Annuity Investments

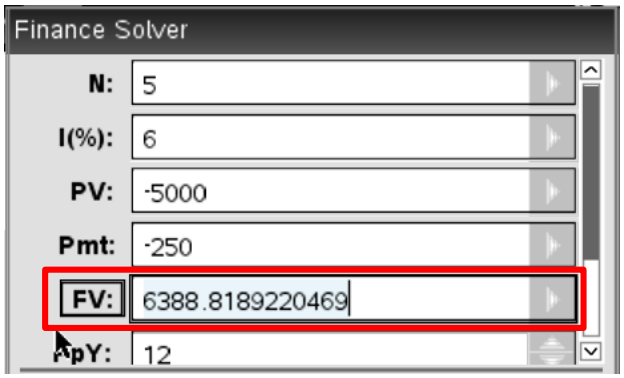
An **annuity investment** is similar to a reducing balance loan, only the balance of your investment **increases** with the **added interest**.

An annuity investment consists of an **initial deposit** plus **regular deposits** made over a period of time.

Consider the following scenario:

An initial **deposit of $5000** was made on an investment taken out over **5 years** at a rate of **6.0% p.a.** (**interest calculated monthly**) and an additional deposit of **$250** is made **each month**. Complete the below table for the first 5 months and calculate the interest earned over this time.

n+1	V_n	***Interest***	$V_n + I$	***Deposit***	V_{n+1}
1	$5000.00	$5000(\frac{0.5}{100})= \$25$	$5025.00	$250	$5275.00
2	$5275.00	$5275(\frac{0.5}{100})= \$26.38$	$5301.38	$250	$5551.38
3	$5551.38	$5551.38(\frac{0.5}{100})= \$27.76$	$5579.14	$250	$5829.14
4	$5829.14	$5829.14(\frac{0.5}{100})= \$29.15$	$5858.29	$250	$6108.29
5	$6108.29	$6108.29(\frac{0.5}{100})= \$30.54$	$6138.83	$250	$6388.83



The TI-Nspire CAS Financial Solver can also be used to predict the final value of the loan after 5 months with the addition of annual deposits paid of $250.00

NB: Rounding can account for the 1 cent discrepancy.

Therefore the balance of the account after 5 months would be $6388.82

Interest = Account balance – (Initial deposit + total monthly deposits)
= 6388.82 – ($5000 + 5×250)
= **$138.82**

The total interest earned over the first 5 months was $138.82.

Annuity investments can also be considered as a recurrence relation:

$$V_{n+1} = V_nR + d$$

$$V_{n+1} = V_n(1 + \frac{r}{100}) + d$$

Where: V_{n+1} = amount after n + 1 payments
V_n = amount at time n
r = interest rate per period
d = deposit amount

The Annuities Formula

The amount owing in a loan account for n repayments is given by the annuities formula:

$$V_n = V_0R^n + \frac{d(R^n - 1)}{R - 1}$$

Where:

V_0 = the amount borrowed (principal)
R = the compounding or growth factor for the amount borrowed
= $1 + \frac{r}{100}$ (r = the interest rate per repayment period)
d = the amount of the regular payments made per period
n = the number of payments
V_n = the amount owing after n payments

Alternatively, the TI-Nspire CAS "Finance Solver" is always available for annuity investments.

Example.1

An initial deposit of $3000 was made on an investment taken out over 10 years at a rate of 7.0% p.a. (interest calculated monthly), and an additional deposit of $200 is made each month. What is the accounts balance at the end of the term? How much interest has been earned over the 10 years?

Task.1

V_0 = $3000

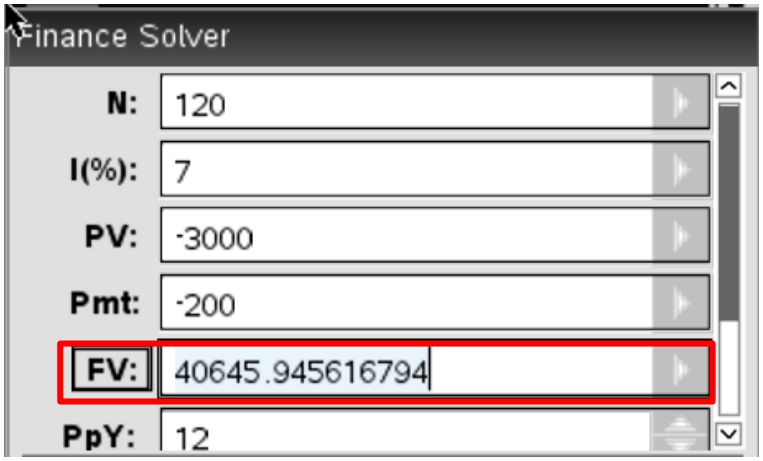
n = 10 years

= 10 × 12 = 120 months

R = $(1 + \frac{7/12}{100})$ or $(1 + \frac{7}{1200})$

d = $200

$$V_n = V_0 R^n + \frac{d(R^n - 1)}{R - 1}$$



$$\therefore V_{120} = 3000 \times (1 + \frac{7}{1200})^{120} + \frac{200((1 + \frac{7}{1200})^{120} - 1)}{(1 + \frac{7}{1200}) - 1}$$

= **$40645.95**

At the end of the 10 year investment term the account balance would be $40645.95.

Task.2

Interest = Account balance – (Initial deposit + total monthly deposits)

= 40645.95 – ($3000 + 120×200)

= **$13645.95**

The total interest earned over the first 5 months was $13645.95.

Example.2

John is aged 40 and is planning to retire at 65 years of age. He estimates that he needs $1,000,000 to retire comfortably. His current superannuation fund has a balance 0f $80,000 and is delivering 8% p.a. compounded monthly.

1. Find the monthly contributions needed to meet the retirement lump sum target.

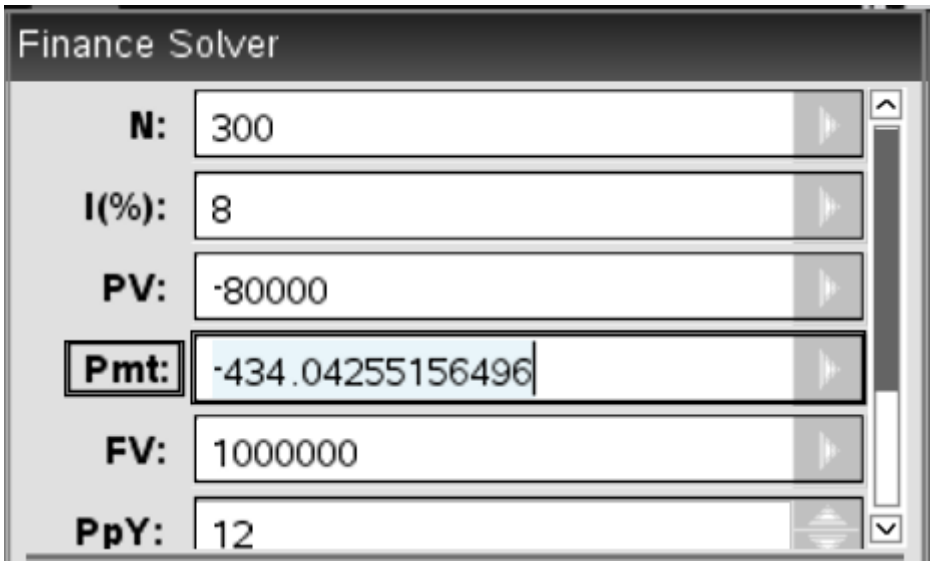
2. If in the final ten years before retirement, John doubles his monthly contribution calculate the new lump sum amount available for his retirement.

3. How much extra could John expect if the interest rate from part 1 is increased to 10% p.a. (for the final 10 years) compounded monthly?

Task.1

Using the TI-Nspire CAS "Finance Solver"

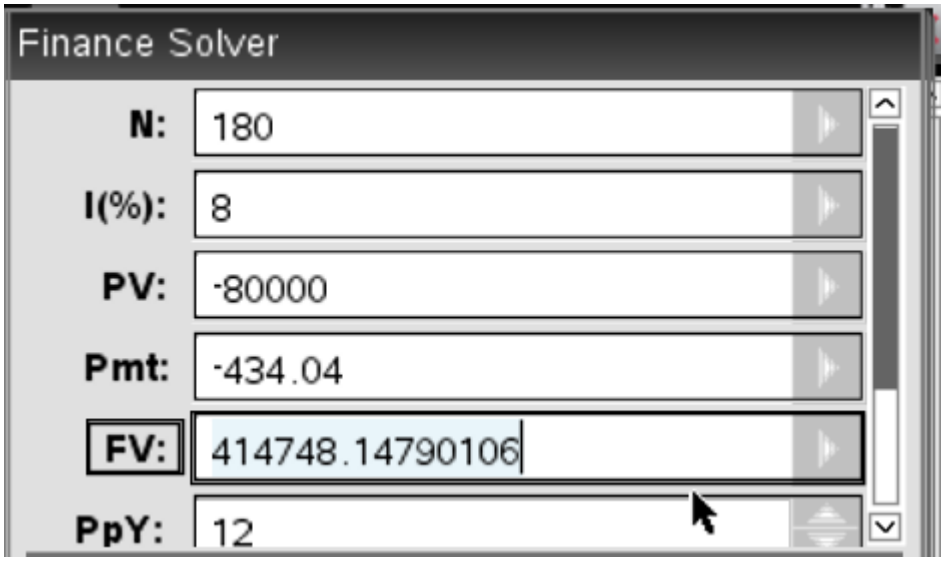
NB: 65yrs – 40yrs = 25yrs = 25 x 12 = 300 months



So in order for John's $80,000 superannuation fund to reach $1,000,000 in the next 25 years at a rate of 8% p.a. compounding monthly, he would need to make monthly payments of approximately $434.04.

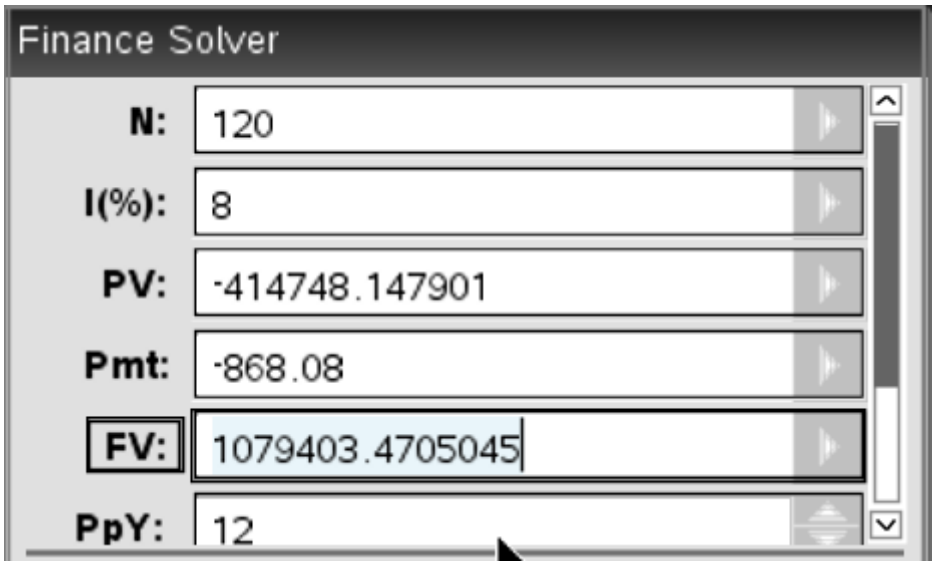
Task.2

Step.1 – Find the superannuation balance 10 years before the investment matures.
ie. 15 years into the 25 year investment (10 years remaining)



The balance after 15 year (with 10 years remaining) was $414748.15

Step.2 – Now use this balance as the starting point for the final 10 years and double the monthly payments from $434.08 to $868.08.



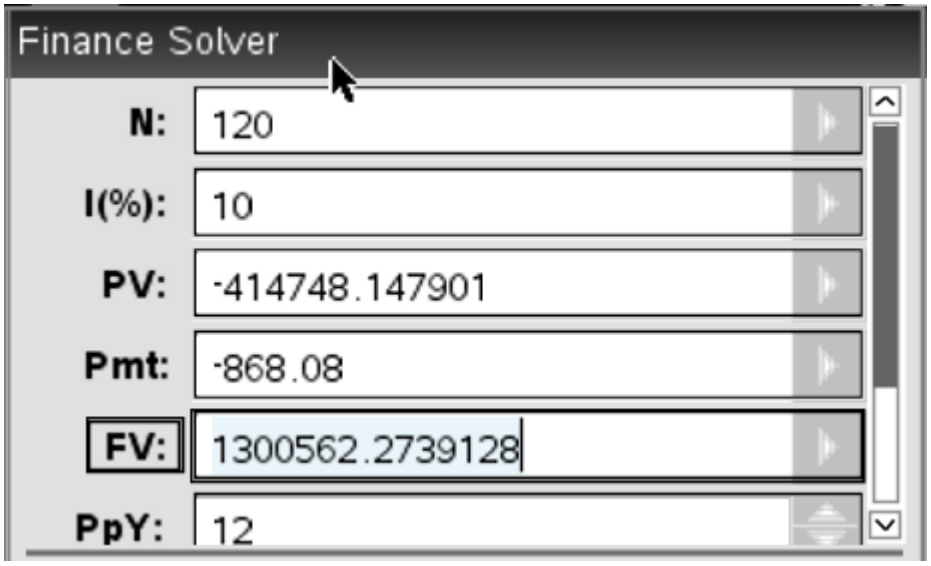
So if the monthly contributions were doubled for the final 10 years of the investment, the account balance upon maturity would be $1,079,403.47

Task.3

Step.1 – Find the superannuation balance 10 years before the investment matures.
ie. 15 years into the 25 year investment (10 years remaining)

From the previous question (ie Task.2) the balance after 15 years was found to be $414,748.15

If the interest rate were now increased to 10% p.a. compounded monthly, and the new doubled monthly deposit of $868.08 maintained, what will be the new account balance upon maturity?



So if the monthly contributions remained at $868.08 and the interest rates were increased to 10% p.a. compounded monthly for the final 10 years of the investment, the account balance upon maturity would now be $1,300,562.27.