Reducing Balance Loans

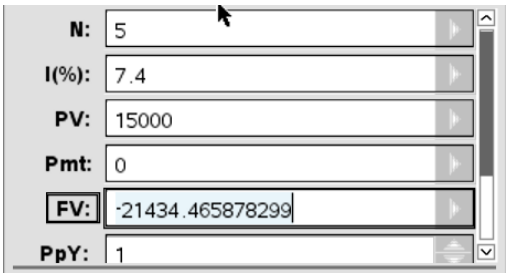
When you take out a loan with a bank, or other financial institute, they will provide you access to funds and charge you interest for the service.

Just as a savings account gains interest over time, therefore making your savings grow. A loan account with be charged interest making your debt grow.

In fact if you took out a loan say for **$15000** at **7.4% p.a**. **compounded annually** and made no additional payments, your debt would continue to grow each and every year.

n+1	***V_n***	***Interest***	***V_{n+1}***
1	$15000.00	$15000(\frac{7.4}{100})= $1110	$16110.00
2	$16110.00	$16110(\frac{7.4}{100})= $1192.14	$17302.14
3	$17302.14	$17302.14(\frac{7.4}{100})= $1280.36	$18582.50
4	$18582.50	$18582.50(\frac{7.4}{100})= $1375.11	$19957.61
5	$19957.61	$19957.61(\frac{7.4}{100})= $1476.86	$21434.47

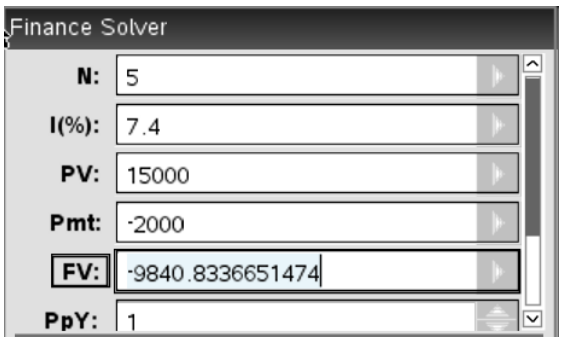
From the above table it can be seen that your debt has risen from an original (V_0) of $15000 to a value of $21434.47 after 5 years ($V_5$).



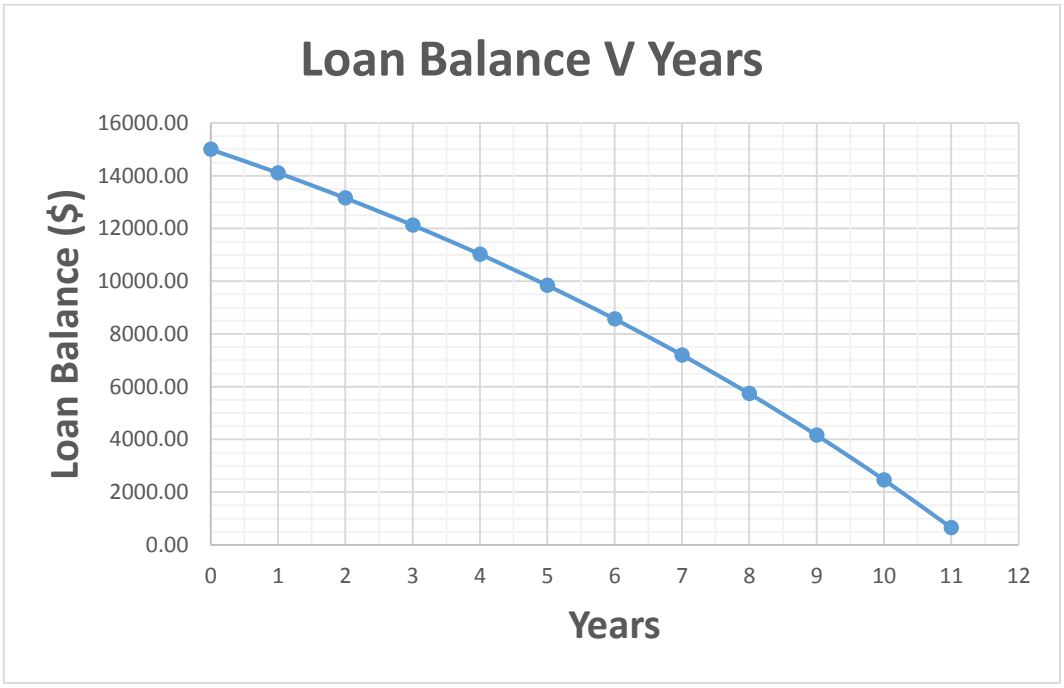
The TI-Nspire CAS Financial Solver can also be used to predict the final value of the loan after 5 years

What would happen if at the end of every year we paid a deposit (instalment) of $2000.00?

n+1	***V_n***	***Interest***	***$V_n + I$***	***Deposit***	***V_{n+1}***
1	$15000.00	$15000(\frac{7.4}{100})= $1110	$16110.00	$2000	$14110.00
2	$14110.00	$14110(\frac{7.4}{100})= $1044.14	$15154.14	$2000	$13154.14
3	$13154.14	$13154.14(\frac{7.4}{100})= $973.41	$14127.55	$2000	$12127.55
4	$12127.55	$12127.55(\frac{7.4}{100})= $897.44	$13024.99	$2000	$11024.99
5	$11024.99	$11024.99(\frac{7.4}{100})= $815.85	$11840.84	$2000	$9840.84



The TI-Nspire CAS Financial Solver can also be used to predict the final value of the loan after 5 years with the addition of annual deposits paid of $2000.00



Clearly the balance of the loan can be reduced over time via periodic payments/instalments.

Payment Comparison

1st Year Payment		5th Year Payment		11th Year Payment	
Instalment	$2,000.00	Instalment	$2000.00	Instalment	$2000
V_0	15,000.00	V_4	$11024.99	V_{10}	$2468.57
V_1	14,110.00	V_5	$9840.84	V_{11}	$651.24
Amount paid	$890.00	Amount paid	$1184.15	Amount paid	$1817.33
Interest paid	$1110.00	Interest paid	$815.85	Interest paid	$182.67

The further into the loan you get:

- the larger the proportion of your instalment contributes towards paying off the loan.
- the smaller the proportion of your instalment contributes towards paying interest.

The Annuities Formula

The amount owing in a loan account for n repayments is given by the annuities formula:

$$V_n = V_0 R^n - \frac{d(R^n - 1)}{R - 1}$$

Where:

V_0 = the amount borrowed (principal)

R = the compounding or growth factor for the amount borrowed

$= 1 + \frac{r}{100}$ (r = the interest rate per repayment period)

d = the amount of the regular payments made per period

n = the number of payments

V_n = the amount owing after n payments

Alternatively, the TI-Nspire CAS "Finance Solver" is always available for reducing balance loans questions.

Example.1

A loan of $60 000 is taken out over 20 years at a rate of 12% p.a. (interest debited monthly) and is repaid with monthly instalments of $660.65. Find the amount still owing after:

1. 5 years
2. 10 years
3. 15 years

Task.1

V_0 = $60000

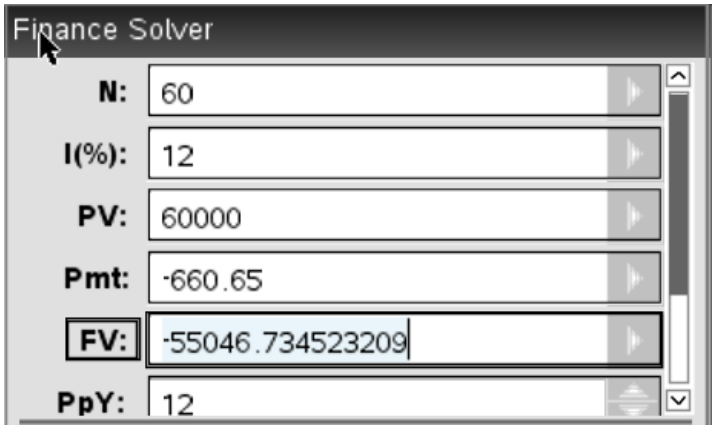
n = 5 years

$= 5 \times 12 = 60$ months

R = $(1 + \frac{1}{100})$

$= 1.01$

d = $660.65



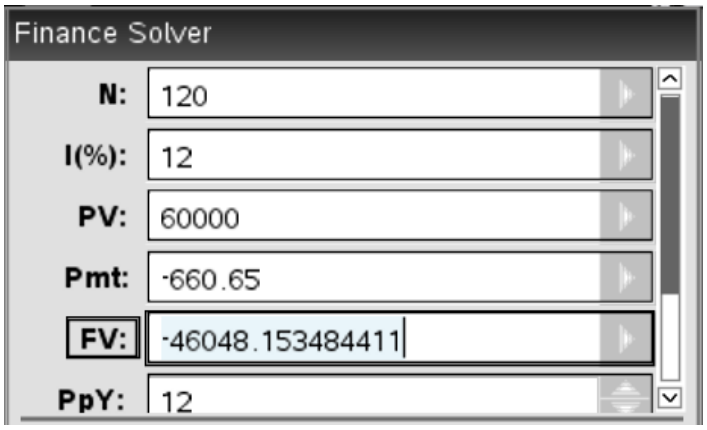
$$V_n = V_0 R^n - \frac{d(R^n - 1)}{R - 1}$$

$$\therefore V_{60} = 60000 \times (1.01)^{60} - \frac{660.65(1.01^{60} - 1)}{1.01 - 1}$$

= **$55046.73**

Task.2

V_0 = \$60000
n = 10 years
= 10 × 12 = 120 months
R = $(1 + \frac{1}{100})$
= 1.01
d = \$660.65



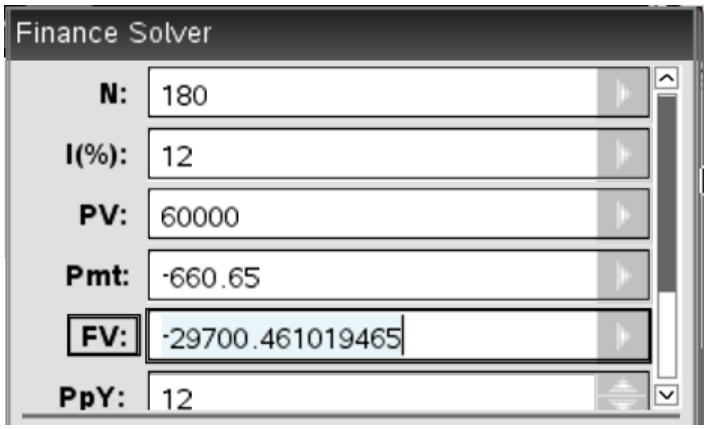
$$V_n = V_0 R^n - \frac{d(R^n - 1)}{R - 1}$$

$$\therefore V_{120} = 60000 \times (1.01)^{120} - \frac{660.65(1.01^{120} - 1)}{1.01 - 1}$$

= **\$46048.15**

Task.3

V_0 = \$60000
n = 15 years
= 15 × 12 = 180 months
R = $(1 + \frac{1}{100})$
= 1.01
d = \$660.65



$$V_n = V_0 R^n - \frac{d(R^n - 1)}{R - 1}$$

$$\therefore V_{180} = 60000 \times (1.01)^{180} - \frac{660.65(1.01^{180} - 1)}{1.01 - 1}$$

= **\$29700.46**

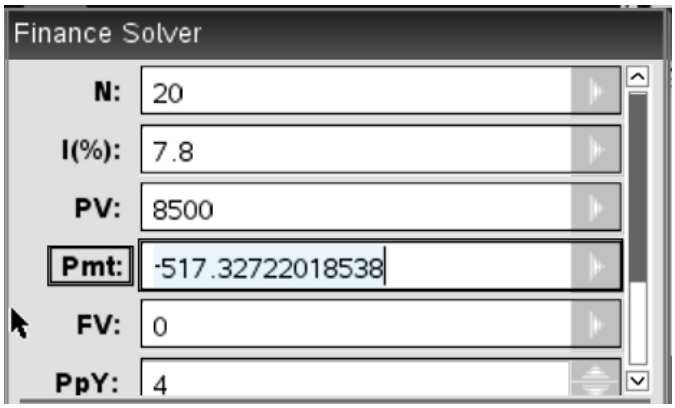
So after 5 years the balance of the loan was \$55046.73, after 10 years the balance on the loan was \$46048.15 and after 15 years the balance on the loan was \$29700.46.

Example.2

Lisa borrows $8500 for a cinema room upgrade. She agrees to repay the loan over 5 years with quarterly instalments at 7.8% p.a. (adjusted quarterly). Find:

1. The instalment value
2. The principal repaid during the 3rd and 15th repayment

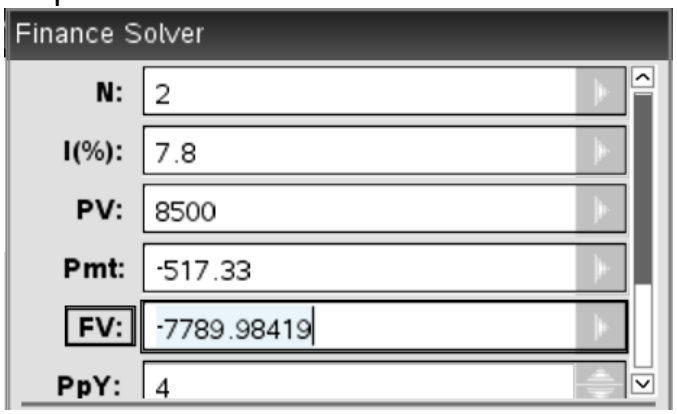
Task.1



Using the TI-Nspire CAS "Financial Solver" the value of the instalment would be $517.33

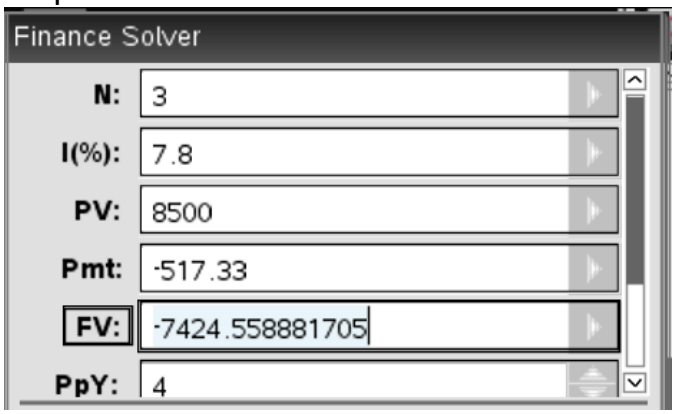
Task.2 – Part A

Step.1 Calculate the balance of the loan after 2 repayments (quarters)



Loan balance after 2 quarters = $7789.98

Step.2 Calculate the balance of the loan after 3 repayments (quarters)



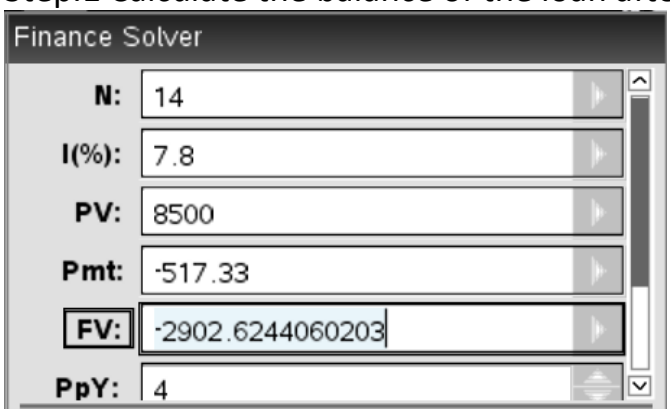
Loan balance after 3 quarters = $7424.56

Principal paid during 3rd repayment (quarter)
= Balance (after 2 qtrs) – Balance (after 3 qtrs)
= $7789.98 - $7424.56
= **$365.42**

Principal repaid during the 3rd repayment was $365.42

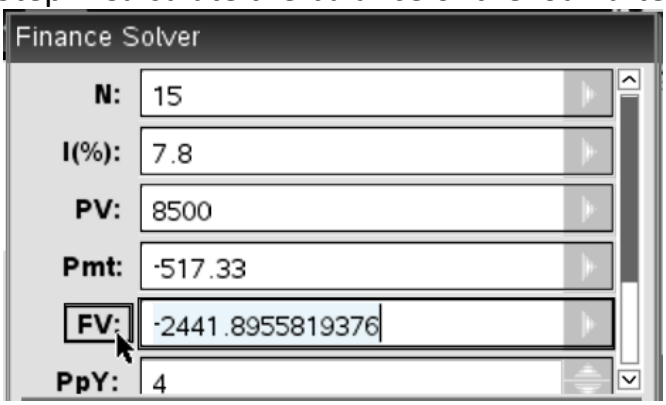
Task.2 – Part B

Step.1 Calculate the balance of the loan after 14 repayments (quarters)



Loan balance after 14 quarters = $2902.62

Step.2 Calculate the balance of the loan after 15 repayments (quarters)



Loan balance after 15 quarters = $2441.90

Principal paid during 15th repayment (quarter)
= Balance (after 14 qtrs) – Balance (after 15 qtrs)
= $2902.62 - $2441.90
= **$460.72**

Principal repaid during the 15th repayment was $460.72