Section 3.1.1 – Gravitational Fields

In this area of study (Unit 3 Area of Study 1) you will be examining three different fields:

- Gravitational field
- Magnetic field
- Electric field

You will analyse how certain objects interact with these fields, what effects these fields have upon objects and examine the various applications of each field.

Gravitational Fields

Gravitational force, just like electrical and magnetic forces are examples of **non-contact forces**. Such a force wouldn't exist if there weren't a gravitational field. In simple terms a field is a **region of space** in which **objects experience a force**. When considering a gravitational field a **point mass** is considered as a test subject. One can then test what happened to a point mass at any location within a gravitational field.

Often it is much easier to visualise a field rather than to describe it. Consider the following gravitational field lines for Earth as shown in **Figure 1**

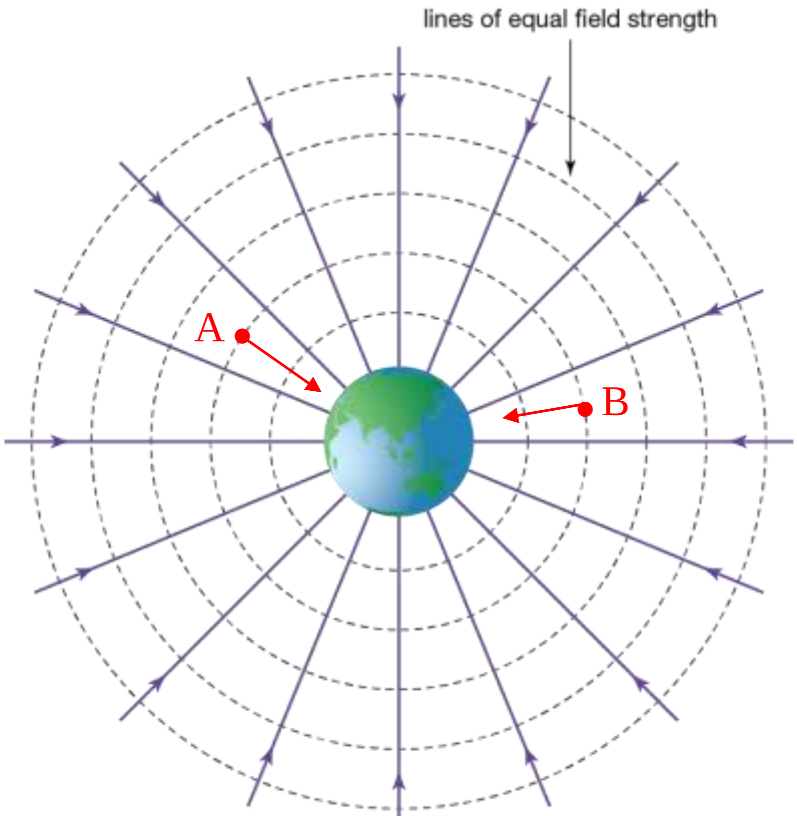


Figure 1 The Earth's gravitational field

NB: A mass will fall to the ground because of the force generated by the Earth's gravitational field.

Points of interest

- The Earth's gravitational field is **directed radially inwards** towards the centre of the planet
- The Earth's gravitational field **attracts** all objects of mass towards it
- The strength of the gravitational field is much **stronger closer to the surface of the Earth** (This can be shown with the arrow drawn **field lines** being closer together near the Earth's surface and further apart the more one moves away from the surface of the Earth)
- Object located at the **same radius** from the centre of the Earth experience **the same gravitational field strength**
  (Eg. Objects A & B both experience the same gravitational field strength in the previous diagram)

NB: The direction of the field is the direction of the field line arrows
Field lines never cross or intersect
If the field lines are closer together the field is strong
If the field lines are further apart the field is weak

Because fields have both **direction** and **magnitude** they are **vector quantities** rather than scalar quantities.

Describing gravitational fields

Figures 2a, 2b & 2c show that the Earth's gravitational filed lines are perpendicular to the Earth's surface. When this is examined some distance above the Earth's surface (ie. **Figures 2a & 2b**) it can be seen that the field lines are **unevenly distributed** and are classified as a **non-uniform field**.

However, upon the surface of the Earth (**Figure 2c**) the field lines can be thought of as running **parallel** to one another are a classified as a **uniform field.**

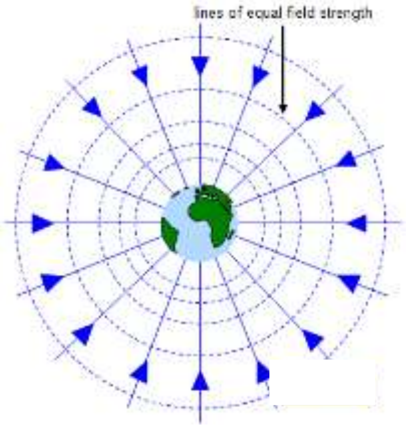


Figure 2a

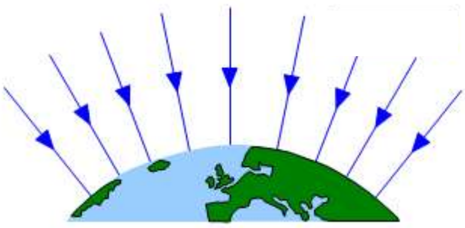


Figure 2b

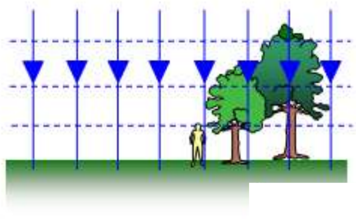


Figure 2c

Newton's Law of Universal Gravitation

Newton's Law of Universal Gravitation states that:

"Any objects that possesses mass experience a force of attraction, called gravity."

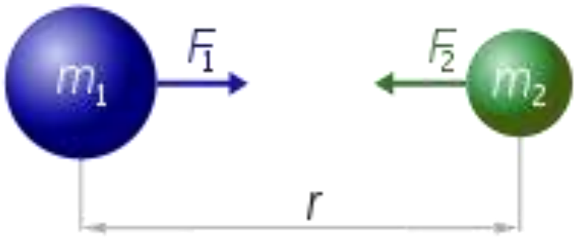


Figure 3

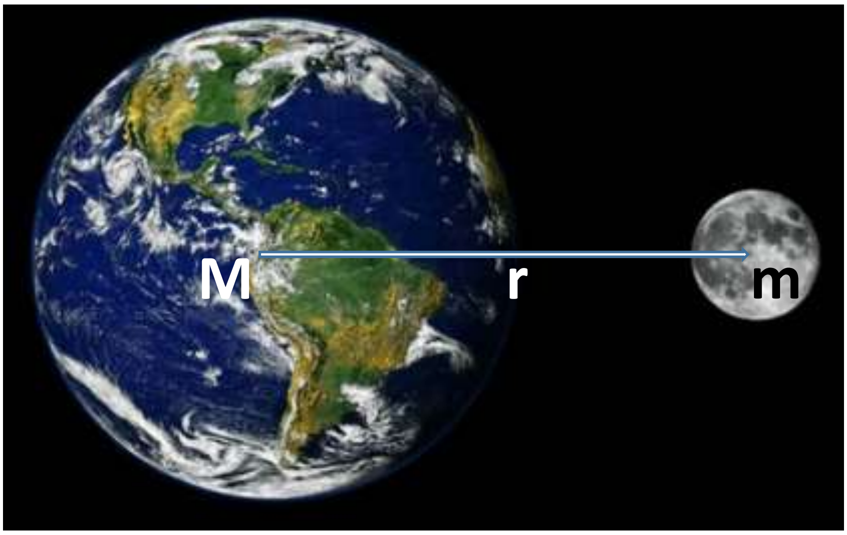
- m_1 exerts a force F_2 on m_2
- m_2 exerts a force F_1 on m_1
- $F_1 = F_2$ (Newton's 3rd Law)

$$\boxed{F_g = \frac{Gm_1m_2}{r^2}}$$

Where F represents the force due to gravity (N)
m_1 represents 1st mass (kg)
m_2 represents 2nd mass (kg)
r represents the radius of separation (m)
G represents the universal gravitational constant
= 6.67×10^{-11} (Nm^2kg^{-2})

Example.1

Calculate the gravitational force that exists between Earth and the moon.



F = ?
G = 6.67×10^{-11} Nm^2kg^{-2}
M = 5.98×10^{24} kg (mass of Earth)
m = 7.36×10^{22} kg (mass of moon)
r = 3.84×10^{8} m (from centre to centre)

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67\times10^{-11}\times5.98\times10^{24}\times7.36\times10^{22}}{(3.84\times10^{8})^2}$$

$$= \underline{\mathbf{1.99 \times 10^{20}\ N}}$$

Calculating the Gravitational Field Strength

Def^n: "A gravitational field is a region in space which an object experiences a force due to its mass"

On **Earth** our **gravitational field strength (g)** is approximately **9.8Nkg^{-1}.**

Scenario:
Consider an object near the surface of the Earth. The force acting upon this object by the Earth's gravitational field can be expressed using two equations

$$\boldsymbol{F_g = mg} \qquad \text{[Equation 1]}$$

$$\boldsymbol{F_g = \frac{GMm}{r^2}} \qquad \text{[Equation 2]}$$

Examine 1 and equation2. As L.H.S are equal, so too R.H.S must be equal:

$$\therefore \boldsymbol{mg = \frac{GMm}{r^2}} \qquad \text{[Now divide both sides by the mass of the object (m)]}$$

$$\boxed{\boldsymbol{g = \frac{GM}{r^2}}}$$

Where g represents the gravitational field strength (Nkg^{-1})
M represents mass of Earth (kg)
r represents the radius of separation (m)
G represents the universal gravitational constant (6.67×10^{-11} Nm^{2}kg^{-2})

It can be seen that the gravitational field strength (g) of a planet is:

1. independent of the objects mass, and;
2. inversely proportional to the radius of the separation squared (ie. $g \propto \frac{1}{r^2}$)

Example.2
Calculate the Earth's gravitational field strength at an altitude of 1000 km.

g = ?
G = 6.67×10^{-11} Nm^{2}kg^{-2}
M = 5.98×10^{24} kg (mass of Earth)
r = R_E + Altitude (altitude is distance above the surface of the Earth)
= $6.37 \times 10^6 + 1.0 \times 10^6$
= 7.37×10^6

$$g = \frac{GM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(7.37 \times 10^6)^2}$$

= **7.34 Nkg^{-1}**

Inverse Square Law

An inverse-square law is any physical law stating that a specified physical quantity or intensity is **inversely proportional** to the **square of the distance** from the source of that physical quantity.

In the case of the gravitational field strength (g) it can be stated that:

$$\boxed{g \propto \frac{1}{r^2}}$$

[The gravitation field strength is inversely proportional to the square of the radius]

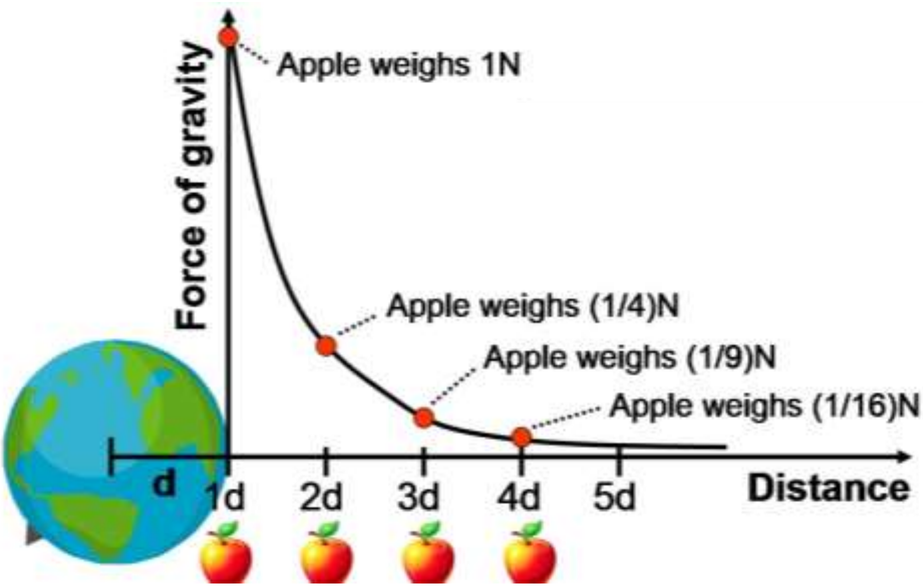


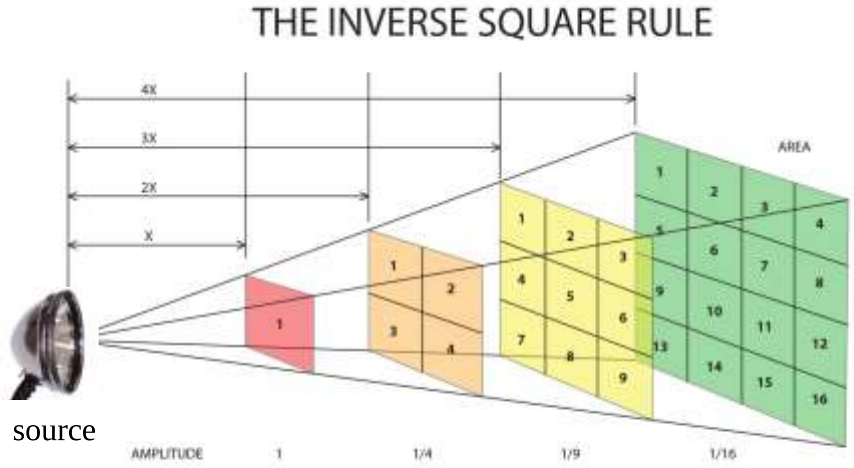
Figure 4

Radius	Radius^2	$1/r^2$
d (x1)	$1^2 = 1$	1/1
d (x2)	$2^2 = 4$	1/4
d (x3)	$3^2 = 9$	1/9
d (x4)	$4^2 = 16$	1/16
d (x5)	$5^2 = 25$	1/25

Light Analogy
Consider a spot light that illuminates a dark room. The light produced spreads at a fixed angle from the globe.

At an arbitrary distance of 1 unit, it covers an arbitrary surface area of 1 squared unit.
At a distance of 2 units, the light now covers a surface area of 4 units
At a distance of 3 units, the light now covers a surface area of 9 units
At a distance of 4 units, the light now covers a surface area of 16 units

However, as the same amount of light energy is found at each of the four different distances, the intensity (or amplitude) of the light across each surface area is reduced by ratios of 1/4th, 1/9th, & 1/16th respectively.



Example.3

Calculate the Earth's gravitational field strength at a **radius** of :

1. 10000 km.
2. 20000 km

@ Radius of 1000 km

g = ?
r = 10000 x 10^3 m
M = 5.98 x 10^{24} kg
G = 6.67 x 10^{-11} Nm^2kg^{-2}

$$g = \frac{GM}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(10 \times 10^6)^2}$$
= **<u>3.99 Nkg^{-1}</u>**

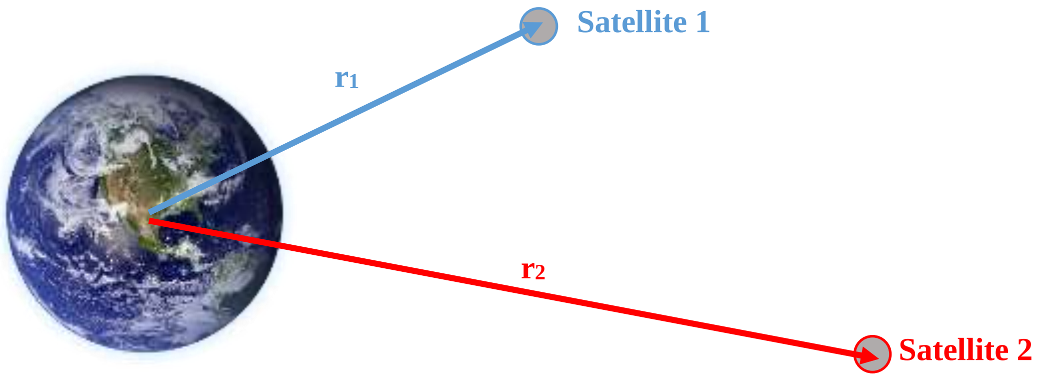
@ Radius of 2000 km

g = ?
r = 20000 x 10^3 m
M = 5.98 x 10^{24} kg
G = 6.67 x 10^{-11} Nm^2kg^{-2}

$$g = \frac{GM}{r^2}$$
$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(20 \times 10^6)^2}$$
= **1.00 <u>Nkg^{-1}</u>**

NB: The radius was increased by a factor of two (x2) and the gravitational field strength was decreased by a factor of four (x1/4). This is predicted by the inverse square law.

Task:
Derive an equation relating the radius and gravitational field strength of two satellites orbiting around a common planet.



$$g_1 = \frac{GM}{r_1^2}$$

$\therefore\ g_1 r_1^2 = GM$ [Equation 1 for satellite 1]

$$g_2 = \frac{GM}{r_2^2}$$

$\therefore\ g_2 r_2^2 = GM$ [Equation 2 for satellite 2]

Examine 1 and equation2. As L.H.S are equal, so too R.H.S must be equal:

$$\therefore\ g_1 r_1^2 = g_2 r_2^2$$

Or

$$\boxed{\frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2}$$

Example.3
Two satellites, Sat 1 (S1) and Sat 2 (S2) orbit around the Earth. S2's radius is 3 time that of S1. Given that S2 experiences a gravitational field strength (g) of 0.5 Nkg^{-1}, calculate the gravitational field strength experienced by S1?

$r_1 = R$
$r_2 = 3R$
$g_2 = 0.5$ Nkg^{-1}
$g_1 = ?$

$$\frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$$

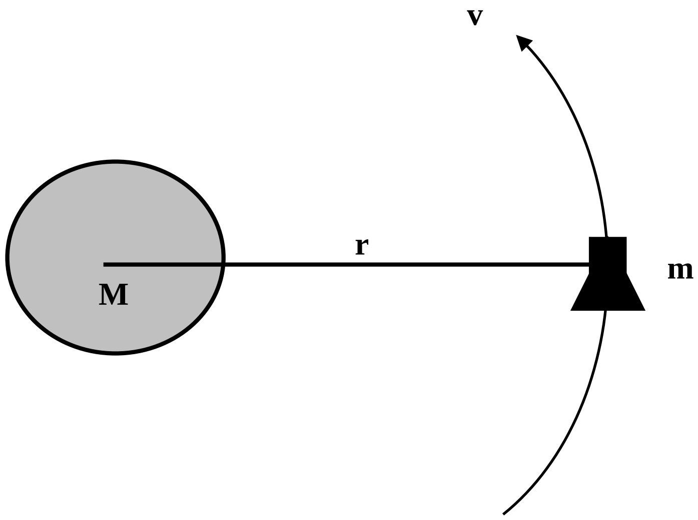
$$g_1 = g_2\left(\frac{r_2}{r_1}\right)^2$$

$$g_1 = 0.5\left(\frac{3R}{R}\right)^2$$

$g_1 = 0.5(3)^2$ = **4.5 Nkg^{-1}**

Circular Orbits

The gravitational attraction between Earth and its satellites results in many satellites having circular orbits. Accordingly, the gravitational force between **Earth** and **satellites** can be examined as a **centripetal force.**



The following centripetal force expressions can be used in calculations for circular path satellites:

$a = \frac{4\pi^2 r}{T^2}$,

$v = \frac{2\pi r}{T}$, and;

$F_C = \frac{mv^2}{r}$

Satellites of any planet can be analysed in terms of forces due to Newton's Law of Universal Gravitation, weight and centripetal force.

Equations of force upon a satellite:

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r} = \frac{4\pi^2 r\, m}{T^2} = mg$$

Newton's 2nd Law of Motion states that F = ma, or rather that a = F/m. To calculate the acceleration of a satellite of a planet, simply divide the force expression by the mass of the satellite (m_S).

Equations of acceleration of a satellite:

$$a = \frac{GM}{r^2} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = g$$

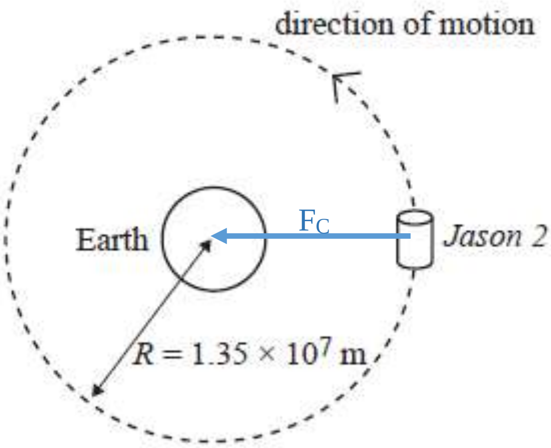
Example.4

The *Jason 2* satellite reached its operational circular orbit of radius 1.35×10^7 m in 2008 and then began mapping Earth's oceans.

DATA: mass of Earth = 5.98×10^{24} kg; mass of *Jason 2* = 525 kg; $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

Question.1

On the figure below, draw one or more labelled arrows to show the direction of any force(s) acting on the satellite *Jason 2* as it orbits Earth. You can ignore the effect of any astronomical bodies other than Earth.



Question.2

Calculate the period of orbit of the *Jason 2* satellite.

$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}
$M = 5.98 \times 10^{24}$ kg
$m = 525$ kg
$r = 1.35 \times 10^7$ m

Step.1 Select two of the equations of planetary acceleration relevant to this problem:

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

Step.2 Transpose, substitute values and solve

$$\therefore T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\therefore T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$= \sqrt{\frac{4\pi^2 (1.35 \times 10^7)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$$

= <u>**1.53×10^4 sec**</u>

Question.3

Calculate the speed of the *Jason 2* satellite

v = ? $\quad v = \frac{2\pi r}{T}$

r = 1.35 x 10^7 m $\quad = \frac{2 \times \pi \times (1.35 \times 10^7)}{1.53 \text{ x } 10^4}$

T = 1.53 x 10^4 sec $\quad = 5544 \text{ ms}^{-1}$

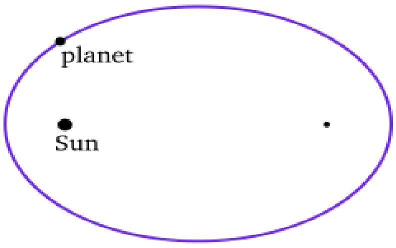
$= \underline{\mathbf{5.5 \text{ x } 10^3 \text{ ms}^{-1}}}$

Johannes Kepler
1571 – 1630

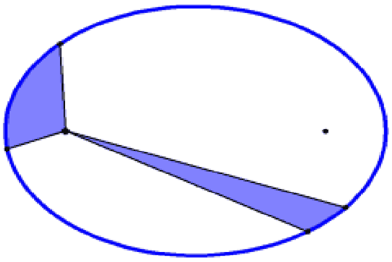
"I used to measure the Heavens, now I measure the shadows of Earth. The mind belonged to Heaven, the body's shadow lies here."

Kepler's Planetary Laws

1. **The Ellipse Law** – "The orbit of every planet is an ellipse with the sun at one focus."



2. **The Area Law** – "A line joining a planet and the sun sweeps out equal areas during equal intervals of time."



3. **$T^2 \alpha\, r^3$ – "**The square of the orbital periods of planets are directly proportional to the cube of the radii."

Derivation of Kepler's 3rd Law
Examine the following two components of the previous acceleration equations:

$$\frac{GM_P}{r^2} = \frac{4\pi^2 r}{T^2}$$

Now, a little mathematical manipulation:

$$\frac{GM_P}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\frac{GM_P}{r^3} = \frac{4\pi^2}{T^2}$$ Group like terms

$$\frac{GM_P}{4\pi^2} = \frac{r^3}{T^2}$$ Rearrange all **<u>constants</u>** to the LHS of equation

Kepler's 3rd Law

$$\boxed{\frac{GM_P}{4\pi^2} = \frac{r^3}{T^2}}$$

Kepler's 3rd Law states that:

"For a given planet, all satellites have the same value for the ratio of $\frac{r^3}{T^2}$"

Mathematically, it can be seen that all values upon the LHS of the above expression are constant, therefore the RHS too ($\frac{r^3}{T^2}$) must equate to a constant value.

Example.5

Two moons of Saturn are Atlas and Helene. Atlas has a radius of 1.37 x 10^5 km and a period of 0.602 days. Given that Helene has a radius of 3.77 x 10^5 km, calculate its period of revolution (days).

Atlas	**Helene**	**constants**
r_A = 1.37 x 10^8 m	r_H = 3.77 x 10^8 m	M_S
T_A = 0.602 days	T_H = ?	G
= 0.602 x 24 x 60 x 60		
= 5.201 x 10^4 sec		

NB: Take Kepler's 3rd Law as a starting point, as both satellites (moons) revolve around the one planet (Saturn)

$$\frac{GM_P}{4\pi^2} = \frac{r^3}{T^2}$$

Consider this equation for both satellites.
LHS are equal for both, $\therefore$ RHS **must** also **be equal.**

$$\therefore \frac{r_A{}^3}{T_A{}^2} = \frac{r_H{}^3}{T_H{}^2}$$

$$\therefore T_H{}^2 = \frac{r_H{}^3 \times T_A{}^2}{r_A{}^3}$$

$$\therefore T_H = \sqrt{\frac{r_H{}^3 \times T_A{}^2}{r_A{}^3}}$$

$$T_H = \sqrt{\frac{(3.77\times10^8)^3 \times (5.201\times10^4)^2}{(1.37\times10^8)^3}}$$

T_H = 2.37 x 10^5 sec
= **2.75 days**

NB: Several man made satellites are designed to be located above a particular point on the Earth permanently. Such satellites are describes as **geostationary.** They have exactly **the same period of rotation** as that of the **Earth** (i.e. 24 hours).

Example.6

Two satellites S_1 and S_2 are in circular orbits around the Earth. Their respective orbital radii are R & 2R respectively. The mass of S_1 is twice that of S_2.

Calculate the value of the following ratios:

1. orbital speed of S_1 : orbital speed of S_2
2. orbital period of S_1 : orbital period of S_2
3. acceleration of S_1 : acceleration of S_2

S₁	**S₂**	**constants**
$r_1 = R$	$r_2 = 2R$	M_E
$m_1 = 2M$	$m_2 = M$	G

1. $v_1 : v_2 = v_1/v_2 = ?$

$$\left.\frac{GM_E}{r^2} = \frac{v_S^2}{r}\right\}$$ From the equations of acceleration of satellites

$$\therefore \frac{GM_E}{r} = v_S^2$$

$\therefore v_S^2 r = GM_E$ Consider this equation for both satellites.
RHS are equal for both, $\therefore$ LHS **must** also **be equal.**

$\Rightarrow v_1^2 r_1 = v_2^2 r_2$

$$\therefore \frac{v_1^2}{v_2^2} = \frac{r_2}{r_1}$$

Substitute values for r_1 and r_2

$$\text{or } \left(\frac{v_1}{v_2}\right)^2 = \frac{r_2}{r_1}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{2R}{R}} = \sqrt{2} = 1.4$$

---

2. $T_1 : T_2 = T_1/T_2 = ?$

$$\left.\frac{GM_E}{4\pi^2} = \frac{r^3}{T^2}\right\}$$ Kepler's 3rd Law
Consider this equation for both satellites.
LHS are equal for both, $\therefore$ RHS **must** also **be equal.**

$$\therefore \frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2}$$

$$\therefore \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\text{or } \left(\frac{T_1}{T_2}\right)^2 = \frac{r_1^3}{r_2^3}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\left(\frac{r_1}{r_2}\right)^3} = \sqrt{\left(\frac{R}{2R}\right)^3} = \sqrt{\left(\frac{1}{2}\right)^3} = \sqrt{\frac{1}{8}} = 0.35$$

3. $a_1 : a_2 = a_1/a_2 = ?$

$a = \dfrac{GM_E}{r^2}$ } From the equations of acceleration of satellites

$\therefore ar^2 = GM_E$

Consider this equation for both satellites.
RHS are equal for both, $\therefore$ LHS **must** also **be equal.**

$\therefore a_1 r_1^2 = a_2 r_2^2$

$$\therefore \frac{a_1}{a_2} = \frac{r_2^{\,2}}{r_1^{\,2}}$$

Substitute values for r_1 and r_2

$$\therefore \frac{a_1}{a_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2R}{R}\right)^2 = (2)^2 = 4$$

Energy Changes in a Uniform Gravitational Field

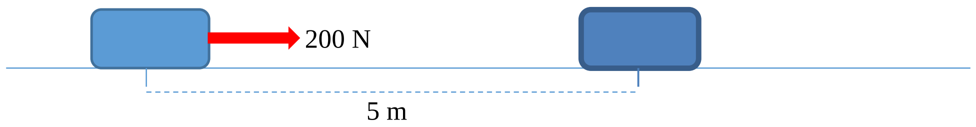
Recall:
Work is equal to the product of force (N) and distance (m)

$\boxed{W = Fs\cos\theta}$

Where W represents Work (Joules)
F represents Force (N)
s represents displacement (m)
θ represents the angle between the force and displacement (°)

NB: 1 Joule = 1Nm

Example.7
Calculate the work done upon an object when a force of 200N is applied, causing it to displace 5m.



W = ?
F = 200 N
s = 5 m

W = Fs
= 200 x 5
= **1000 J**

NB: As both the force and displacement are parallel, cos(0°) = 1

Scenario
Consider a 5 kg bowling ball that is raised a height of 2 m above a person's head. What is the work done on the bowling ball?

W = ?
F = mg
= 5×9.8
= 49 N
s = 2 m
θ = 0° (as the force and displacement are parallel)

W = Fscos(θ)
= $49 \times 2 \times \cos(0)$
= **98 Joules**

NB: Work done on the bowling ball = gain in gravitational potential energy
$W = \Delta E_{gp}$
$Fs = E_{gp}$
$mgs = E_{gp}$

This is commonly expressed as:

$\boxed{E_{gp} = mgh}$

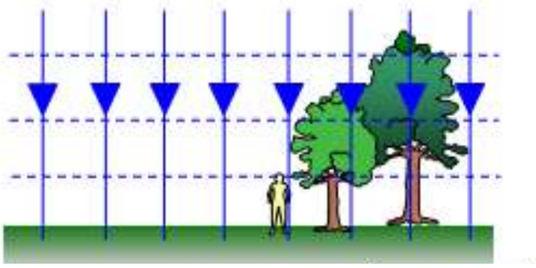
or

$\boxed{\Delta E_{gp} = mg\Delta h}$

Where E_{gp} represents gravitational potential energy (J)
m represents mas (kg)
g represents the gravitational field strength (Nkg^{-1})
h represents the elevated height (m)

NB: By lifting the object above the surface of the Earth, work has been done against gravity and the object has now gained gravitational potential energy.

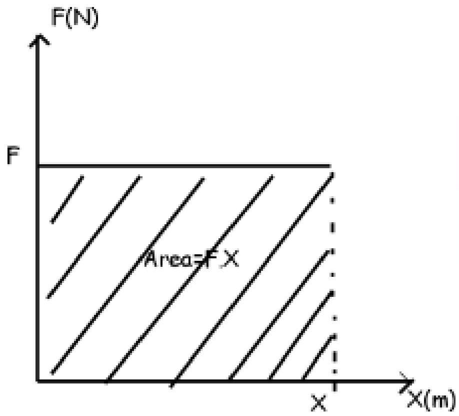
One can apply $E_{gp} = mgh$ near the Earth's surface as the gravitational field can be considered as **uniform or constant.**



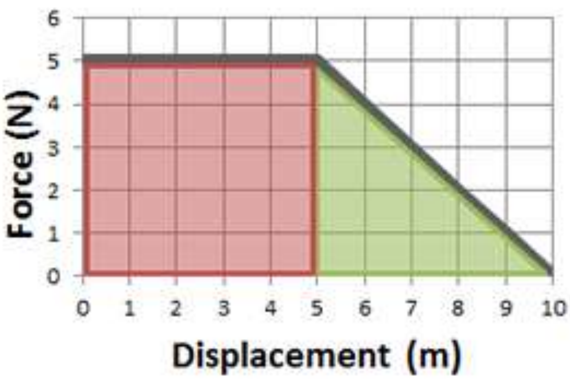
Also recall:

If the force applied to an object is graphed against the objects distance, then the area under the graph will be equal to the work done on the object, or the change in energy.

Graph 1 shows a constant force applied upon an object over a distance x
Graph 2 shows a changing force applied upon an object over a distance of 10 m



Graph 1



Graph 2

NB: Area under a F-x graph is equal to the work done (W) or the change in energy (ΔE).

Example.8

Calculate the work done upon the object as displayed by the Force – Displacement graph shown in Graph 2.

W = ?
Area = **Area 1 (square)** + **Area 2 (triangle)**
= [5N $\times$ 5m] + [$\frac{1}{2}$5N $\times$ 5m]
= 25 J + 12.5 J
= 37.5 J

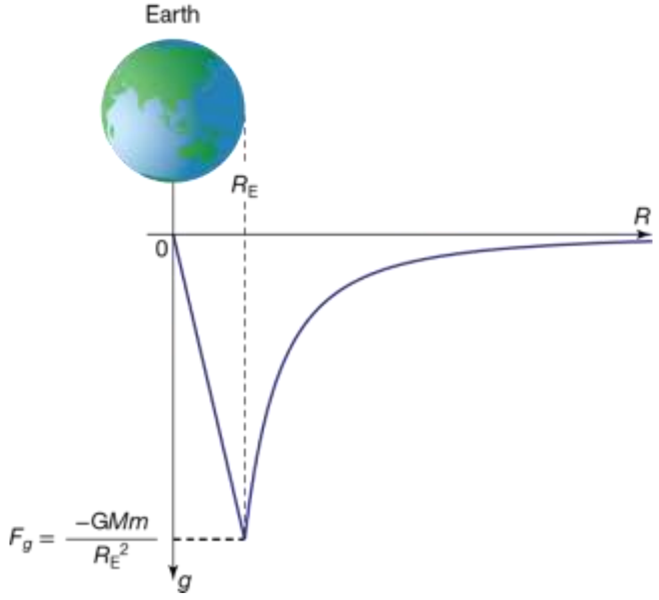
W = Area under graph
= **<u>37.5 J</u>**

Energy Changes in a Non - Uniform Gravitational Field

As objects move towards or away from planet Earth over considerable distances, the planets gravitational field changes. This follows the previously examined inverse square law. Accordingly, one cannot simply use $\Delta E_{gp} = mg\Delta h$, as the gravitational field strength (g) changes with the radius of separation. This is shown in Graph 3 below.

NB: Graph 3 can either be presented upright or, as in the above case upside down.

It is drawn upside down as the direction of the radius and that of the force upon an object are opposite. The inverted graph reflect the true vector nature of both quantities.

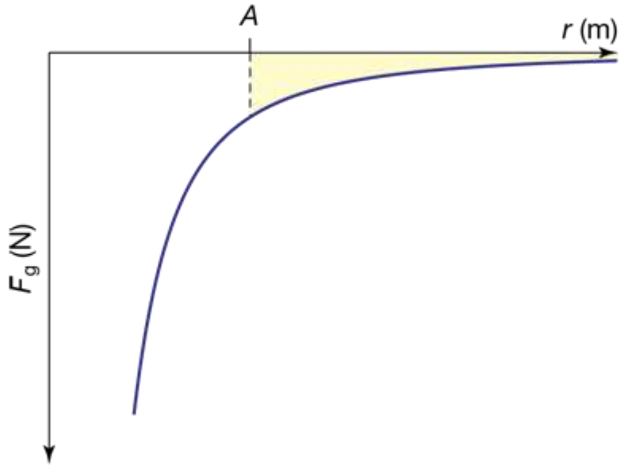


Graph 3

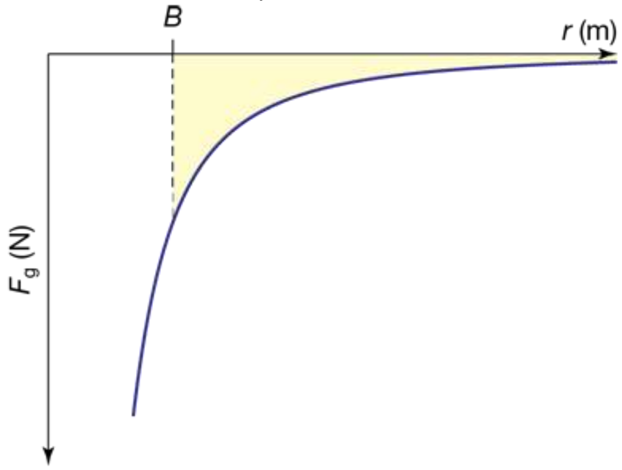
However, the area under a Force – Displacement graph can still be sued to calculate the work done or change in energy of an object that moves either towards or away from a planet.

Scenario

Consider a satelite at a radius (A) from the centre of Earth, as shown in Graph A below. It then moves to a smaller radius (B) from the centre of Earth, as shown in Graph B below.



Graph A

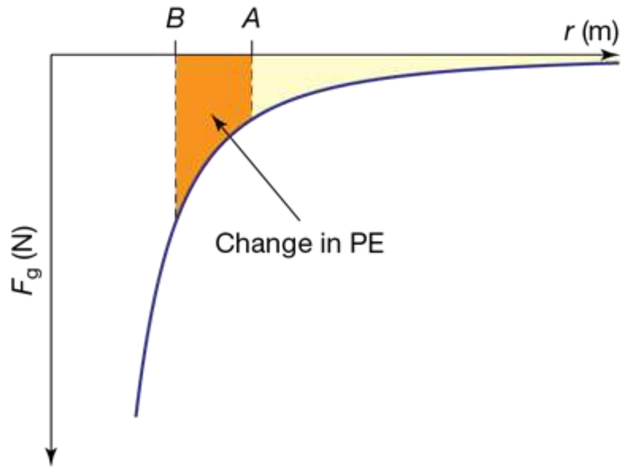


Graph B

By moving though the Earths gravitational field work has been done. Accordingly, both the kinetic energy and gravitational potential energy has changed. This change in energy can be calculated by finding the area under the graph.

NB: To find the area under a curved graph you can either break areas into rectangles, triangles or trapeziums.
Or alternatively, you can approximate the number of squares covered from a grid and multiply this by the area of a single square.

Graph C

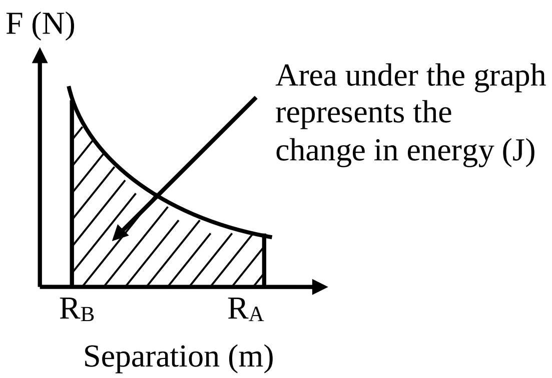


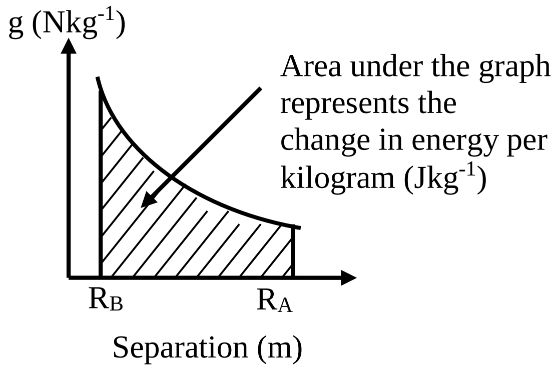
Graph C

Recall

W = ΔE

If the satellites moves from A to B, it will be closer to Earth: ∴ E_k ↑ and E_{gp} ↓
If the satellites moves from B to A, it will be further from Earth: ∴ E_k ↑ and E_{gp} ↓





NB: Both the force and gravitational field strength are at a maximum at the shortest radius

NB: The total mechanical energy (ΣE_{mech}) remains constant
$\Sigma E_{mech} = E_p + E_k$

Exam Style Questions

Question 1

A satellite is in orbit around a planet (not Earth). Each orbit takes 32 hours to complete, and the radius of the satellite's orbit is 8.6×10^7 m. Calculate the mass of the planet.

M = ?
T = 32 x 60 x 60
= 115200 sec
$r = 8.6 \times 10^7$ m
$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

Use Kepler's 3rd Law

$$\frac{GM}{4\pi^2} = \frac{r^3}{T^2}$$

$$\therefore M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 (8.6x10^7)^3}{6.67x10^{-11} x (115200)^2}$$

= ***2.84 x 10^25^ kg***

$$\boxed{2.84 \times 10^{25} \text{ kg}}$$

Question 2

A comet is in orbit around the sun. The mass of the comet is 3.7×10^{22} kg, and the radius of the comet's orbit around the sun is 7.2×10^9 m. The mass of the sun is 1.99×10^{30} kg.

Calculate the period of the comet's orbit. Be sure to include appropriate units when giving your answer

T = ?
$M = 1.99 \times 10^{30}$
$r = 7.2 \times 10^9$ m
G = 6.67 x 10-11 Nm^{2}kg^{-2}

Use Kepler's 3rd Law

$$\frac{GM}{4\pi^2} = \frac{r^3}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\therefore T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$= \sqrt{\frac{4\times\pi^2\times(7.2\times10^9)^3}{6.67\times10^{-11}\times1.99\times10^{30}}} = \underline{3.4 \times 10^5 \text{ seconds}}$$

$$\boxed{3.4 \times 10^5 \text{ sec}}$$

The following information relates to Questions 3–5

The International Space Station (ISS) is currently under construction in Earth orbit. It is incomplete, with a current mass of 3.04×10^5 kg. The ISS is in a circular orbit of 6.72×10^6 m from the centre of Earth.

In the following questions the data below may be needed.
Mass of ISS 3.04×10^5 kg
Mass of Earth 5.98×10^{24} kg
Radius of Earth 6.37×10^6 m
Radius of ISS orbit 6.72×10^6 m
Gravitational constant 6.67×10^{-11} N m^2 kg^{-2}

Question 3
What is the weight of the ISS in its orbit?

$$\boldsymbol{F_g = \frac{GMm}{r^2}}$$

$$\boldsymbol{F_g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 3.04 \times 10^5}{(6.72 \times 10^6)^2}}$$

$$\boldsymbol{F_g = 2.69 \times 10^6\ N}$$

2.69 x 10^6 N

Question 4
What is the period of orbit of the ISS around Earth?

$$\boldsymbol{F = \frac{m4\pi^2 r}{T^2}}$$

$$\boldsymbol{\therefore T = \sqrt{\frac{m4\pi^2 r}{F}}}$$

$$\boldsymbol{\therefore T = \sqrt{\frac{3.04 \times 10^5 \times 4 \times \pi^2 \times 6.72 \times 10^6}{(2.69 \times 10^6)}}}$$

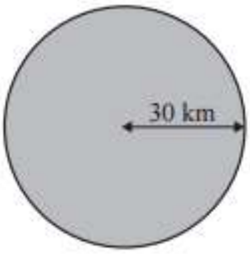
5480 s

Question 5
When the ISS is completed, its mass will have increased to 3.70×10^5 kg. Will the period of orbit of the ISS around Earth then be greater, the same, or less?

Same

Period is independent of mass.

The following information relates to Questions 6–8



Assume that somewhere in space there is a small spherical planet with a radius of 30 km. By some chance a person living on this planet visits Earth. He finds that he weighs the same on Earth as he did on his home planet, even though Earth is so much larger.

Earth has a radius of 6.37×10^6 m and a mass of 5.98×10^{24} kg
The acceleration due to gravity (g), or the gravitational field, at the surface of Earth, is 9.8 Nkg^{-1}
The universal gravitational constant, $G = 6.67 \times 10^{-11}$ Nm^2kg^{-2}

Question 6
What is the value of the gravitational field on the surface of the visitor's planet?

For him to have the same weight force, the gravitational field strength is the same as that of Earth.

9.8 Nkg^{-1}

Question 7
What is the mass of the visitor's planet?
Explain your answer by showing clear working.

$$g = \frac{GM}{r^2}$$
$$\therefore M = \frac{gr^2}{G}$$

$$M = \frac{9.8 \times (30 \times 10^3)^2}{6.67 \times 10^{-11}}$$

$$= 1.35 \times 10^{11} \text{ kg}$$

1.35×10^{11} kg

The visitor's home planet is in orbit around its own small star at a radius of orbit 1.0×10^9 m. The star has a mass of 5.7×10^{25} kg.

Question 8

What would be the period of the orbit of the visitor's planet? Show working.

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\therefore T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$\therefore T = \sqrt{\frac{4 \times \pi^2 \times (1.0 \times 10^9)^3}{6.67 \times 10^{-11} \times 5.7 \times 10^{25}}}$$

$= 3.2 \times 10^6$ sec

3.2×10^6 sec

Question 9

Before the spacecraft Apollo 11 landed on the moon, it travelled around the moon in an orbit with a period of 2.0 hours.

Calculate the height of Apollo 11 above the moon's surface during its orbit of the moon. Take the orbit to be circular.

Take G = 6.67×10^{-11} Nm^2kg^{-2}; M_{moon} =7.36×10^{22} kg; R_{moon} = 1.74×10^6 m.

$$\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$$

$$\therefore R = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$R = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22} \times (2 \times 3600)^2}{4\pi^2}}$$

$= 1.86 \times 10^6$ m

Height (altitude) above the moon's surface = R - R_{moon}

Height = $1.86 \times 10^6 - 1.74 \times 10^6$

$= 1.2 \times 10^5$ m

1.2×10^5 m

Question 10
Students measure the gravitational force between two masses of 1.0 kg and 100 kg, placed 10 cm apart. The universal gravitational constant, G, is 6.67×10^{-11} N m^2 kg^{-2}.
Which one of the following best gives the gravitational force of attraction between the two masses?

A. 1.0×10^{-3} N
B. 6.7×10^{-5} N
C. 6.7×10^{-7} N
D. 1.0×10^{6} N

$$F_g = \frac{GMm}{r^2}$$
$$F_g = \frac{6.67 \times 10^{-11} \times 1 \times 100}{(0.10)^2}$$
= **6.7×10^{-7} N**

C

The following information relates to Questions 11 & 12

The mass of the planet Mars is 6.4×10^{23} kg.
The radius of Mars is 3.4×10^{6} m.
The universal gravitational constant, G is 6.67×10^{-11} N m^2 kg^{-2}.

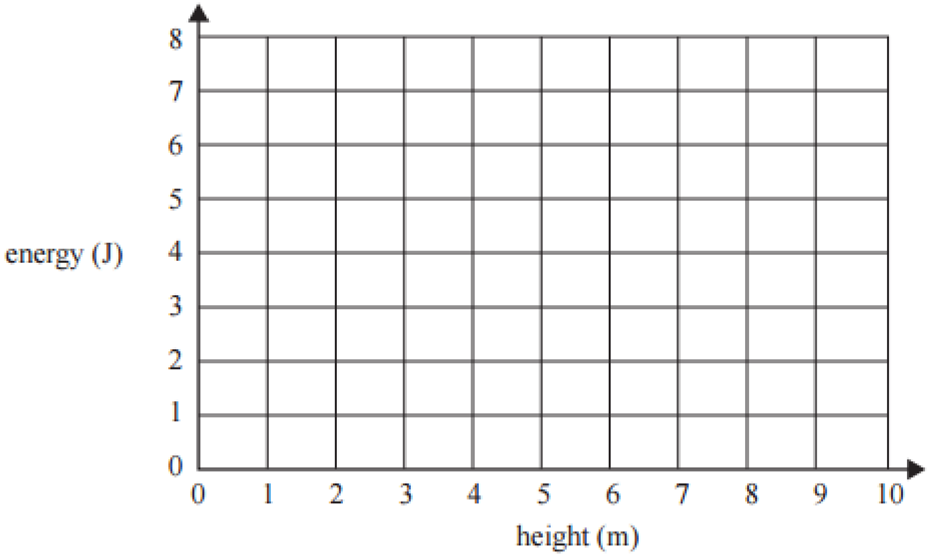
Question 11
Calculate the acceleration due to gravity at the surface of Mars. Show your working.

$$a = g = \frac{GM}{r^2}$$
$$a = g = \frac{6.67 \times 10^{-11} \times 6.4 \times 10^{23}}{(3.4 \times 10^6)^2}$$
= **3.69 ms^{-2}**

3.69 ms^{-2}

Question 12
A probe of mass 20 kg is released from a height of 10 m above the surface of Mars. Assume that the gravitational field strength is uniform (the same as at the surface). Ignore air resistance.

Sketch the gravitational potential energy of the probe as a function of height above the surface of Mars on the axes provided below and label this as U_g. Take potential energy at the surface of Mars as zero. Include the initial potential energy value on the energy axis. On the same axes, sketch the kinetic energy of the probe and label this as E_K.


energy (J)
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5
6
7
8
9
10
height (m)