Velocity

Displacement not only states a distance but also a direction. Similarly, the **average velocity** of an object, symbol v_{av}, is a statement of the average speed of the object together with its direction of travel. For example, if you are driving north at an average speed of 60 km/h, then your average velocity is stated as:

v_{av} = 60 km/h N

This means that velocity is calculated using displacement instead of distance in the formula. This formula is shown next.

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}}$$

In symbols: $v_{av} = \dfrac{\Delta x}{t}$

Going to town on zero velocity? WOW!

Suppose you drive 25 km from home to the city and then return home. The whole journey takes 1 hour. This means that your average speed is 50 km/h. But what is your average velocity? Since your displacement is zero (you are back where you started from), your average velocity is also zero!

Calculations involving average velocity

SKILLS

Example

Consider a tram travelling along a north–south tram track.

The arrows in Figure 7.13 show that the tram left the depot and travelled to point P, 20 km due south of the depot. Then it travelled 10 km back towards the depot in a northerly direction.

Assuming that the tram did not stop at any time, and that it took 30 minutes to travel from depot to point P and 15 minutes to travel back towards the depot, calculate:

1 the tram's velocity on the first leg of its journey
2 the tram's average velocity on the whole journey.

Solutions

1 At the end of the first leg of its journey, the tram has moved from the depot to 20 km south of the depot. So its displacement Δx is 20 km S. This leg took 30 minutes.

$$v_{av} = \frac{\Delta x}{t} = \frac{20}{0.5} = 40 \text{ km/h S}$$

Δx = 20 km S

$t = 30 \text{ min} = \frac{30}{60} \text{ h} = 0.5 \text{ h}$

v_{av} = ? km/h S

Answer: The average velocity of the tram on the first leg of its journey is 40 km/h S.

2 At the end of its journey, the tram is 10 km south of the depot. So its displacement Δx is 10 km S. The journey took 45 minutes.

$$v_{av} = \frac{\Delta x}{t} = \frac{10}{45} = 22 \text{ km/h S}$$

Δx = 10 km S

$t = 45 \text{ min} = \frac{45}{60} \text{ h} = 0.75 \text{ h}$

v_{av} = ? km/h S

Answer: The average velocity of the tram on the whole journey is 22 km/h S.

Note

- Using the formula for average speed (page 155), you will find the average speed of the tram on the whole journey is 40 km/h. Check this out for yourself! The only time these answers will match is if the moving object does not change direction during the journey.
- Instead of using bearings such as north and south, it is often convenient to define one direction of travel as positive and the other as negative. For example, if v_{av} = +10 m/s means going at an average velocity of 10 m/s to the right, then v_{av} = −5 m/s means going at an average velocity of 5 m/s to the left.
- The expression 'moving at a constant velocity' means that both the speed and the direction are not changing.
- Average velocity can also be determined from the gradient of a position–time graph.

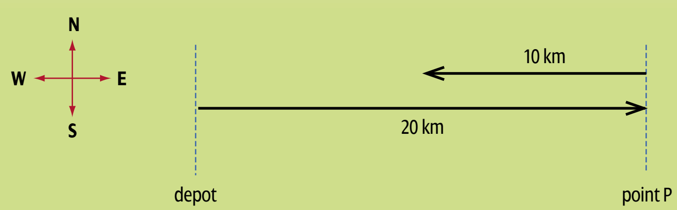

Figure 7.13 A diagram of the tram's journey

Acceleration

Figure 7.14 Acceleration

Suppose two cars are stopped next to each other at a traffic light. They are both travelling in the same direction. When the light turns green, one car reaches 60 km/h in 3 seconds but the other takes 6 seconds to reach the same velocity (Figure 7.14).

An observer on the street might comment on how much the first car accelerated – that is, sped up. 'Speeding up' is our common understanding of the meaning of acceleration. But physicists define the **acceleration** of an object, symbol ***a***, as a measure of the rate at which velocity changes. The two most common units for acceleration are metres per second per second, or m/s/s (which usually is written as m/s^2 or ms^{-2}) and kilometres per hour per second, km/h/s.

As the first car achieved a change of velocity from 0 to 60 km/h in 3 seconds, we say its acceleration was 20 km/h/s. The acceleration of the second car was 10 km/h/s – half that of the first car.

The formula for calculating acceleration is as follows.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

In symbols: $a = \dfrac{\Delta v}{t}$

where $\Delta v = v - u$

v = final velocity

u = initial velocity

Calculations involving acceleration

SKILLS

Example 1

Calculate the acceleration of a car travelling along a straight stretch of freeway if it accelerates from 60 km/h to 110 km/h in 4 seconds.

Solution

$\Delta v = v - u = (110 - 60)$ km/h $= 50$ km/h

$t = 4$ s

$a = ?$ km/h/s

$$a = \frac{\Delta v}{t} = \frac{50}{4} = 13 \text{ km/h/s}$$

Answer: The acceleration of the car was 13 km/h/s.

Example 2

How long would it take a racing car to accelerate from 150 km/h to 240 km/h if it accelerates at 10 km/h/s?

Solution

$\Delta v = v - u = (240 - 150)$ km/h $= 90$ km/h

$a = 10$ km/h/s

$t = ?$ s

$$a = \frac{\Delta v}{t}$$

$$\text{Hence } t = \frac{\Delta v}{a} = \frac{90}{10} = 9 \text{ s}$$

Answer: It will take the car 9 seconds to accelerate from 150 km/h to 240 km/h.

Example 3

A car travelling at 15 m/s along a straight road accelerates at 2 m/s^2 for 3 seconds. What is the final velocity of the car?

Solution

$a = 2\ m/s^2$

$t = 3$ s

$\Delta v = ?$ m/s

$u = 15$ m/s

$$a = \frac{\Delta v}{t}$$

Hence $\Delta v = a \times t = 2 \times 3 = 6$ m/s

Now $\Delta v = v - u$

Hence $v = \Delta v + u = 6 + 15 = 21$ m/s

Answer: The final velocity of the car is 21 m/s.

Note

- As acceleration takes direction into account, an object is considered to have accelerated if it maintains constant speed but changes direction.
- If an object travelling in a particular direction slows down, its acceleration will have a negative value. **Deceleration** is a term also used to mean slowing down.

glossary

Velocity–time graphs

Suppose a car is travelling east along a straight road that lies in the east–west direction in the city. When we start timing it, the car is stationary, but at that instant has got a green light. The car accelerates from rest until it reaches 13 m/s, which is just under the speed limit for that road. This takes 5 seconds. It stays at that speed for 45 seconds when the driver encounters another red light. The driver slows the car down steadily until it stops at the light. This takes 10 seconds.

A **velocity–time graph** shows the velocity of a moving object against time. The graph for the situation we have just considered is shown in Figure 7.15.

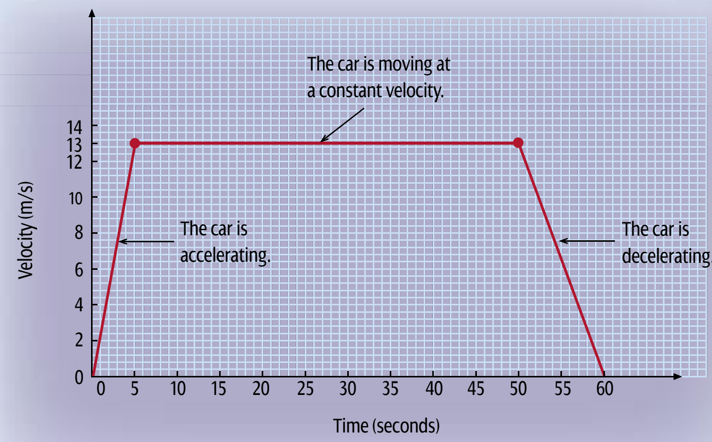

Figure 7.15 A velocity–time graph

If you examine this graph, you can see that the horizontal section of the graph has a very different meaning to that in a position–time graph (Figure 7.5). In this kind of graph it means that the velocity of the car has reached a constant value. For a moving object that does not change direction, we know that when the graph slopes upwards, the car must be accelerating (speeding up) and when the graph slopes downwards, it must be decelerating (slowing down).

Acceleration–time graphs

Let us consider the velocity–time graph in Figure 7.15. This data can also be represented as an **acceleration–time graph** – a graph that shows the acceleration of an object against time. The conversion from one to the other is shown in Figure 7.16.

Note

- As the car has not changed its direction of movement, the negative value of acceleration in the last period of time tells us the car is decelerating (slowing down).
- The sections of the graph are horizontal, since in each section of the motion the acceleration is a constant value.
- In the middle section acceleration is zero, since the car is moving at constant velocity.

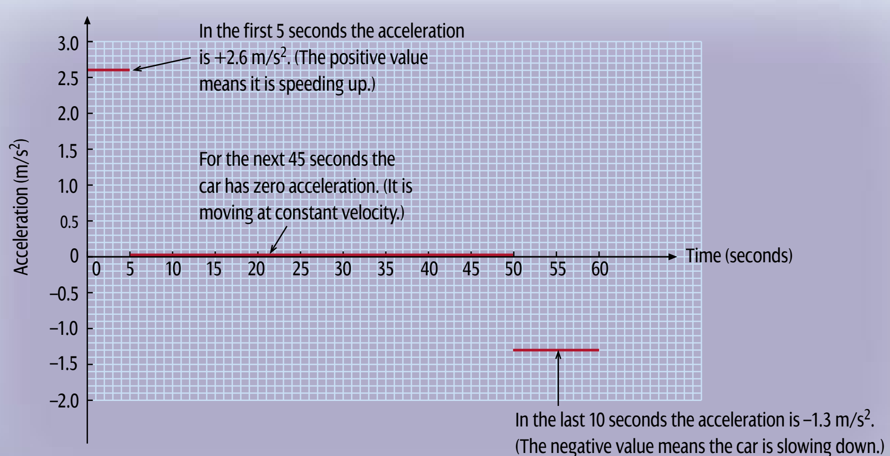

Figure 7.16 The acceleration–time graph for the car

All that from one graph? WOW!

The velocity–time graph in Figure 7.15 can be used to tell a great deal more than you might imagine. The gradients of the three lines give the value of the acceleration of the car in those time periods. A negative gradient gives us a negative value for acceleration, which simply means the car is decelerating. Where the line is horizontal, the gradient is zero. This is to be expected, as the car is travelling at constant velocity. Its acceleration therefore is zero.

But that is not all! The areas under the graph lines tell us the distance travelled by the car in each time period. The area under the whole graph tells us the distance between the two traffic lights. This area is worked out using the formulae for areas of triangles and rectangles.

BLM 7.3 On the move
BLM 7.4 Braking times

glossary

Questions 7.3

The following information refers to questions 1 to 6.

Mary set out to walk due east to the local milk bar and back. She left home and walked at a steady pace for 3 minutes until she met her friend Jane. They stopped and chatted for 5 minutes before Mary hurried on, taking another 4 minutes to reach the milk bar. It took 2 minutes to buy milk and then she walked home, without stopping, in 6 minutes.

Figure 7.17 shows Mary's journey. For all the following questions, assume that Mary's house is the reference point.

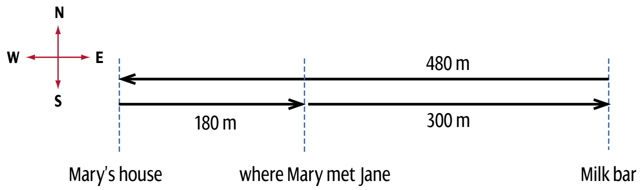

Figure 7.17

1 For each of the following positions, state Mary's displacement:
  a where Mary stopped to talk to Jane
  b at the milk bar
  c Mary's house
2 Draw a position–time graph to represent Mary's walk to the milk bar and back.
3 Calculate Mary's average velocity, in m/s, for each of the following parts of her journey:
  a from her house to where she stopped to talk
  b from her house to the milk bar
4 Calculate Mary's average velocity, in m/s, over the whole journey.
5 Calculate Mary's average speed, in m/s, over the whole journey.
6 Explain why the answers to questions 4 and 5 are different.
7 Calculate the acceleration in m/s^2 for each of the following situations:
  a a car travelling at 5 m/s accelerates to a speed of 25 m/s in 5 s
  b an ambulance travelling at 20 m/s accelerates to 30 m/s in 4 s
8 Calculate the final speed of a car that accelerates at 2 m/s^2 for 10 s from rest (that is, from zero velocity).
9 Calculate the initial speed of a car that has a final speed of 25 m/s after accelerating at 2.5 m/s^2 for 4 s.
10 A car is stopped at a set of traffic lights along a straight road that lies in a north–south direction. The lights change and the car accelerates northward at a steady rate to a speed of 15 m/s in 5 seconds. It continues at this speed for the next 10 seconds before slowing down at a steady rate to stop at the next set of lights in 3 seconds.
  a Draw a velocity–time graph for the car's journey.
  b Use the graph to determine the velocity of the car at 2 seconds.
  c On your graph, label the parts that show when the car has stopped.
  d Highlight the part of the graph that shows the car is decelerating and explain what this term means.

Puzzle

11 For the situation described in Question 10, use the graph you drew to calculate the following. (Hint: See the Wow! box on page 161.)
  a The acceleration of the car in the first 5 seconds
  b The acceleration of the car in the last 3 seconds
  c How far the car travelled in the first 5 seconds
  d The total distance between the two traffic lights
12 From your graph in Question 10, predict what the acceleration–time graph might look like.

Investigate

13 One way to determine the speed of moving objects, such as people going on a power walk, is to time how long they take to travel a certain distance, such as 10 m. Two parallel lines are drawn 10 m apart in chalk along a straight track. Timing starts when the person walks over the first line and stops when they walk over the second line. Try this and measure the speed of some students in your class or members of your family going for a power walk. Record your findings. Identify one limitation or practical difficulty of this method.