Section 4.2.1 – The Photoelectric Effect

What is the photoelectric Effect?

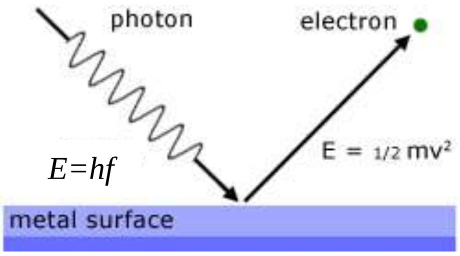


Figure 1

The basics of the photoelectric effect involves the ejection of an **electron** from a **metal surface** as a result of **exposure to light**, or other electromagnetic radiation.

A material that can exhibit this phenomena is said to be **photoemissive**, and the ejected electrons are called **photoelectrons**. This is shown in Figure 1.

The photocell

A photocell is a device used to investigate and measure quantities involved with the photoelectric effect. It consists of an **evacuated tube**, a **cathode** and **anode**, an **ammeter** and **variable voltage** (as shown in Figure 2).

Under the correct conditions, incident light is strike the photocell and photoelectrons are ejected from the metal cathode. They are then accelerated by the electric field to the anode and measured by the ammeter as a current.

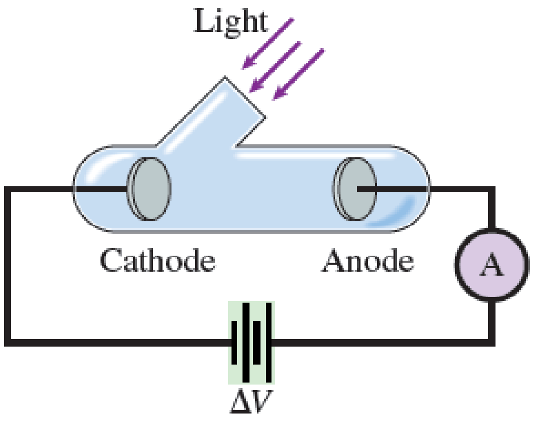


Figure 2

The Work Function (ϕ)

Electrons in a metal are bound by the positively charged protons. In order for these electrons to escape the surface of the metal they require energy. Light carries energy that surface electrons can absorb and use to become free.

The energy required to release the **least bound electron** from a cathode is the **work function, ϕ.** Each metal has its own unique work function.

Any energy "left over" from the incident light is given to the released photoelectron as kinetic energy (E_k).

$$\boxed{E_k = E_{light} - \phi}$$

Example.1

An electron on the surface of a sodium cathode absorbs 4.8 eV from incident light. The work function of sodium is 2.3 eV. Will the electron be released and, if so, how much kinetic energy will it have?

$E_k = ?$
$\phi = 2.3$ eV
$E_{light} = 4.8$ eV

$$E_k = E_{light} - \phi$$
$$= 4.8 - 2.3$$
$$= 2.5 \text{ eV}$$

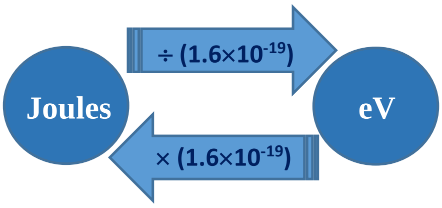
$\therefore$ The light has sufficient energy to release the photoelectron.

The electron Volt

In the macroscopic world the unit, Joule, is used to measure the energy of everyday objects such as trains and planes. However, in the microscopic world such as the energy of electrons, the electron volt is the energy unit of choice.

<u>Recall</u>

1 eV = 1.6×10^{-19} Joules



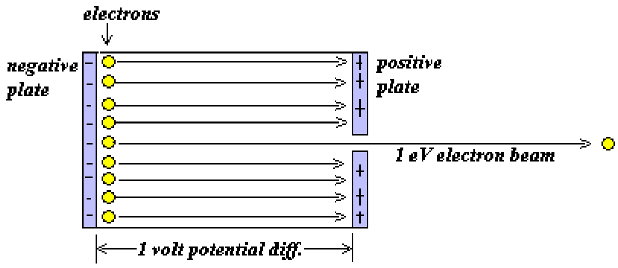


Figure. 4 An electron-volt

Electrons accelerated across a potential of 1V will gain 1.6×10^{-19} J or 1 eV of energy.

Example.2

Express your answer to example.1 in units of Joules

E_k = 2.5 eV
= $2.5 \times 1.6 \times 10^{-19}$
= 4.0×10^{-19} Joules

Variables under consideration

The photoelectric effect experiment is designed to determine relationships **between two independent variables** and **two dependent variables.** The variables being examined are as follows:

Independent variables	Dependent variables
Intensity (brightness) of incident light	Rate of ejection of electrons from surface
Frequency (colour) of incident light	Maximum kinetic energy (E_k) of ejected electrons

Photocurrent

If a photoelectron has enough kinetic energy to reach the other side (anode) a current will form in the circuit. This current is called a **photocurrent**.

NB: The **photocurrent** is a measure of the **rate of ejection of electrons**.

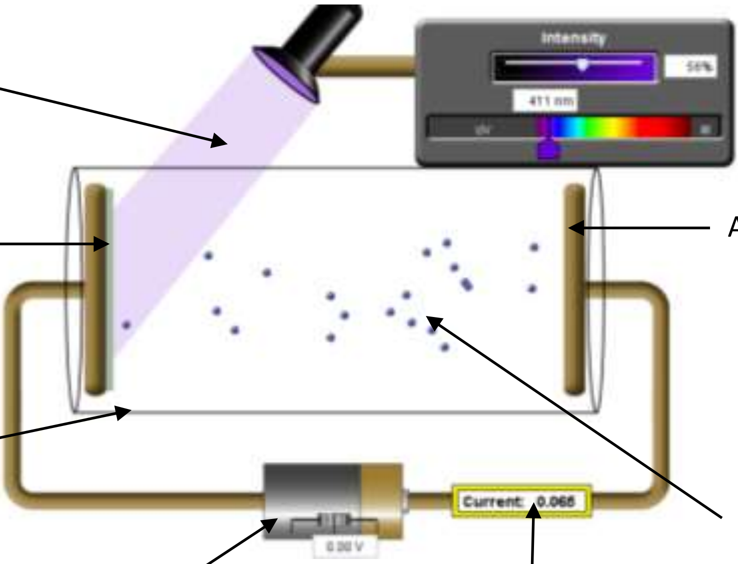


Figure 3 Photocell

Stopping Voltage

__Recall__

There is a change of energy (ΔE) associated with a charged particle when moved through an electric field. Depending upon the polarity of both the charged particle and the electric field a particle can either be accelerated or decelerated.

The **work done**, or gain in **kinetic energy** (E_k) upon a charged particle by an electric potential can be expressed as:

$$\boxed{W = \Delta E_k = qV}$$

Where W is the work done (Joules or eV)
ΔE_k is the change in kinetic energy (Joules or eV)
q is the charge (Coulombs)
V is the potential difference (Volts)

Example.2

Calculate the gain in kinetic energy of an electron accelerated across a potential of 5.0 V. Calculate the velocity of this electron?

$\Delta E_k = ?$
$q = 1.6 \times 10^{-19}$ C
$V = 5.0$ V
$m_e = 9.1 \times 10^{-31}$ kg

Qn.1

$$\Delta E_k = qV = 1.6 \times 10^{-19} \times 5.0 = 8.0 \times 10^{-19} \text{ J}$$

Qn.2

$$\Delta E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\Delta E_k}{m}} = \sqrt{\frac{2\times8.0\times10^{-19}}{9.1\times10^{-31}}} = 1.3 \times 10^{6} \text{ ms}^{-1}$$

In Figure 3, the released photoelectrons shown have sufficient kinetic energy to move between the cathode to the anode.

- If a positive potential difference is applied to the anode, all released photoelectrons will be accelerated between the two electrodes at high speeds.
- However, if a negative (or reverse) potential is placed across the anode then the electric field will oppose the motion of the released photoelectrons.

Eventually the **reverse potential** will reach a magnitude at which there is **no photocurrent** generated (ie. the photoelectrons stop reaching anode). This is called the **stopping voltage, V_0**.

The stopping voltage is a measure of the **maximum kinetic energy** of the ejected electrons.

$$\boxed{E_k(max) = eV_0}$$

Where $E_k(max)$ is the maximum kinetic energy of a photoelectron (Joules or eV)
e is the charge of an electron (Coulombs)
V_0 is the stopping voltage (Volts)

Example.3
Calculate the maximum kinetic energy of a photoelectron whose stopping voltage is 3.0 Volts. State your answer in both Joules and eV.

$E_k(max) = ?$
$e = 1.6 \times 10^{-19}$ C
$V_0 = 3.0$ Volts

$E_k(max) = eV_0$
$= 1.6 \times 10^{-19} \times 3.0$
$= 4.8 \times 10^{-19}$ Joules

$E_k(max) = 4.8 \times 10^{-19}$ Joules
$= \dfrac{4.8 \times 10^{-19}}{1.6 \times 10^{-19}}$
$= 3.0$ eV

NB: A stopping voltage of 3.0 Volts measures the $E_k(max)$ of 3.0 eV.

Example.4
When conducting the photoelectric effect experiment with a calcium cathode ($\phi = 2.9$ eV) students measure a stopping voltage of 1.5 V.

1. What is the maximum kinetic energy, in eV, of ejected electrons in this experiment?
2. What is the energy, in eV, of the incident light?

Qn.1
$E_k(max) = ?$
$\phi = 2.9$ eV
$V_0 = 1.5$ V

$E_k(max) = eV_0$
$= 1.6 \times 10^{-19} \times 1.5$
$= 2.4 \times 10^{-19}$ J
(1.5 eV)

Qn.2
$E_k = E_{light} - \phi$
$\therefore E_{light} = E_k + \phi$
$= 1.5 + 2.9$
$= 4.4$ eV

The Photoelectric Effect – Changing Light Intensity

Let's consider the effect of **changing the intensity,** or brightness of light. A brighter or more intense light has more energy than a dimmer or less intense light.

The wave model, which associates intensity with wave amplitude, would expect a more intense light to deliver more energy to the electrons. Accordingly, the wave model would predict a more intense light should produce photoelectrons with higher kinetic energies and therefore a larger stopping voltage (V_0).

However, as can be seen in Figures 4a & 4b, for a given metal (in this case Sodium) and for an incident light energy > than the work function (ϕ):

- The photocurrent is directly proportional to the intensity: (I_{ph}) α Intensity
- The stopping voltage is not affected by the intensity $V_0 \not\propto$ Intensity

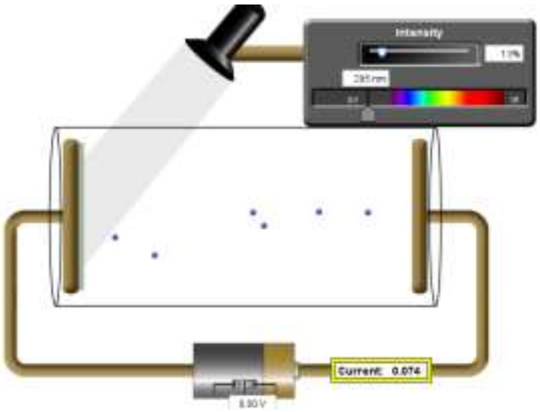


Figure 4a Sodium cathode & low intensity

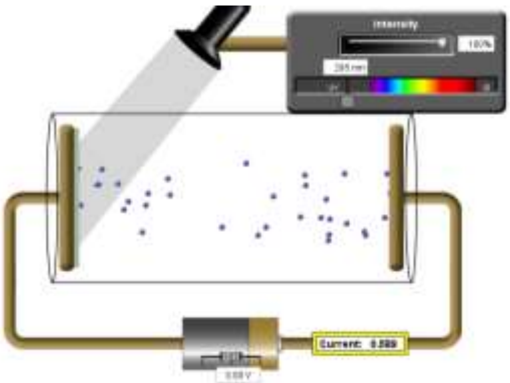


Figure 4b Sodium cathode & high intensity

NB:↑ intensity and ↑I_{ph}, but no change in E_k or V_0.

As can be seen from the I-V graph in Figure 5, the higher the light intensity the higher the photocurrent produced. However, regardless of the light intensity the stopping voltage (V_0) remains the same.

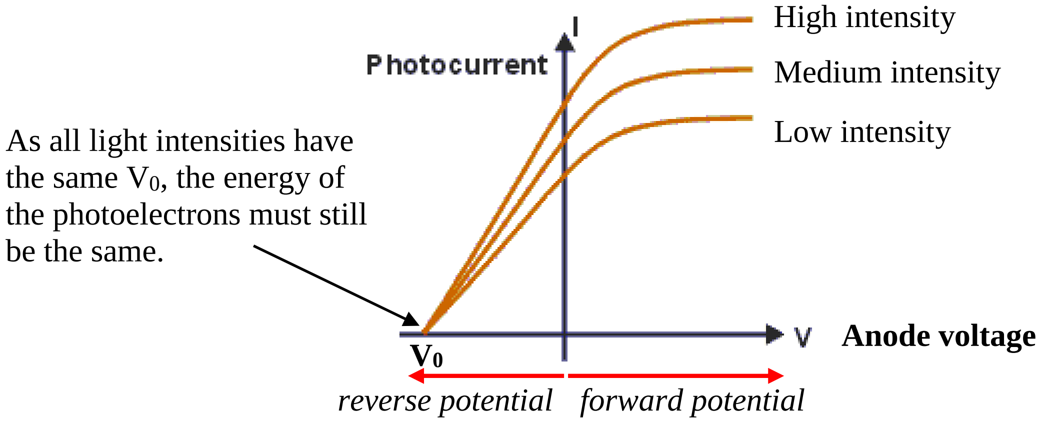


Figure 5 I-V Graph for the Photoelectric Effect with varying intensity

NB: The wave model of light could not explain the results for the change of light intensity.

The Photoelectric Effect – Changing Light Frequency

In the previous are of study we have always described the colour of light as a function of wavelength (λ). For this area of study we shall refer to the colour of **light as a function of frequency (*f*).**

Recall from our knowledge of the electromagnetic spectrum that higher frequency light has more energy than lower frequency light.
(Eg. X-Rays have high frequency and high energy)

The results from the varying the frequency of the incident light upon a given metal cathode are as follows:

- There is a minimum frequency below which no photocurrent is measured. This is called the **threshold frequency (f_0)**
- Above f_0, the **kinetic energy** of photoelectrons **increases proportionally** with the **frequency**: $E_k \propto f$
- The photocurrent is <u>not affected</u> by the frequency: $I_{ph} \not\propto f$

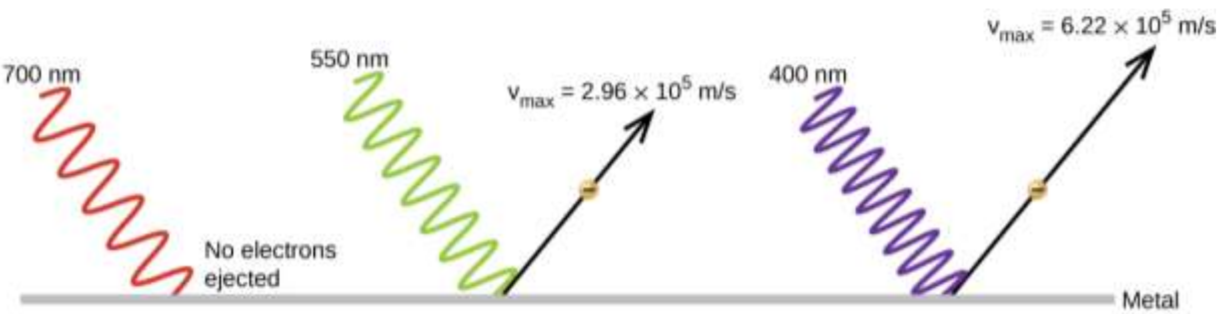


Figure 6 The effect of frequency (f) upon kinetic energy (E_k)

Figures 7a & 7b show that the higher the frequency the higher the kinetic energy, but not the photocurrent.

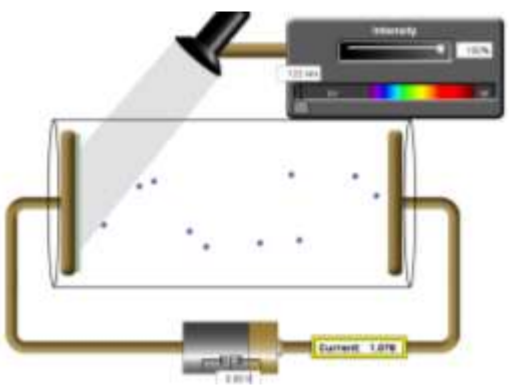


Figure 7a Sodium cathode & λ 122 nm

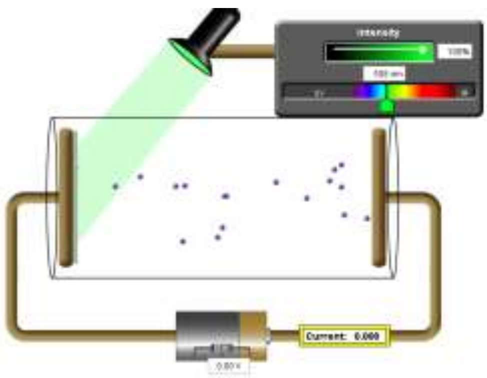


Figure 7b Sodium cathode & λ 508 nm

NB:$\uparrow$ frequency the $\uparrow E_k$, but no change in I_{ph}

Task

Convert the wavelengths (λs) from Figure 6a & 6b into frequencies

$c = \lambda f$ $\qquad \lambda_1 = 122\ nm$ $\qquad \lambda_2 = 508\ nm$

$\therefore f = \frac{c}{\lambda}$ $\qquad f = \frac{3.0 \times 10^8}{122 \times 10^{-9}}$ $\qquad f = \frac{3.0 \times 10^8}{508 \times 10^{-9}}$

$= 2.46 \times 10^{15}$ Hz $\qquad = 5.91 \times 10^{15}$ Hz

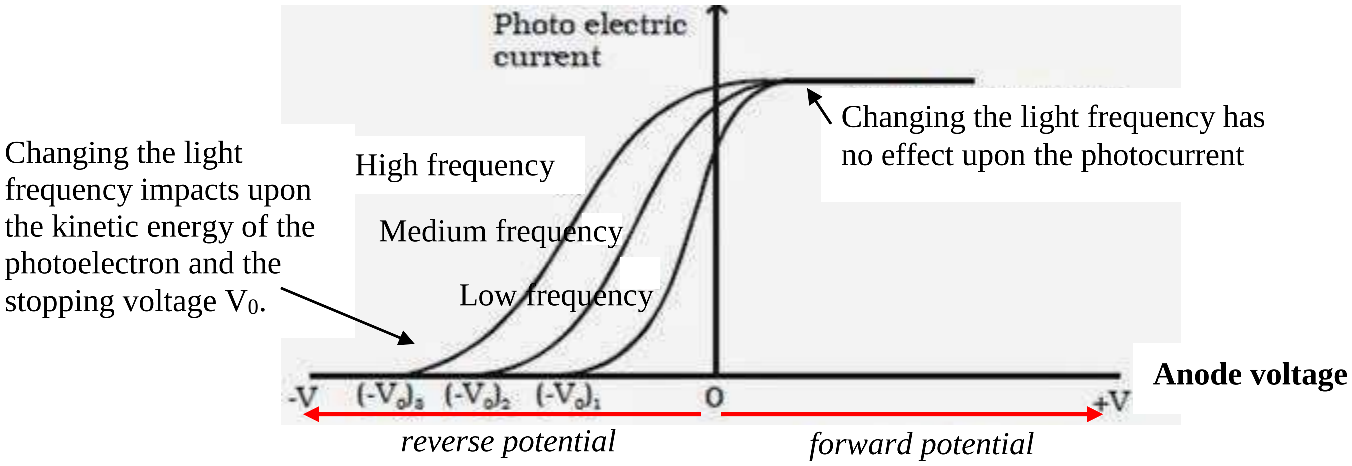


Figure 8 I-V Graph for the Photoelectric Effect with varying intensity but same intensity

As can be seen from the I-V graph in Figure 8, the higher the light frequency the higher the stopping voltage (V_0) required. However, regardless of the light frequency the photocurrent remains the same when a positive anode voltage is applied.

Kinetic energy vs frequency graphs

The graph shown in Figure 9 below shows the kinetic energy of a photoelectron as a function of light frequency for three different metals (potassium, beryllium & titanium)

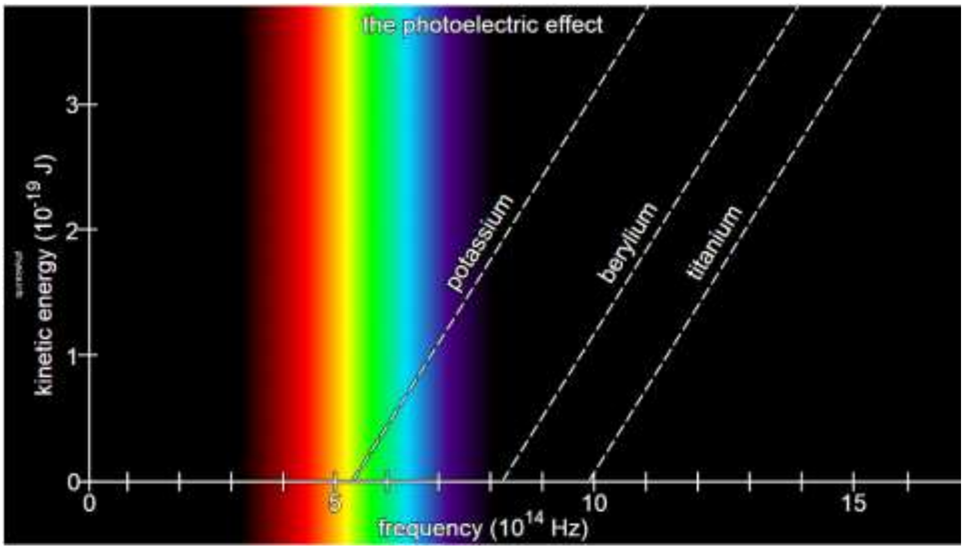


Figure 9 E_k – f graph

There is much to be found from the above graph.

- Each metal cathode has its own unique **threshold frequency (f_0)** represented by the **x intercept**.
  f_0 (K) ≈ 5.3 × 10^{14} Hz, f_0 (Be) ≈ 8.2 × 10^{14} Hz & f_0 (Ti) ≈ 9.9 × 10^{14} Hz
- The higher the frequency the greater the kinetic energy of the released photoelectron
- Each line graphed has an identical gradient. The gradient of each line is known as Planck's constant (h = 6.63 × 10^{-34} Js or h = 4.14 × 10^{-15} eVs)

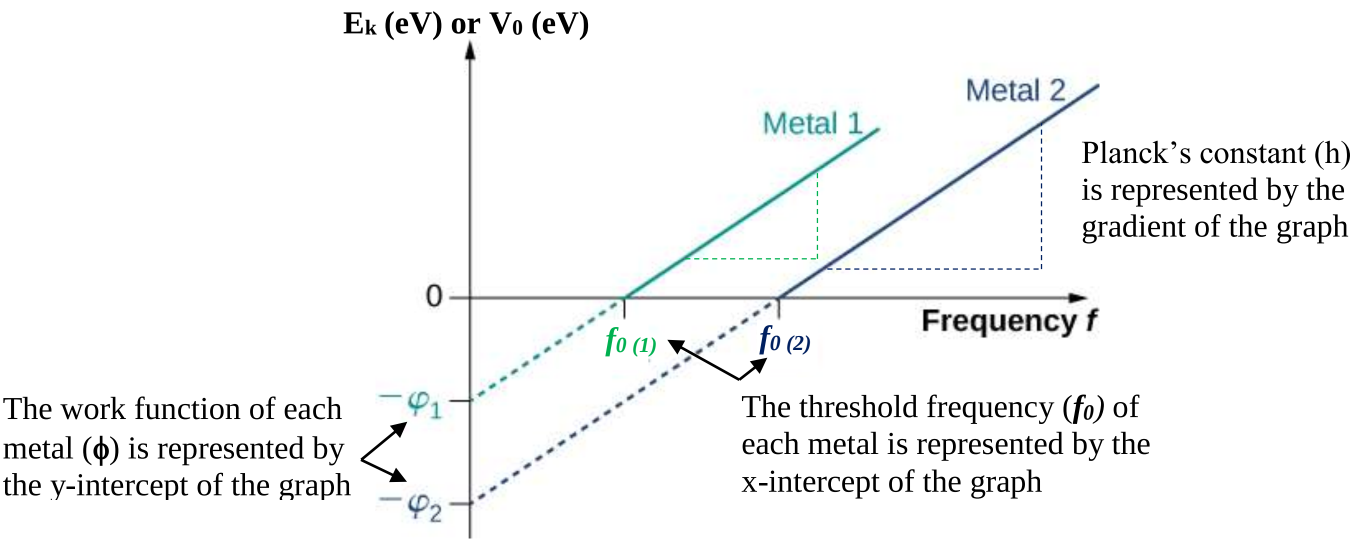


Figure 10 Extended E_k – f graph

Figure 10 provides additional information regarding the E_k – frequency graph.

- The positive values along the y-axis represent the kinetic energy of the released photoelectrons
- The negative values along the y-axis represent not kinetic energy (ie. positive energy) but work done (ie. negative energy).
- The traced back "dashed" line intercepts the y-axis at the work function (ϕ) value for that particular metal.

Analysing the linear relationship from the graph displayed in Figure 10 you get:

$y = mx + c$ [the equation for a straight line]

$E_k = m \times f + c$ [Substitute in both the dependent and independent variables]

$E_k = hf + c$ [Substitute Planck's constant h for the gradient]

$E_k = hf + (-\phi)$ [Substitute the work function for the y-intercept]

$\boxed{E_k = hf - \phi}$

Or

$\boxed{E_k = hf - hf_0}$

Where E_k is the kinetic energy of the photoelectron (Joules or eV)
h is Planck's constant (6.63×10^{-34} Js or 4.14×10^{-15} eVs)
f is the frequency (Hz)
f_0 is the threshold frequency (Hz)
ϕ is the work function (Joules or eV)

Example.5

The threshold frequency of sodium is 5.6×10^{14} Hz

- **a.** Calculate the work function of sodium
- **b.** A sample of sodium metal is exposed to radiation of frequency 7.5×10^{14} Hz. What is the maximum kinetic energy of the ejected electron?
- **c.** What is the stopping voltage for this metal?

a. $f_o = 5.6 \times 10^{14}$ Hz
$h = 6.63 \times 10^{-34}$ Js
$\phi = ?$

$\phi = hf_o$
$W = 6.63 \times 10^{-34} \times 5.6 \times 10^{14}$
$= \mathbf{\underline{3.7 \times 10^{-19}\ J\ (2.32\ eV)}}$

b. $f = 7.5 \times 10^{14}$ Hz
$\phi = 3.7 \times 10^{-19}$ J
$E_k(max) = ?$

$E_P = \phi + E_k(max)$
$\therefore E_k(max) = E_P - \phi$
$= hf - \phi$
$= (6.63 \times 10^{-34} \times 7.5 \times 10^{14}) - 3.7 \times 10^{-19}$
$= \mathbf{\underline{1.27 \times 10^{-19}\ J\ (0.79\ eV)}}$

c. $E_k(max) = 1.27 \times 10^{-19}$ J
$e = 1.6 \times 10^{-19}$ C
$V_o = ?$

$E_k(max) = eV_o$
$\therefore V_o = E_k(max)/e$
$= 1.27 \times 10^{-19}/1.6 \times 10^{-19}$
$= \mathbf{\underline{0.79\ V}}$

Why the wave model fails ?

The wave model predicts that the **energy of a wave can be increased** by either:

- Increasing its amplitude
- Increasing its frequency

So in terms of the Photoelectric Effect investigation, the wave model **incorrectly predicts** that:

- Increasing the intensity of light should result in greater kinetic energy of photoelectrons
- Any frequency of light should be able to overcome the work function to eject electrons, as long as the intensity is great enough and/or the light shines for a long enough time

The wave model falls short in its explanation of the Photoelectric Effect as **it cannot explain**:

- Independence of photoelectron kinetic energy and light intensity
- Existence of a threshold frequency
- No time delay for electron ejection

The Photoelectric Effect Explained

The Photoelectric Effect can be explained in term of light acting as **particles or photons**. A photon is a **discrete particle of light**, or a **packet of energy**.

A photon can transfer all of its energy to a single electron, via a **1 to 1 interaction**, that is 1 photon transfers all its energy to 1 electron. It cannot transfer only some of its energy and it cannot transfer its energy to multiple electrons.

The "frequency" (or colour) of light is a property of each photon. The **energy of a photon is determined by its frequency.**

$$\boxed{E_{\text{photon}} = hc \text{ or } \frac{hc}{\lambda}}$$

The intensity (or brightness) of light relates to the **amount of photons**. It is not a property of individual photons.

If we model light as a photon, it helps us to explain the results of the photoelectric effect:

Point 1 - Independence of photoelectron EK and light intensity

- Increasing the intensity of light at the same frequency only increases the number of photons.
- It does not change the energy of each photon.
- This means more electrons will absorb energy and be released, but each electron absorbs the same amount of energy.
- Greater photocurrent, but E_K does not change

Point 2 - Threshold frequency

- An electron is released only if it absorbs at least as much energy as the work function.
- This means the photon energy must be greater than or equal to the work function.
- Photon energy is determined by its frequency.
- So a minimum (or threshold) frequency exists which is required to release a photon.

If photon energy is greater than work function, extra energy becomes E_K of photoelectron.

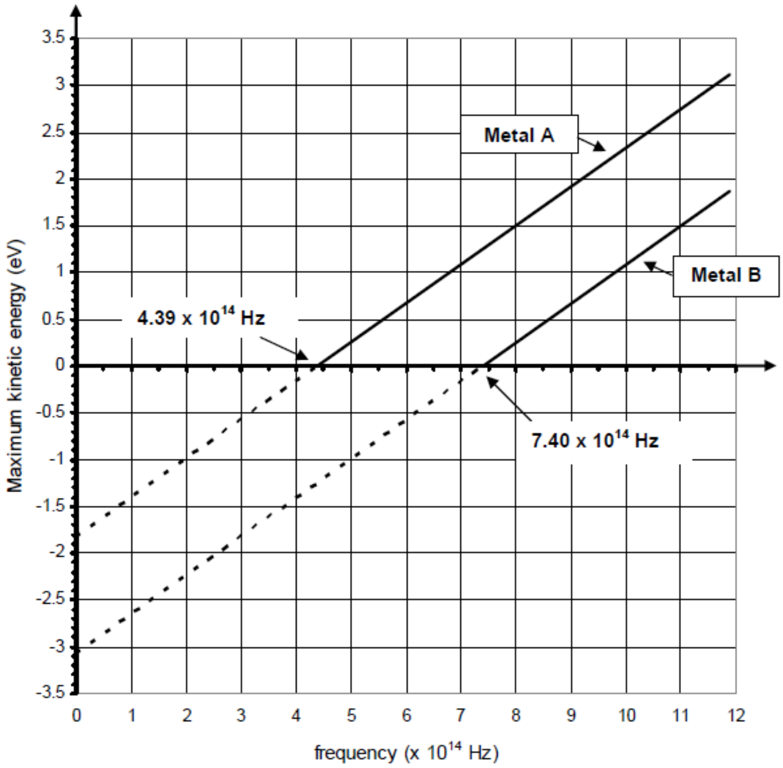
Point 3 - No time delay

- A photon either has more energy than the work function or it does not.
- Electrons are released immediately if they absorb a photon with more energy than the work function or they are not released at all if there is insufficient energy.
- It does not take time for the electron to gather enough energy.

Exam Style Questions

Questions 1 to 5 relate to the following information.

Robert Millikan performed an early experiment to investigate the photoelectric effect. The graph given below represents two of his experimental results using two different anode metals. He shone light of different frequencies onto the surface of different metals and measured the kinetic energy of the photo-ejected electrons.



Question 1.
What is the minimum energy, in eV, of a photon that can eject an electron from Metal A

The minimum energy is measured by the work function which is given by the modulus of the y-intercept of the graph for metal A. This value is clearly seen as 1.8 eV.

Or by calculation:

$E = hf_0 = 4.14 \times 10^{-15} \times 4.39 \times 10^{14} = 1.8$ eV

1.8 eV

Question 2.
What is the cut-off **wavelength** for Metal B?

For metal B, $f_0 = 7.40 \times 10^{14}$ Hz
$c = \lambda f, \therefore \lambda = c/f = 3.0 \times 10^8/(7.4 \times 10^{14}) = 4.054 \times 10^{-7}$

4.05×10^{-7} m

Question 3.

Planck's constant is given as 4.14×10^{-15} eV s

Explain, with calculations, how this value could be obtained from the given graph.

Any suitable gradient calculation for either metal A or metal B, typically:

From metal A's graph h = gradient = $\frac{rise}{run} = \frac{1.8}{4.39 \times 10^{14}} = 4.10 \times 10^{-15}$

From metal B's graph h = gradient = $\frac{rise}{run} = \frac{3.1}{7.4 \times 10^{14}} = 4.19 \times 10^{-15}$

Note: neither of these give exactly 4.14×10^{-15}

Question 4.

The work functions for a number of different metals are given in the table below.

Metal	Joules ($\times 10^{-19}$)
aluminium	6.56
magnesium	5.77
zinc	4.91
sodium	3.65
caesium	2.92

Use this table and the previously provided information to name the two metals A and B.

The work function is given by the modulus of the y-intercept of the graph for each metal. This value needs to be converted into Joules

A: $W = 1.8 \times 1.6 \times 10^{-19} = 2.88 \times 10^{-19}$ *J ∴ close to caesium*

B: $W = 3.1 \times 1.6 \times 10^{-19} = 4.96 \times 10^{-19}$ *J ∴ close to zinc*

Metal A: caesium

Metal B: zinc

Question 5.

The intensity of the light source providing the photons of different frequencies used by Milikan in his experiment is reduced by 25%. The energy of the individual photoelectrons emitted from the surface of the metal will:

A. be unchanged

B. be reduced by 75%

C. be reduced by 25%

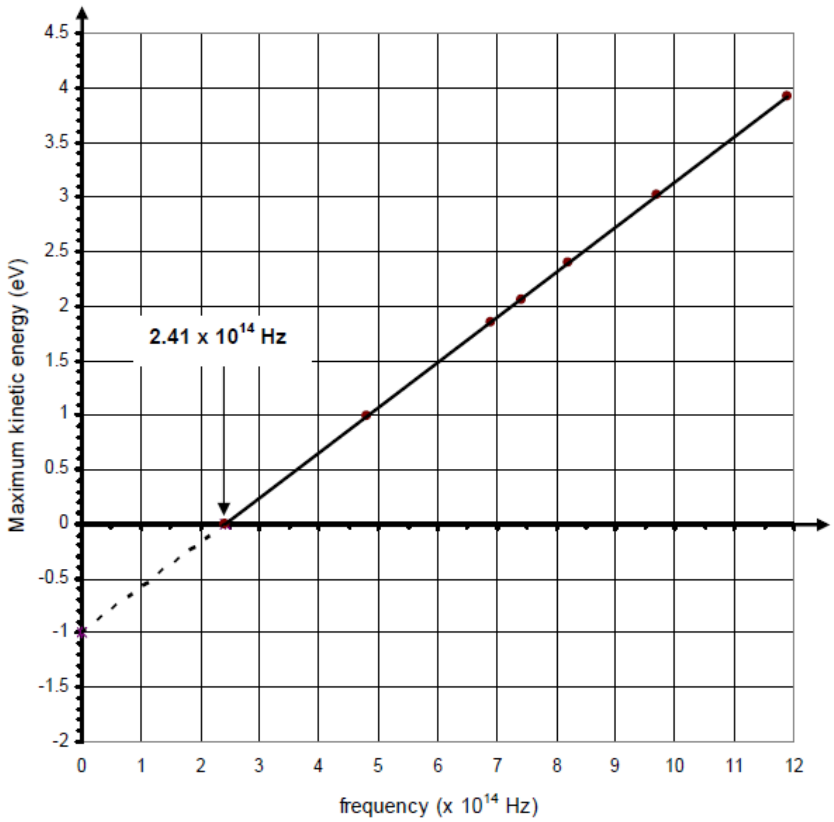
D. not be able to be determined from the given data

A

Decreasing the intensity of the light means less photons are emitted and therefore less photoelectrons are ejected from the metal surface. This will mean lower photocurrent, but the energy of the photocurrent is only affected by the frequency of the light and not by the intensity of that light.

Questions 6 to 10 relate to the following information.

Robert Millikan performed an early experiment to investigate the photoelectric effect. The graph given below represents the results of one of his experiments. He shone light of different frequencies onto the clean surface of a metal and measured the kinetic energy of the ejected photoelectrons.



Question 6.

Use the graph to determine the work function for this metal.

The work function is the modulus of the y-intercept when the line is extended. NOTE, -1eV is not acceptable as negative energies do not exist.

1.0 eV

Question 7.

Use the graph to determine the threshold frequency for this metal.

The threshold frequency is given by the x-intercept of the graph.

2.41×10^{14} Hz

Question 8.
Use the graph to determine the maximum kinetic energy of the electrons ejected when light of frequency 8.0×10^{14} Hz is shone on the metal.

Reading from the graph gives this value. (No other value allowed)

2.3 eV

Question 9.
What is the significance of the gradient of this graph and what numerical value should it be close to? (No calculation required)

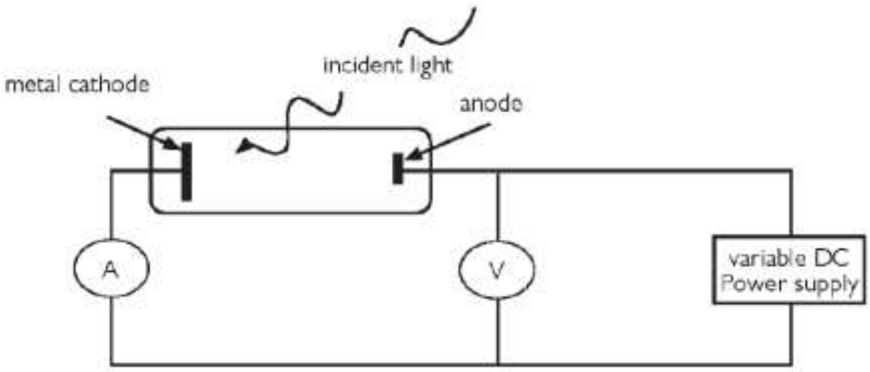
The gradient of the graph represents Plank's constant
Its value should be approximately 4.14 x 10^{-15} (since the maximum kinetic energy is measured in eV). [No calculations required]

Question 10.
Explain why no electrons are ejected at the frequency of 2.0×10^{14} Hz.

This frequency is below the threshold frequency.
Light of this frequency does not have enough energy to eject electrophotons.

Questions 11 to 14 relate to the following information.

Robert Millikan performed an early experiment to investigate the photoelectric effect using apparatus similar to that shown below.



He shone light of different frequencies onto the clean surface of a metal and measured the kinetic energy of the ejected photoelectrons. The table below represents the typical results of one of these types of experiments.

Frequency of incident light ($\times 10^{14}$ Hz)	Maximum kinetic energy (eV)	Maximum current (mA)
7.3	0.9	4.2

Question 11.

Use the table values to determine the work function for this metal.

$W = E_{light} - E_k\ max = hf - E_k\ max = (4.14 \times 10^{-15} \times 7.3) \times 10^{14} - 0.9$
$W = 3.0 - 0.9 = 2.1$ eV

2.1 eV

Question 12.

Determine the threshold frequency for this metal.

$W = hf_0,\ \therefore f_0 = W/h$
$f_0 = 2.1/(4.14 \times 10^{-15}) = 5.1 \times 10^{14}$ Hz

5.1×10^{14} Hz

Question 13.
What is the **minimum** number of photons contained in the incident light beam per second?

Each photon in the incident light beam releases on photoelectron from the metal which then constitutes the 4.2 mA photocurrent. The number of electrons per second in the current equates to the minimum number of photons per second in the light beam:

4.2 mA is a flow of 4.2×10^{-3} C of charge per second which is $4.2 \times 10^{-3}/(1.6 \times 10^{-19})$ electrons per second or 2.6×10^{16} electrons per second, liberated by 2.6×10^{16} photons per second.

$\boxed{2.6 \times 10^{16} \text{ electrons per second}}$

Question 14.
The original light source is replaced by a light of frequency of 2.0 × 1014 Hz. What will be observed? Explain your answer.

Nothing will happen with this new frequency light as it is below the threshold frequency of 5.1×10^{14} Hz.