Section 7 – Momentum & Impulse

Momentum

Momentum is the product of an objects mass (kg) and its velocity (ms^{-1}).

$p = mv$

Where p represents momentum ($kgms^{-1}$)
m represents mass (kg)
v represents velocity (ms^{-1})

Example.1

A car of mass 1500 kg is travelling at 100 kmh^{-1}, what is its momentum?

$m_c = 1500$ kg
$v_c = 100$ kmh^{-1}
$= 27.8$ ms^{-1}
$p = ?$

$p = mv$
$= 1500 \times 27.8$
$= 41700$ $kgms^{-1}$
$= \underline{\mathbf{4.17 \times 10^4\ kgms^{-1}}}$

Conservation of Momentum

The conservation of momentum states:

> *"For a collision occurring between object 1 and object 2 in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision."*

So for a 2 body system (ie. object a & object b) the following holds true:

p (before collision) = p (after collision)

$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

Example.2

A car of mass 750 kg is travelling east at a velocity of 100 kmh^{-1} when it collides into a stationary truck of mass 2 tonne. Following the collision, the car remains stationary, what velocity does the truck have?

Before Collision **Collision** **After Collision**

$m_c = 750$ kg
$u_c = 100$ kmh^{-1}
$= 27.8$ ms^{-1}

$m_t = 2000$ kg
$u_t = 0$ ms^{-1}

$m_c = 750$ kg
$v_c = 0$

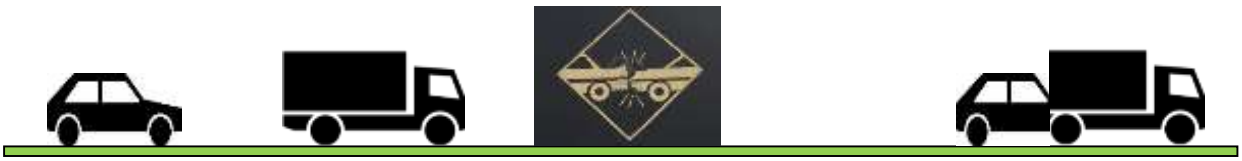
$m_t = 2000$ kg
$v_t = ?$

p (before collision) = p (after collision)

$$m_c u_c + m_t u_t = m_c v_c + m_t v_t$$
$$(750 \times 27.8) + (2000 \times 0) = (750 \times 0) + (2000 \times v_t)$$
$$20850 = 0 + 2000 v_t$$
$$\therefore v_t = 20850/2000 = \underline{\mathbf{10.4\ ms^{-1}}}$$

Example.3

What would have been the velocity of the truck in the previous example had it entangled with the car and moved from the collision joined together?



Before Collision **Collision** **After Collision**

$m_c = 750$ kg
$u_c = 100$ kmh^{-1} = 27.8 ms^{-1}

$m_t = 2000$ kg
$u_t = 0$ ms^{-1}

$(m_c + m_t) = 750 + 2000 = 2750$ kg
$v_{c+t} = ?$

p (before collision) = p (after collision)

$$m_c u_c + m_t u_t = m_{c+t} v_{c+t}$$
$$(750 \times 27.8) + (2000 \times 0) = (2750 \times v_{c+t})$$
$$20850 = 2750 v_{c+t}$$
$$\therefore v_{c+t} = 20850/2750 = \underline{\mathbf{7.6\ ms^{-1}}}$$

Example.4

A had pistol of mass 1.5 kg fires a bullet of mass 50 g at a speed of 300 ms^{-1} in an easterly direction. Use the conservation of momentum to calculate the pistols recoil velocity.

Before Collision **Collision** **After Collision**

$m_p = 1.5$ kg
$u_p = 0$ ms^{-1}

$m_b = 0.05$ kg
$u_b = 0$ ms^{-1}

$m_p = 1.5$ kg
$v_p = ?$

$m_b = 0.05$ kg
$v_b = 300$ ms^{-1}

p (before collision) = p (after collision)

$$m_p u_p + m_b u_b = m_p v_p + m_b v_b$$
$$(1.5 \times 0) + (0.05 \times 0) = (1.5 \times v_p) + (0.05 \times 300)$$
$$0 = 1.5 v_p + 15$$
$$1.5 v_p = -15$$
$$\therefore v_p = -15/1.5 = -10 \text{ ms}^{-1} \text{ (0r 10 ms}^{-1} \text{ west)}$$

Impulse

When a golf club strikes a golf ball it gives some of its momentum to the ball. In turn the club applies a force over a relatively short period of time.

Impulse is the physics term used to describe the product of an applied force over a period of time.

Impulse = $F\Delta t$	Where Impulse is measure (Ns) F represents force (N) Δt represents time (sec)

Commonly called the "impulse delivered"

A quick derivation:

Newton's 2nd Law: $F = ma$

$= m\left(\frac{\Delta V}{\Delta t}\right)$

$\therefore F\Delta t = m\Delta v$

Or rather "the impulse delivered is equal to the change of momentum".

Example.5

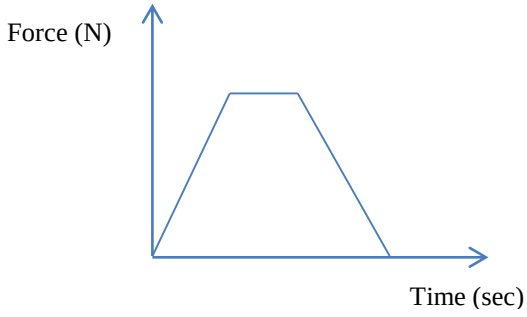
During a golf shot a force of 2kN is applied upon a 75 g golf ball for a period of 1 ms. Calculate the change of velocity of the golf ball.

$F = 2 \times 10^3$ N
$m = 75 \times 10^{-3}$ kg
$\Delta t = 1 \times 10^{-3}$s
$\Delta v = ?$

$F\Delta t = m\Delta v$
$2 \times 10^3 \times 1 \times 10^{-3} = 75 \times 10^{-3} \times \Delta v$
$2 = 75 \times 10^{-3} \times \Delta v$
$\therefore \Delta v = 2/75 \times 10^{-3}$
$= \underline{\mathbf{26.7\ ms^{-1}}}$

Force – Time Graph

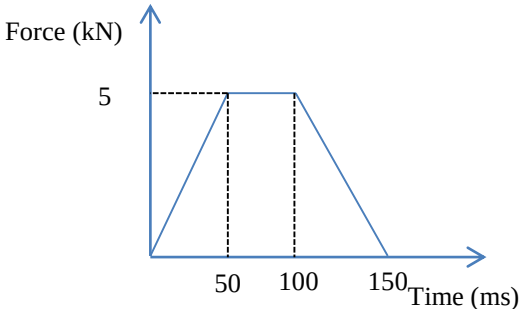
The area under of Force-Time graph is equal to the impulse delivered from one object to another.



Area = Impulse delivered = ΔP

Example.6

John foolishly rides his bike into a tree. The following Force-Time graph is generated. Given the combined mass of John and his bike is 100 kg calculate the impulse delivered to the bike/rider and the change in velocity experienced.



Impulse delivered = Area under graph
$= ½(50 \times 10^{-3} + 150 \times 10^{-3}) \times 5000$
$= 500$ Ns

$\Delta v = ?$ $\Delta p = m\Delta v$
$m = 100$ $\Delta v = \Delta p/m$
$\Delta p = 500$ kgms^{-1} $= 500/100 =$ **5 ms^{-1}**

Exam Style Questions

Questions 1 to 3 refer to the following information

James is riding his bicycle along a flat level road. He is travelling at a constant speed of 10 m s^{-1}. The traffic lights turn red and he applies his brakes.

James brings his bike to a halt in 12 m.
James and his bike have a mass of 80 kg.

Question 1.
What is the magnitude of the momentum of James and his bike before he applies the brakes?

p = ?
m = 80 kg
v = 10 ms^{-1}

p = mv
= 80 x 10
= 800 kgms^{-1}

800 kgms^{-1}

Question 2.
How long does it take for James to come to rest?

u = 10 ms^{-1}
v = 0 ms^{-1}
x = 12 m
t = ?

x = ½(u + v)t
12 = ½(10 + 0)t
12 = 5t
∴t = 12/5 = 2.4 s

2.4 s

Question 3.
What is the magnitude of the average braking force applied by the brakes as the bike comes to rest?

Δp = F$_{av}$ t, ∴F$_{av}$ = Δp/t
= 800 / 2.4
= 333 N

OR

u = 10 ms^{-1}
v = 0 ms^{-1}
t = 2.4 s
a = ?

v = u + at
0 = 10 + 2.4a
-10 = 2.4a
∴a = -10/2.4
= -4.167 ms^{-2}

F = ma = 80 x 4.167 = 333 N

333 N