometer. The following data was recorded:

$T_{measured} = 20$ °C
Uncertainty = ± 1 °C
$T_{true} = 20.2$°C



Express the measured temperature and uncertainly in one mathematical statement. State the error in this measurement.

$T_{measured} = 20 \pm 1$°C
Error = $T_{measured} - T_{true}$
= 20 – 20.2
= -0.2 °C or 0.2°C for an absolute value.

Uncertainties can be expressed as:

- Absolute error, in the form of $\boldsymbol{x} \pm \Delta\boldsymbol{x}$
- Percentage error, in the form $\boldsymbol{x} \pm \left(\frac{\Delta x}{\boldsymbol{x}}\right) \times \mathbf{100}\%$

Example.9

Students measure the temperature of a solution to be 23.5°C and allocate an uncertainty of 0.2°C. Express this measurement as both an absolute and percentage error.

Absolute error: $23.5 \pm 0.2°C$

Percentage error: $23.5 \pm (\frac{\mathbf{0.2}}{\mathbf{23.5}} \times \mathbf{100}\%)$

NB: You can reduce the amount of uncertainty by:

- Taking repeated measurements and using higher precision devices (random error)
- Correctly calibrating your measuring device (systematic error)

Calculations Combining Uncertainties

[NB: More relevant at the Year 12 level of study]

When combining data to generate a calculated answer, the impact of combining uncertainties must be treated via the following rules:

1. **<u>Addition and Subtraction:</u> <u>ADD the Absolute Uncertainties</u>**:

(x ± Δx) - (y ± Δy) = (x - y) ± (Δx + Δy)
(x ± Δx) + (y ± Δy) = (x + y) ± (Δx + Δy)

Example.10
Consider the numbers: (6.5 ± 0.5) m and (3.3 ± 0.1) m

Add: (6.5 ± 0.5) m + (3.3 ± 0.1) m = (9.8 ± 0.6) m
Subtract: (6.5 ± 0.5) m - (3.3 ± 0.1) m = (3.2 ± 0.6) m

2. **<u>Multiplication and Division:</u> <u>ADD the Percentage Uncertainties</u>**:

(x ± *Δx*) / (y ± *Δy*) = (x / y) ± (*Δx* + *Δy*)
(x ± *Δx*) × (y ± *Δy*) = (x × y) ± (*Δx* + *Δy*)

Example.11
Consider the numbers: (5.0 N ± 4.0%) and (3.0 m ± 3.3%)

Multiply: (5.0 N ± 4.0%) x (3.0 m ± 3.3%) = 15.0 Nm ± 7.3%)
Divide: (5.0 N ± 4.0%) / (3.0 m ± 3.3%) = (1.7 N/m ± 7.3%)

3. **For a number raised to a power**: **MULTIPLY the Percentage Uncertainty by the power**.

$(x \pm \Delta x)^n = (x^n \pm n\Delta x)$

Example.12
Consider the number: (2.0 m ± 1.0%)

Cube: $(2.0\text{ m} \pm 1.0\%)^3 =$ m^3 ± 3.0%)
Square Root: $(2.0\text{ m} \pm 1.0\%)^{1/2} = (1.4\text{ m}^{1/2} \pm 0.5\%)$

4. **For multiplying a number by a constant** (there are 2 options)

Absolute Uncertainty:
$c(x \pm \Delta x) = cx \pm c(\Delta x)$

Example.13
Consider: 1.5(2.0 ± 0.2) m = (3.0 ± 0.3) m

Percentage Uncertainty:
$c(x \pm \Delta x) = cx \pm \Delta x$

Example.14
Consider: 1.5(2.0 m ± 1.0%) = 3.0 m ± 1.0%)

NB: The Relative Uncertainty is not multiplied by the constant

Example .15
Calculate the displacement and percentage uncertainty, with correct significant figures, from the following quantities:

s = ?
u = 10.2 ± 0.2 ms^{-1}
a = 5.00 ± 0.01 ms^{-2}
t = 8.20 ± 0.01 sec

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$
$$= (\mathbf{10.2 \times 8.20}) + \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{5.00} \times (\mathbf{8.20})^2$$
= 251.74 m

∴ s = 252 ± 2 m

NB: As u is limited to 3 sig figs, ∴ s too is limited to 3 sig figs.

Likewise Δu is limited to 1 sig fig, ∴ Δs is limited to 1 sig fig.

Evaluate uncertainties for this equation

Section 1
Δut = (Δu + Δt)
$= (\frac{.2}{10.2} \times 100) + (\frac{0.01}{8.20} \times 100)$
= 2.1%
$= (\frac{2.1}{100} \times (10.2 \times 8.20)) = 1.76$

Section 2
$\Delta\frac{1}{2}\boldsymbol{at^2} = \frac{1}{2}(\Delta a + 2\Delta t)$
$= \frac{1}{2}[(\frac{.01}{5.00} \times 100) + 2(\frac{0.01}{8.20} \times 100)]$
= 0.22%
$= [\frac{0.22}{100} \times \left(\frac{1}{2} \times 5.00 \times 8.20^2\right)] = 0.37$

Combine uncertainties
$\Delta s = \Delta ut + \Delta\frac{1}{2}\boldsymbol{at^2}$
= 1.76 + 0.37 = 2.13

Further Examples

Example.16

The period of a pendulum is given by $\boldsymbol{T = 2\pi\sqrt{\frac{l}{g}}}$

Here, *l* = 0.24m is the pendulum length and g = 9.81m/s^2 is the acceleration due to gravity.

✘**WRONG:** *T* = 0.983269235922s
✓**RIGHT**: *T* = 0.98s

Your calculator may report the first number, but there is no way you know *T* to that level of precision. When no uncertainties are given, report your value with the same number of significant figures as the value with the smallest number of significant figures.

Example.17

The mass of an object was found to be 3.56g with an uncertainty of 0.032g.

✘**WRONG:** *m* = 3.56 ± 0.032 g
✓**RIGHT:** *m* = 3.56 ± 0.03 g

The first way is wrong because the uncertainty should be reported with one significant figure.

Example.18

The length of an object was found to be 2.593cm with an uncertainty of 0.03cm.

✘**WRONG** *L* = 2.593 ± 0.03 cm
✓**RIGHT:** *L* = 2.59 ± 0.03 cm

The first way is wrong because it is impossible for the third decimal point to be meaningful.

Example.19

The velocity was found to be 2. 45m/s with an uncertainty of 0.6m/s.

✘**WRONG:** *v* = 2.5 ± 0.6 cm
✓**RIGHT:** *v* = 2.4 ± 0.6 cm

The first way is wrong because the first discarded digit is a 5. In this case, the final digit is rounded to the closest even number (i.e. 4)

Example.20

The distance was found to be 45600m with an uncertainty around 1m

✘**WRONG:** *d* = 45600 cm
✓**RIGHT:** *d* = 4.5600 × 10^4 m

The first way is wrong because it tells us nothing about the uncertainty. Using scientific notation emphasizes that we know the distance to within 1m.

Validity

Validity describes "How well do the findings provide a response to research question?" Generally if a single variable has been manipulated and an accurate outcome has been measured then the experiment would have high validity.

If other extraneous variables have impacted or other explanations are possible then the experiment would have low validity.

Validity can be improved with better methodology, either in terms of precision of data collection or manipulation of variables

Example.21

Consider an investigation that compares different golf clubs in terms of maximum driving range.

Variable control to improve investigation validity:

- Multiple golfers – reduce to single person
- Amateur player – replace with professional for more consistency
- Testing over time – test at under same weather / environmental conditions
- Testing one after the other – factor in tiredness of server etc…..

You need to continually review and appraise your methodology to ensure your investigation has a high degree of validity.