Section 3.1.2 – Electric Fields

There are many similarities between **gravitational fields** and **electric fields.** These include the shape of the fields, the equations used to make calculations, the inverse square nature of both fields and the way objects interact when in the presence of each field.

Electric Fields

When examining a gravitational field a **point mass** was considered as a test subject. Likewise when examining an electric field a **point charge** is used as a test subject. One can then test what happened to a point charge at any location within an electric field.

Unlike gravitational fields, which only create attraction between object, an electric field can create both an **attractive** or **repulsive force** depending on the charge located within the field. An electric field is a **vector**, it has both **magnitude and direction.**

Electric field lines travel from a **positive charge to a negative charge**. An electric field can exert a force on charges within it.

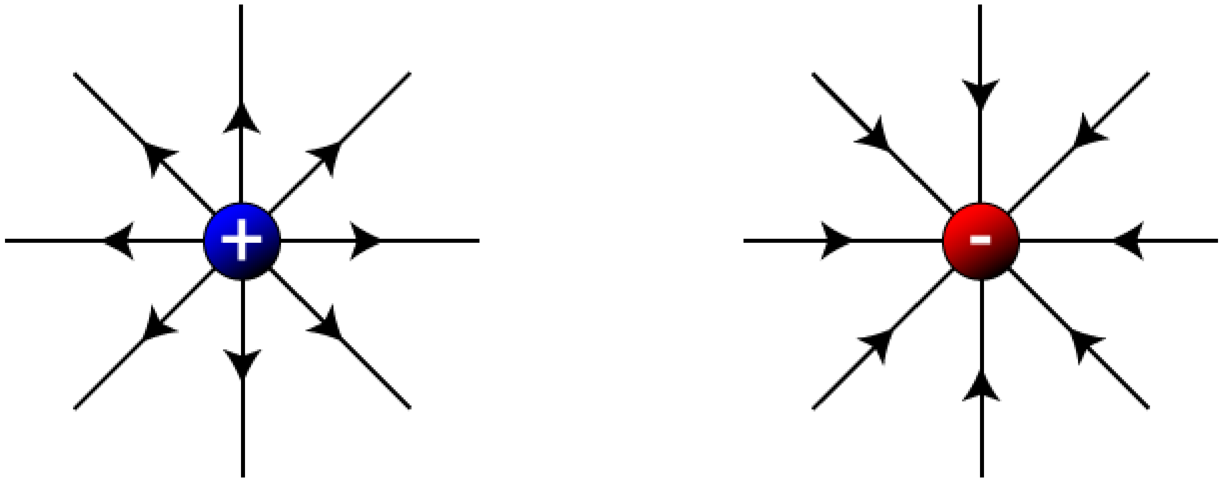


Figure 1 The electric field created by a positive and negative charge respectively

NB: A positive test charge, when placed in an electric field, will move in the same direction as that of the electric field.

Recall:
The **fundamental charge** is allocated to both an electron and proton.

Charge of an electron (e^-) : -1.6×10^{-19} C
Charge of a proton (p^+) : $+1.6 \times 10^{-19}$ C

All other charges are multiples of the fundamental charge.

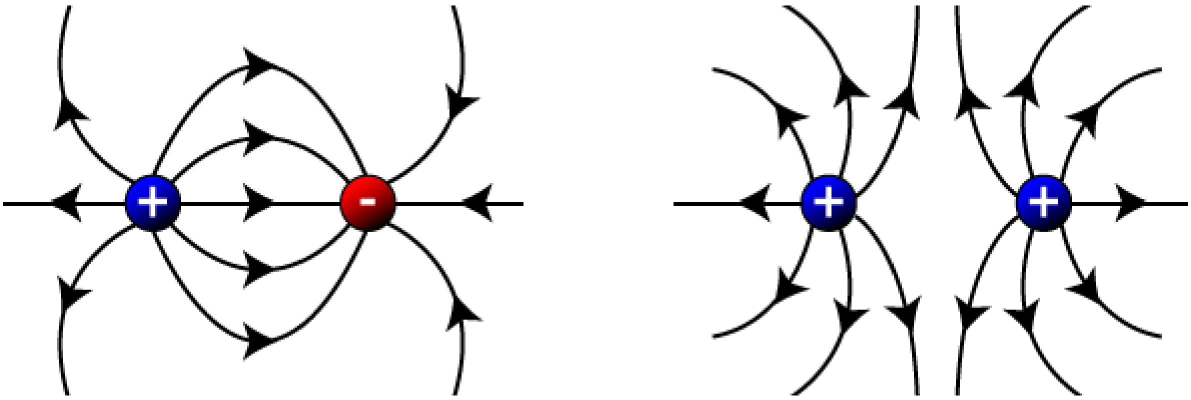


Figure 2 Like charges repel, whereas opposite charges attract

Coulomb's Law

You will recall Newton's gravitational law:

$$F = \frac{GMm}{r^2}$$

Similarly, Coulomb's Law states that:

$$\boxed{F = \frac{kq_1q_2}{r^2}}$$

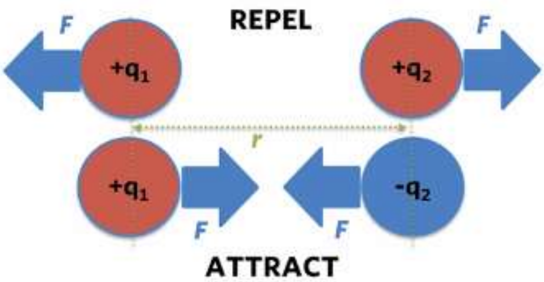
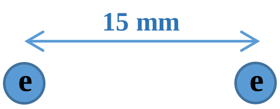


Figure 3 Coulomb's force between like and opposite charges

Where F represents the Coulomb force (N)
q_1 represents 1st charge (C)
q_2 represents 2nd charge (C)
r represents the radius of separation (m)
k represents Coulomb's constant
= 8.99 × 109 (Nm2C-2)

Example.1

Two electrons are located 15 mm apart. Calculate the magnitude and direction of the force the electrons exert on one another.



F = ?
$q_1 = -1.6 \times 10^{-19}$ C
$q_2 = -1.6 \times 10^{-19}$ C
$r = 15 \times 10^{-3}$ m
$k = 8.99 \times 10^{9}$ Nm^{2}C^{-2}

$$F = \frac{kq_1q_2}{r^2} = \frac{8.99 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(15 \times 10^{-3})^2} = 1.0 \times 10^{-24} N$$

As both charges are electrons (ie. like charges), this is a repulsive force.

Electric Field Strength

Again you will recall that the gravitational field strength (g) is a measurement of the force (in Newtons) experienced by a unit mass (ie. per kilogram). The equation used to calculate the gravitational field strength was:

$$g = \frac{GM}{r^2}$$

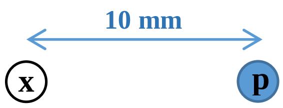
Similarly, the electric field strength E is a measurement of the force (in Newtons) experienced by a unit charge (ie. per Coulomb). The equation used to calculate the electric field strength is:

$$\boxed{E = \frac{kQ}{r^2}}$$

Where E represents the electric field strength (NC^{-1})
Q represents the charge (C)
r represents the radius of separation (m)
k represents Coulomb's constant
= 8.99×10^9 (Nm^2C^{-2})

Example.2

Calculate the magnitude and direction of the electric field at a point X, which is at a distance of 10 mm to the left of a single proton. Use k = 8.99×10^9 Nm^2C^{-2}, the charge of a proton is 1.6×10^{-19}C.



E = ?
Q = 1.6×10^{-19} C
r = 10×10^{-3} m
k = 8.99×10^9 Nm^2C^{-2}

$$F = \frac{kQ}{r^2} = \frac{8.99 \times 10^9 \times 1.6 \times 10^{-19}}{(10 \times 10^{-3})^2} = 1.4 \times 10^{-5} NC^{-1}$$

As the charge of the protons positive, all electric fields lines will move out from the centre of the proton. So at point X, the field will be moving to the left.

Alternatively the electric field strength can be calculated using the following equation:

$$\boxed{E = \frac{F}{q}} \quad or \quad \boxed{F = Eq}$$

Where E represents the electric field strength (NC^{-1})
F represents the force (N)
q represents the charge (C)

Field Distance Graphs

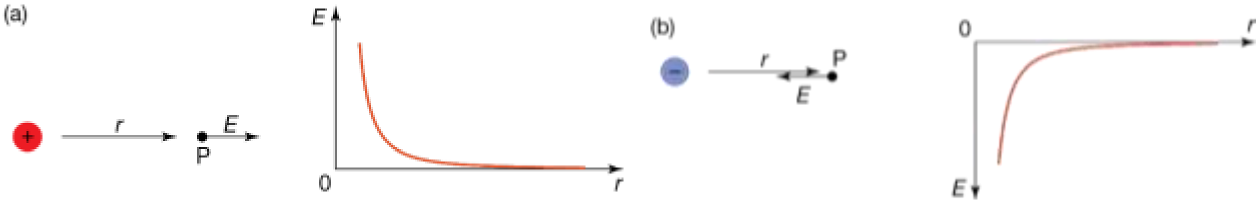


Figure 4 Diagrams and field–distance graphs for the electric field around (a) a positive charge and (b) a negative charge

Both of the above graphs shown in Figure 4 represent the electric field (E) against the radius of separation.

Graph A
When the central charge is positive, then the electric field radiates away from the central charge in the same vector direction as the radius. Accordingly both the electric field and radius are graphed as positive quantities.

Graph B
When the central charge is negative, then the electric field radiates towards the central charge in the opposite vector direction to the radius. Accordingly the electric field is a graphed as a negative quantity and the radius as a positive quantities.

NB: The area under both graphs represents the Voltage of (JC^{-1})

Uniform & Non-Uniform Electric Fields

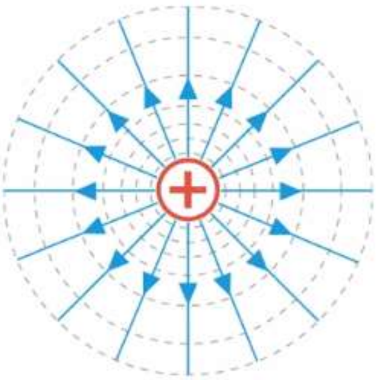


Figure 5 Non-uniform electric field

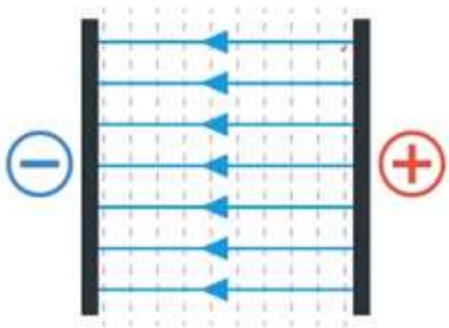


Figure 6 Uniform electric field between 2 plates

Figure 5 represents a radial diverging electric field generated by a central charge.
This is a non-uniform electric field.

Figure 6 represents a evenly spaced and parallel field lines between two charged plates.
This is a uniform electric field.

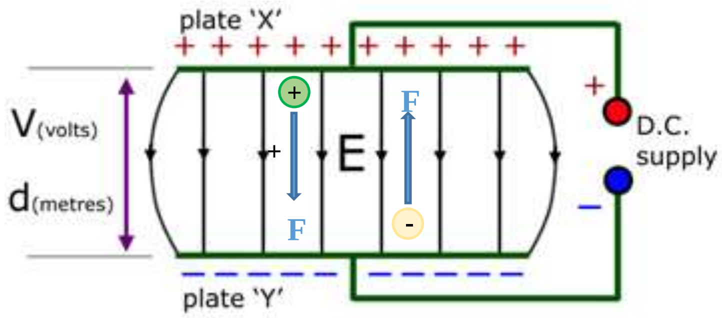


Figure 7 Labelled diagram of electric field between two plates

There is a potential difference between the two charged plates (V). This potential difference creates a uniform electric field (E) between the two plates. If a charge particle is placed between the two plates it will be within an electric field and experience a force. Work will be done upon the charged particle and the particle will gain kinetic energy (E_k).

$$E = \frac{F}{q} \quad \therefore \boxed{F = Eq} \longleftarrow \text{Equation 1}$$

$$\boxed{E = \frac{V}{d}} \longleftarrow \text{Equation 2}$$

Where E represents Electric field (Vm^{-1})
V represents the potential difference between the plates (V)
d represents the plate separation distance (m)

As a charge particle is accelerated between the plates, work is being done by the electric field (E).

$$\boxed{W = Fd} \longleftarrow \text{Equation 3}$$

Substitute Equation1 into Equation 3

$$\therefore W = Eqd \longleftarrow \text{Equation 4}$$

Substitue Equation 2 into Equation 4

$$\therefore W = \frac{V}{d}qd$$

This simplifies to:

$$\boxed{W = Vq}$$

Where W represents the work done by the electric field (Joules)
V represents the potential difference between the plates (V)
q represents charge of the particle placed between the plates (C)

Recall

Work done (W) = Change in Energy (ΔE)

$$\boxed{\boldsymbol{Vq = \frac{1}{2}mv^2}}$$

Example.3

Calculate the magnitude of the uniform electric field that causes a force of 1.28×10^{-19} N on a proton. The charge of a proton is 1.6×10^{-19} C.

E = ?	$E = \frac{F}{q}$
F = 1.28×10^{-19} N	$E = \frac{1.28 \times 10^{-19}}{1.6 \times 10^{-19}}$
q = 1.6×10^{-19} C	E = <u>0.8 NC^{-1}</u>

Example.4

A pair of oppositely charged electric plates are located in a vacuum and separated by a distance of 10 cm and a potential of 2000 Volts. If a single electron is placed near the negative plate, calculate the kinetic energy and velocity it will have when it reaches the positive place. Also calculate the electric field strength in Vm^{-1} between the plates

<u>Part A</u>

V = 2000 V	E_k = W = qV
q = 1.6×10^{-19} C	= $1.6 \times 10^{-19} \times 2000$
E_k = ?	= <u>3.2×10^{-16} Joules</u>

<u>Part B</u>

$E_k = 3.2 \times 10^{-16}$ J	$E_k = \frac{1}{2}mv^2$
m = 9.1×10^{-31} kg	$\therefore v^2 = \frac{2E_k}{m}$
v = ?	$v = \sqrt{\frac{2E_k}{m}}$
	$v = \sqrt{\frac{2 \times 3.2 \times 10^{-16}}{9.1 \times 10^{-31}}}$
	v = <u>2.65×10^7 ms^{-1}</u>

<u>Part C</u>

V = 2000 V	$E = \frac{V}{d}$
d = 0.1 m	$E = \frac{2000}{0.1}$
E = ?	= <u>20 000 Vm^{-1}</u>

Potential Energy in Electric Fields

You would recall that a mass elevated within a gravitational field has gravitational potential energy (E_{gp}). If the gravitational field is allowed to do work on the mass, that is the mass is allowed to fall, the gravitational potential energy will convert into kinetic energy (E_k).

Likewise, a charged particle placed within an electric field contains **electric potential energy**. If it is allowed to move between the two charged plates, then **work is done** on the charged particle by the electric field. The particles electric potential energy is converted into **kinetic energy (E_k).**

Summary

Figure 8 below summarises the various relationships associated with a uniform field generated between two charged plates. Start your analysis of the information at the bottom right corner, labelled **potential (V)**

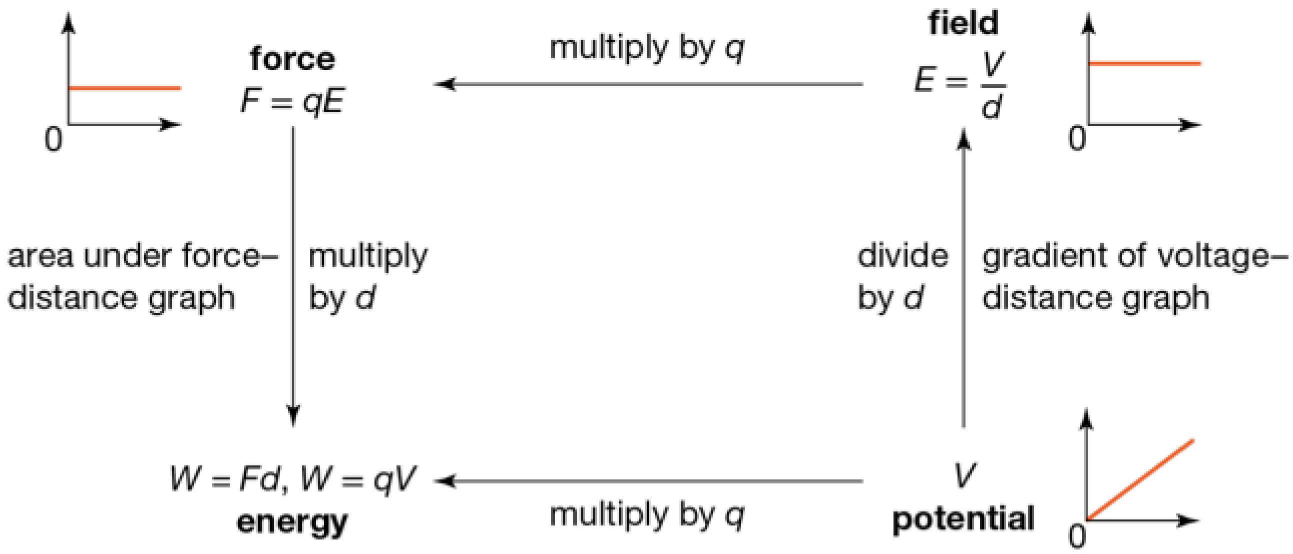


Figure 8 The relationships between force, field strength, energy and potential in a uniform electric field

Figure 9 below summarises the various relationships associated with an electric field around a point charge. Start your analysis of the information at the top left corner, labelled **Force and Coulomb's Law**

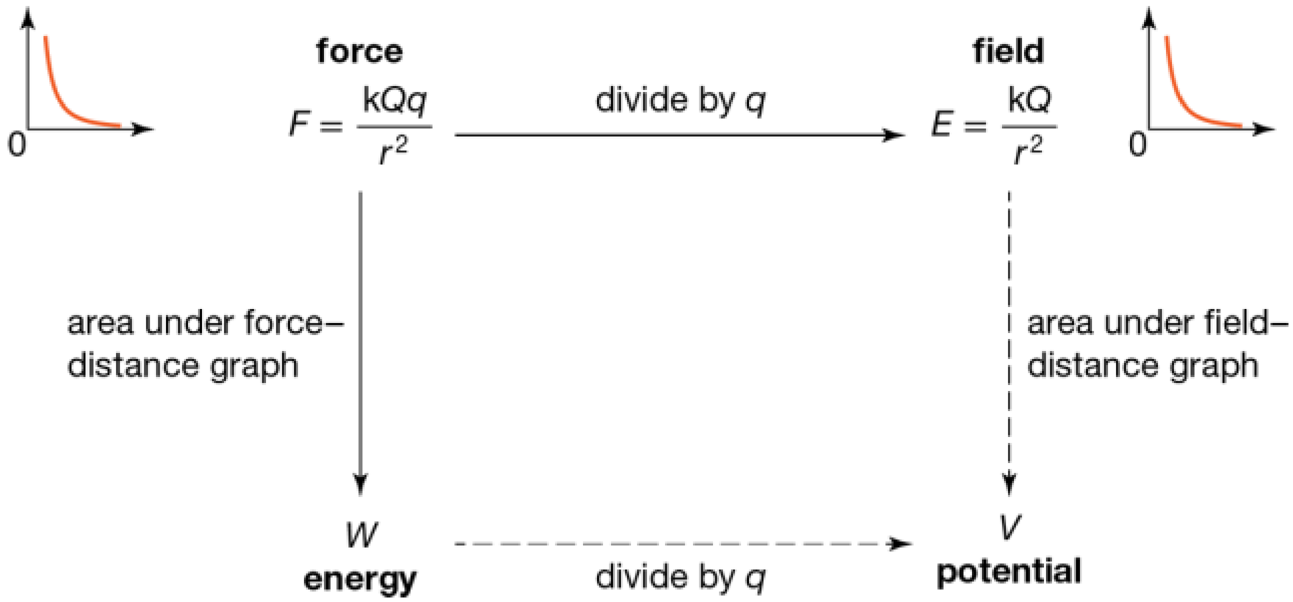
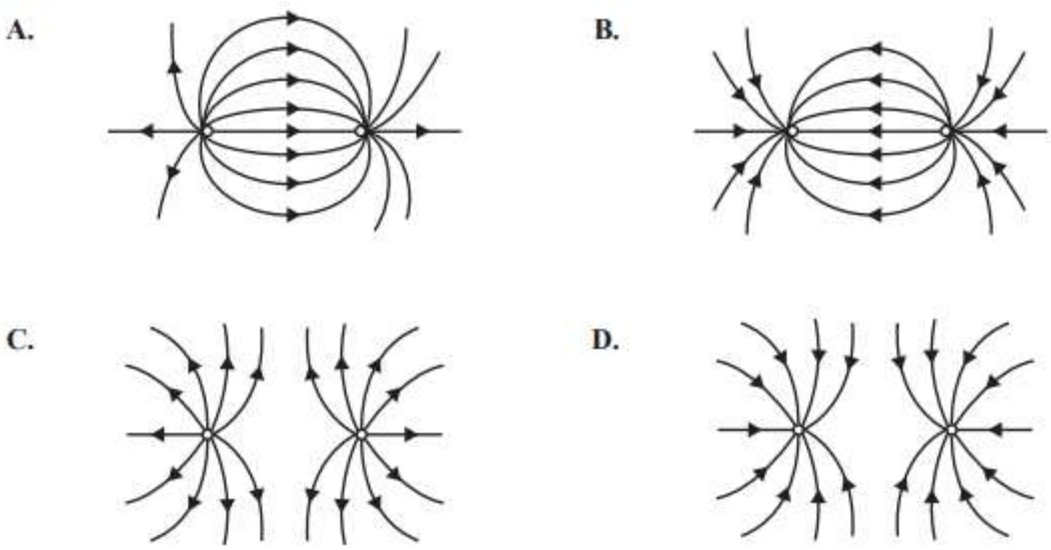


Figure 9 The relationships between force, field strength, energy and potential in an electric field around a point charge

Exam Style Questions

Question 1

Which one of the following diagrams shows the electric field pattern surrounding two equal, positive point charges?



C

Like charges will repel each other, this eliminates option A & B. As the two charges are both positive the electric field lines in both cases will be leaving the charges, $\therefore$ option C.

Question 2

A metal sphere has a charge of 1.0×10^{-8} C on it.
A small sphere with a charge of 1.0×10^{-9} C is placed 30 cm from it.
Assume both can be considered point charges.
Take $k = 9.0 \times 10^{9}$.

Which one of the following best gives the magnitude of the force on the small sphere?

A. 1.1×10^{-14} N
B. 1.0×10^{-6} N
C. 3.0×10^{-6} N
D. 3.0×10^{-5} N

B

$$F = \frac{kq_1q_2}{r^2}$$

$$F = \frac{9.0 \times 10^{9} \times 1.0 \times 10^{-8} \times 1.0 \times 10^{-9}}{(0.30)^2}$$

$= 1.0 \times 10^{-6}$

$\therefore$ B

Question 3

The first stage of the electrons' path through the synchrotron is the electron-gun injector, in which stationary electrons are initially accelerated by an electric field, E.
In a particular synchrotron, the electric field in the injector is 200 kV m^{-1}.

Which one of the following best gives the force on a single electron while it is in the field of the injector?

A. 9.1 × 10–31 N
B. 1.6 × 10–19 N
C. 3.2 × 10–14 N
D. 1.8 × 10–25 N

C

Use F = Eq, F = 200 x 10^3 x 1.6 x 10^{-19} = 3.2 x 10^{-14} N

The following information relates to Questions 4 & 5.

An electron gun is used to inject electrons into the linac of a synchrotron.
Figure 1 shows a schematic diagram of the electron gun.
The mass of the electron is 9.1 × 10^{-31} kg, and the charge on the electron is 1.6 × 10^{-19} C.

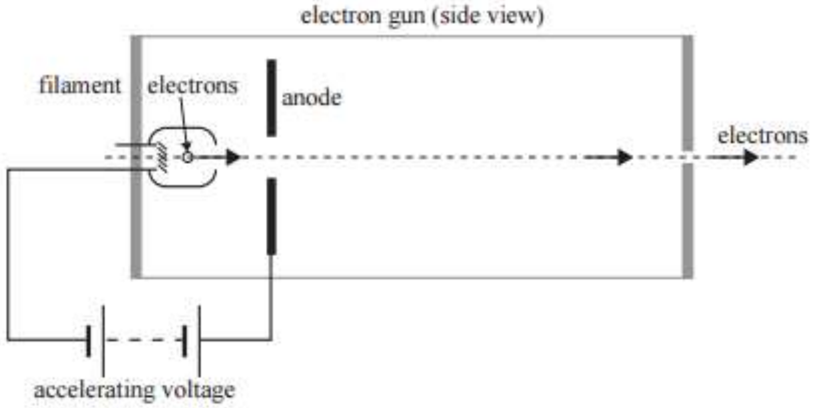


Figure 1

The electron gun is tested by first operating it at a low voltage.
Using this voltage the acceleration of the electrons is 1.8 × 10^{15} m s^{-2}.

Question 4

What is the magnitude of the electric field acting on the electrons?

A. 1 kV m^{-1}
B. 10 kV m^{-1}
C. 100 kV m^{-1}
D. 1000 kV m^{-1}

Using F = ma and F = Eq, E = ma /q = 9.1 x 10^{-31} x 1.8 x 10^{15} / 1.6 x 10^{-19}

E = 10.2 x 10^3 V/m

B

The accelerating voltage of the electron gun is now increased to its maximum value.
The electrons now reach a speed of 4.6×10^7 m s^{-1}.

Question 5
Which of the following best gives the accelerating voltage now?

A. 600 V
B. 2 600 V
C. 6 000 V
D. 260 000 V

C

Using gain in KE = Vq, $V = 1/2\, mv^2/q = 0.5 \times 9.1 \times 10^{-31} \times (4.6 \times 10^7)^2 / 1.6 \times 10^{-19}$
$V = 6.02 \times 10^3$ V

Question 6 (2 2011)
Figure 2 shows the injector gun of a synchrotron.

Figure 2

The electrons enter the injector at X with practically zero speed, and emerge from the injector at Y with a speed of 2.65×10^7 ms^{-1}.

Point X at the entry point of the injector is at a potential of zero volts.
An electron has a mass of 9.1×10^{-31} kg and a charge of -1.6×10^{-19} C.
Which of the following is the best estimate of the potential (voltage) of point Y with respect to point X?

A. +2000 V
B. -2000 V
C. +15000 V
D. -15000V

A

Using $½mv^2 = Vq$, $V = 0.5 \times 9.1 \times 10^{-31} \times (2.65 \times 10^7)^2/(1.6 \times 10^{-19}) = 1997$ V.
Plate Y must be positive to attract the electrons and X negative to repel them, so the answer is A

Use the following information to answer Questions 7 & 8

An electron gun is used to inject electrons into the linac of a synchrotron. **Figure 3** shows a schematic diagram of the electron gun. The mass of an electron is 9.1×10^{-31} kg and the charge on an electron is 1.6×10^{-19} C.

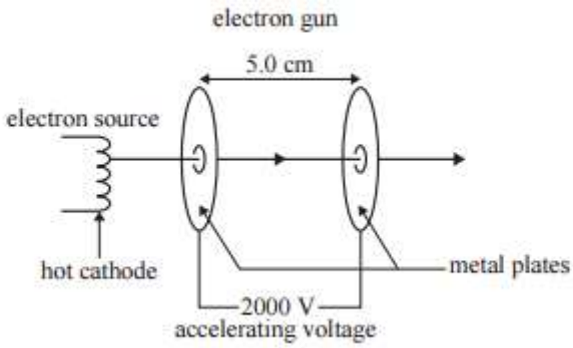


Figure 3

The electron is accelerated by an electric field generated by a voltage of 2000 V applied across the two plates. The plates are 5.0 cm apart.

Question 7
Which of the following is the best estimate of the magnitude of the electric force acting on an electron when it is between the plates?

A. 1.6×10^{-17} N
B. 6.4×10^{-17} N
C. 6.4×10^{-15} N
D. 4.0×10^{4} N

C

Using $F = Eq = Vq/d = 2000 \times 1.6 \times 10^{-19} / (5.0 \times 10^{-2}) = 6.4 \times 10^{-15}$ N

Question 8
The accelerating voltage is now doubled to 4000 V.
Which of the following is the best estimate of the speed of an electron as it leaves the electron gun?

A. 1.35×10^{7} m s^{-1}
B. 2.7×10^{7} m s^{-1}
C. 3.8×10^{7} m s^{-1}
D. 5.4×10^{7} m s^{-1}

C

Voltage is doubled, so speed is increased by sqrt (2) from 2.7 to 3.8×10^{7} m/s